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A constitutive law for rate of earthquake production and its application to earthquake clustering

James Dieterich

U.S. Geological Survey, Menlo Park, California

Abstract. Seismicity is modeled as a sequence of earthquake nucleation events in which the distribution of initial conditions over the population of nucleation sources and stressing history control the timing of earthquakes. The model is implemented using solutions for nucleation of unstable fault slip on faults with experimentally derived rate- and state-dependent fault properties. This yields a general state-variable constitutive formulation for rate of earthquake production resulting from an applied stressing history. To illustrate and test the model some characteristics of seismicity following a stress step have been explored. It is proposed that various features of earthquake clustering arise from sensitivity of nucleation times to the stress changes induced by prior earthquakes. The model gives the characteristic Omori aftershock decay law and interprets aftershock parameters in terms of stress change and stressing rate. Earthquake data appear to support a model prediction that aftershock duration, defined as the time for rates to return to the background seismicity rate, is proportional to mainshock recurrence time. Observed spatial and temporal clustering of earthquake pairs arises as a consequence of the spatial dependence of stress changes of the first event of the pair and stress-sensitive time-dependent nucleation. Applications of the constitutive formulation are not restricted to the simple stress step models investigated here. It may be applied to stressing histories of arbitrary complexity. The apparent success at modeling clustering phenomena suggests the possibility of using the formulation to estimate short- to intermediate-term earthquake probabilities following occurrence of other earthquakes and for inversion of temporal variations of earthquake rates for changes in driving stress.

Introduction

Change of the rate of production of earthquakes is a readily observed and characteristic feature of earthquake occurrence. Physical causes of temporal variations of seismic activity remain poorly understood. However, various attributes of earthquake activity indicate a connection between alteration of stress and earthquake rates. Significant geologic events which modify stress state are often associated with pronounced changes of the rate of production of earthquakes. Examples include aftershocks following large earthquakes, earthquake swarms associated with magma intrusion processes, and systematic variation of background seismicity through the stressing cycle of great earthquakes [e.g., *Ellsworth et al.*, 1981]. Also, changes of effective stress related to impoundment of reservoirs and fluid injection into deep wells are well known to alter seismicity. Finally, at distances generally not associated with aftershocks, *Reasenberg and Simpson* [1992] have reported correlations between small stress changes calculated for the 1989 Loma Prieta, California, earthquake and small changes in the rates of production of background seismicity.

This paper presents a general approach for obtaining rate of earthquake activity resulting from some stressing history. The concept is implemented for faults with rate- and state-dependent constitutive properties which are derived from laboratory fault slip experiments. Previously, some effects of stress perturbations on rate of earthquake occurrence and earthquake probability have been simulated for faults with these properties [*Dieterich*, 1986, 1987, 1988*b*]. Those previous applications employed numerical simulations or limited closed-form solutions. The following presents a general constitutive formulation for rate of earthquake production on faults with rate- and state-dependent friction. The formulation is amenable to exact solution for simple stressing perturbations, and by straightforward numerical implementation it may be applied to stressing histories of arbitrary complexity. To illustrate and test the model some applications to earthquake aftershocks and earthquake clustering are presented. Table 1 lists symbols used in this study and equation numbers where the symbols first appear.

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Model

General formulation. In the following discussion the term "earthquake nucleation" is used to describe the processes and interactions that lead to the initiation of an earthquake instability at some specific place and time. Nucleation is assumed to occur over a restricted region which is referred to as a nucleation source. The essential concept of the analysis is the treatment of a seismically active volume of the Earth as having a population of sources that nucleate successive earthquakes to produce observed seismicity. The objective is to find the time at which each source in the population initiates an earthquake by following the evolution of conditions on the sources when subjected to some stressing history.