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A Comparison of Techniques for Magnetotelluric Response Function Estimation

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Spectral analysis of the time-varying horizontal magnetic and electric field components yields the magnetotelluric (MT) impedance tensor. This frequency dependent 2x2 complex tensor can be examined for details which are diagnostic of the electrical conductivity distribution in the Earth within the relevant (frequency dependent) inductive scale length of the surface observation point. As such, precise and accurate determination of this tensor from the electromagnetic time series is fundamental to successful interpretation of the derived responses. In this paper, several analysis techniques are applied to the same data set from one of the EMSLAB Lincoln Line sites. Two subsets of the complete data set were selected, on the basis of geomagnetic activity, to test the methods in the presence of differing signal-to-noise ratios for varying signals and noises. Illustrated by this comparison are the effects of both statistical and bias errors on the estimates from the diverse methods. It is concluded that robust processing methods should become adopted for the analysis of MT data, and that whenever possible remote reference fields should be used to avoid bias due to uncorrelated noise contributions.

1. INTRODUCTION

EMSLAB has brought together in a cooperative effort many electromagnetic (EM) induction workers with diverse backgrounds and experiences. That the EMSLAB project has many facets is well illustrated by the breadth of the subject matter of the papers in this special section. One such topic has focussed interest on the problem of determining the magnetotelluric (MT) impedance tensor elements from measurements of the time-varying components of the EM field as precisely and as accurately as possible. The availability of synoptic observations of the time-varying EM field over the EMSLAB-Juan de Fuca area motivated examination of the many disparate spectral analysis methods used to analyze similar (or identical) data and also led to the development of new ways of computing MT responses (e.g., robust methods, see below). In an analogous fashion to the objectives of the mini-EMSLAB project [*Young et al.*, 1988], we wished to undertake a comparison exercise to evaluate the relative efficacies of our analysis codes given the same data.

The time-varying EM field components are, by *Maxwell's* [1892] equations, related by linear differential operators, and for certain classes of external source potentials [*Egbert and Booker*, this issue], concepts appropriate for multiple-input/multiple-output linear systems can be appealed to. The estimation of the weighting response functions, or their frequency domain equivalent the transfer functions, for a multiple-input/multiple-output linear system by analyses of the respective input and output time series is a problem that has received much attention over the past century. A tremendous boon occurred with the advent of fast Fourier transformation algorithms during the 1960s, and with some exceptions, these transfer functions are now routinely estimated in the frequency domain. While this is done mainly for computational reasons, direct estimation of the impulse response functions by cross-correlation methods is unwise because of bad statistical properties for the estimates [*Jenkins and Watts*, 1968, pp. 422-429].

For the analysis of MT data, the linear system can be thought of as having two inputs, the horizontal components of the time-varying magnetic field ($b_x(t)$ and $b_y(t)$), and two independent outputs, the horizontal components of the time-varying electric field ($e_x(t)$ and $e_y(t)$), with additive noise components on each channel ($n_{b_x}(t)$, $n_{b_y}(t)$, $n_{e_x}(t)$, and $n_{e_y}(t)$) giving our observable time-varying field components ($\tilde{b}_x(t)$, $\tilde{b}_y(t)$, $\tilde{e}_x(t)$, and $\tilde{e}_y(t)$) (see Figure 1). The true inputs and outputs are related, by a convolution operation, to the four lag-domain weighting functions $z_{xx}(\tau)$, $z_{xy}(\tau)$, $z_{yx}(\tau)$, and $z_{yy}(\tau)$. In the frequency domain, the complex frequency dependent relation between these components can be written (dependence on frequency assumed)

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} B_x \\ B_y \end{bmatrix}$$

where Z is the MT impedance tensor defined initially by *Berdichevsky* [1960, 1964] and *Tikhonov and Berdichevsky* [1966] and where $B_x(\omega)$ is the Fourier transform of $b_x(t)$ and similarly for the other components.

Generally, we have no knowledge of the true components (or latent variables) E_x, E_y, B_x, and B_y or of the noise contributions on these components (N_{E_x}, N_{E_y}, N_{B_x}, and N_{B_y}) but only of our observations of these $\tilde{E}_x$,

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