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Does apparent stress vary with earthquake size?

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Abstract. Seismic energy is distributed across a wide frequency band so that limited bandwidth recording can lead to substantial underestimates of the radiated seismic energy or introduce an artificial upper bound of radiated energy. We estimate an adjustment factor to account for the probable missing energy and apply it to three previously studied data sets with limited recording bandwidth. We find that this adjustment, together with accounting for possibly missing events, eliminates much of the moment dependence of radiated energy found previously. We obtain a nearly constant ratio of radiated energy to seismic moment, 3×10^{-5}, or 1 MPa of apparent stress drop, over 17 orders of seismic moment. This suggests that deviation from similarity of the energy radiation for seismic events essentially the entire observable range of earthquake size may not yet be resolved.

Introduction

The energy radiated from the seismic source is a fundamental source parameter and can be estimated from the energy flux by integrating the squared velocity seismogram in the time or frequency domains [e.g., *Kanamori et al.,* 1993; *Choy and Boatwright,* 1995]. *McGarr* [1999] summarized the results for earthquakes of various sizes and found that the maximum of apparent stress, $\Delta\sigma_a$, defined as the rigidity multiplied by the ratio between radiated energy and seismic moment, is nearly constant over 19 orders of seismic moment. The major data sets in his study are the micro earthquake studies of Underground Research Laboratory, Canada [*Gibowicz et al.*, 1991] and KTB borehole, Germany [*Jost et al.*, 1998], the micro earthquake studies of *Abercrombie* [1995] and the relatively large earthquake studies of *Kanamori et al.* [1993] in California.

Despite the nearly uniform apparent stress over such a wide range of earthquake size, individual studies show strong size dependence. *Kanamori and Heaton* [2000] interpret the combined trend seen in the results of *Abercrombie* [1995] and *Kanamori et al.* [1993] as a difference in frictional behavior during rupture between small and large earthquakes. *Abercrombie* [1995] and *McGarr* [1999] both suggested trends might arise from underestimation of energy due to limited recording bandwidth.

Boore [1986], *Di Bona and Rovelli* [1988], and *Singh and Ordaz* [1994], have previously discussed the influence of finite bandwidth on the estimation of source parameters. They demonstrated that radiated energy can be severely underestimated when high frequencies are not recorded. In this paper, we show that seismic energy is widely distributed in more than a decade of frequency assuming an omega square model, and account for this bias to obtain an adjustment value of the radiated seismic energy.

Effect of Finite Bandwidth on Energy Estimation

We first examine the range of frequencies that make a significant contribution to the estimate of the energy flux. We follow *Boore* [1986], *Di Bona and Rovelli* [1988] and assume a simple omega square model [*Aki*, 1967]. We approximate the velocity spectrum as:

$$\dot{u}(f) \approx M_o f / (1 + (f/f_0))^2 \,, \quad (1)$$

where M_o is seismic moment and f_0 is the corner frequency [*Brune*, 1970]. Squaring and integrating, the estimate of radiated energy E is proportional to:

$$E \approx (1/2) M_o^2 f_0^3 [F(f, f_o)]_0^\infty = (1/4)\pi M_o^2 f_0^3 \quad (2)$$

$$F(f, f_o) = (-f/f_0)/(1 + (f/f_0))^2 + \arctan(f/f_0) \quad (3)$$

In practice, the highest frequency is fixed by some cutoff value determined by instrumental characteristics and/or attenuation. When the upper limit is f_M, equation (2) is changed to

$$(1/2) M_o^2 f_0^3 [F(f, f_o)]_0^{f_M} = (1/2) M_o^2 f_0^3 F(f_M, f_o). \quad (4)$$

The ratio R between the estimated energy and the true energy is a function of f_M and f_0,

$$R(f_M, f_o) = (2/\pi) F(f_M, f_o). \quad (5)$$

This is the same equation derived by *Di Bona and Rovelli* [1988]. Figure 1 shows the shape of this function together with original form of omega-square velocity spectrum. Equation (5) indicates that over 80% of the radiated seismic energy will be carried by waves of higher frequency than the corner frequency. Fig. 1 also shows that integration up to approximately ten times the corner frequency is necessary to approach 90% of the seismic energy. This condition is not often met for seismic observations and suggests that estimates of the seismic energy that don't account for this effect may be biased.

Some studies use a different style of omega-square model [*Boatwright*, 1978] as

$$\dot{u}(f) \approx M_o f / \sqrt{1 + (f/f_0)^4)} \,. \quad (6)$$

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