10.1098/rspa.2003.1123

THE ROYAL SOCIETY

A confidence limit for the empirical mode decomposition and Hilbert spectral analysis

BY NORDEN E. HUANG[1], MAN-LI C. WU[2], STEVEN R. LONG[3], SAMUEL S. P. SHEN[4], WENDONG QU[5], PER GLOERSEN[1] AND KUANG L. FAN[6]

[1]*Code 971, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA* (norden.e.huang@nasa.gov)
[2]*Code 910, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA*
[3]*Code 972, NASA GSFC/Wallops Flight Facility, Wallops Island, VA 23337, USA*
[4]*Department of Mathematical Sciences, University of Alberta, Edmonton T6G 2G1, Canada*
[5]*Engineering Sciences, Mail Stop 104-44, California Institute of Technology, Pasadena, CA 91125, USA*
[6]*Institute of Oceanography, National Taiwan University, No. 1, Sec. 4, Roosevelt Road, Taipei POB 23-13, Taiwan 106, Republic of China*

Received 4 September 2001; accepted 14 January 2003; published online 23 July 2003

The confidence limit is a standard measure of the accuracy of the result in any statistical analysis. Most of the confidence limits are derived as follows. The data are first divided into subsections and then, under the ergodic assumption, the temporal mean is substituted for the ensemble mean. Next, the confidence limit is defined as a range of standard deviations from this mean. However, such a confidence limit is valid only for linear and stationary processes. Furthermore, in order for the ergodic assumption to be valid, the subsections have to be statistically independent. For non-stationary and nonlinear processes, such an analysis is no longer valid. The confidence limit of the method here termed EMD/HSA (for empirical mode decomposition/Hilbert spectral analysis) is introduced by using various adjustable stopping criteria in the sifting processes of the EMD step to generate a sample set of intrinsic mode functions (IMFs). The EMD technique acts as a pre-processor for HSA on the original data, producing a set of components (IMFs) from the original data that equal the original data when added back together. Each IMF represents a scale in the data, from smallest to largest. The ensemble mean and standard deviation of the IMF sample sets obtained with different stopping criteria are calculated, and these form a simple random sample set. The confidence limit for EMD/HSA is then defined as a range of standard deviations from the ensemble mean. Without evoking the ergodic assumption, subdivision of the data stream into short sections is unnecessary; hence, the results and the confidence limit retain the full-frequency resolution of the full dataset. This new confidence limit can be applied to the analysis of nonlinear and non-stationary processes by these new techniques. Data from length-of-day measurements and a particularly violent recent earthquake are used to demonstrate how the confidence limit is obtained and applied. By providing a confidence limit for this new approach, a stable range of stopping criteria for the decomposition or sifting phase

(EMD) has been established, making the results of the final processing with HSA, and the entire EMD/HSA method, more definitive.

Keywords: empirical mode decomposition; EMD/HSA; Hilbert spectral analysis; Hilbert–Huang transform (HHT); nonlinear data analysis; non-steady data analysis

1. Introduction

The two-step method of empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA) introduced by Huang *et al*. (1998*a*, 1999*a*, hereafter referred to as H98 and H99) has proved to be a powerful procedure for analysing non-stationary and nonlinear data. During the years since its introduction, many applications have been found (Huang 2001; Huang *et al*. 1998*b*, 1999*b*, 2000, 2001; Gloersen & Huang 1999; Wu *et al*. 1999; Loh *et al*. 2001; Hu *et al*. 2002) that include analysing acoustic, biological, ocean, earthquake, climate and mechanical vibration data. As versatile as it has proved to be, the method still needs further clarifications and improvements. An aspect in need of development and clarification is the definition and analysis of a confidence limit for the resulting intrinsic mode functions (IMFs) and the Hilbert spectrum.

The confidence limit is a standard measure for results from statistical analysis. Ideally, it should be derived from an ensemble of observations and computed using the ensemble mean and standard deviation from this mean. Assuming the error is normally distributed, the confidence limit is usually defined as a range of values near this mean: one standard deviation is equivalent to 68%, and two standard deviations are equivalent to a 95% confidence limit. For practical reasons, however, only a few of the many statistical analysis studies follow this rule. Most of the studies used a confidence limit computed from one set of observations only, instead of an ensemble; the mean and the standard deviation are actually computed by invoking the ergodicity, assuming that the data are linear and stationary, and the data can be subdivided into statistically independent subsections. The temporal mean from these subsections is then used to approximate the ensemble mean.

The ergodic rule seems straightforward, but there are certain difficulties involved that are consistently overlooked. The first difficulty is that there may not be enough data values to allow subdivision of the dataset into enough subsets so that each will have enough data to represent the process and allow a realistic mean to be computed within each subset. And the more troublesome hurdle is that the data may not be stationary or from a linear process. Once the stationary assumption is abandoned, the ergodicity can no longer be assumed to compute the mean and standard deviation from different subsections of the same given dataset; a true ensemble mean is required. Unfortunately, in the real world, only one realization is available, which gives only one dataset: no one can demand and achieve a repeat of a natural process even once, much less so enough for an ensemble average.

The most serious objection is that natural processes are not only non-stationary but also nonlinear, which involves sensitivity to the initial conditions and reactions to feedback. Thus, most of the underlying physical processes make the ergodic assumption inapplicable, if not irrelevant. Therefore, a statistical measure of the result cannot simply be established by resorting to spatial and temporal averaging; yet finding an alternative is a grand challenge.