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THE ESTIMATION OF MAGNETOTELLURIC IMPEDANCE TENSOR ELEMENTS FROM MEASURED DATA†

W. E. SIMS*, F. X. BOSTICK, JR.‡, AND H. W. SMITH‡

Six different estimates of the magnetotelluric impedance tensor elements may be computed from measured data by use of auto-power and cross-power density spectra. We show that each of the estimates satisfies a mean-square error criterion. Two of the six estimates are relatively unstable in the one-dimensional case when the incident fields are unpolarized. For the remaining four estimates, it is shown that two are unaffected by random noise on the H signal, but are biased upward by random noise on the E signal. The remaining two estimates are unaffected by random noise on the E signal, but are biased downward by random noise on the H signal. Computation of all of the estimates provides a measure of the total amount of noise present, as indicated by a stability coefficient for the estimates. In the absence of additional information as to the relative signal-to-noise ratios of the E and H signals, we suggest that a mean estimate be used. A numerical example is included.

INTRODUCTION

The magnetotelluric sounding method for the determination of subsurface electrical conductivity profiles as proposed by Cagniard (1953) is based upon the assumption of a horizontally stratified layered earth model. For an anisotropic or laterally inhomogeneous earth, the impedance becomes a tensor quantity. Methods for the estimation of the impedance tensor elements have been studied and proposed by many investigators, including Neves (1957), Cantwell (1960), Bostick and Smith (1962), Swift (1967), and Rankin and Reddy (1969). Their methods are generally quite similar and use power spectral density estimates of orthogonal E and H field data.

In the formulation employed both by Swift (1967) and by Rankin and Reddy (1969, equations 16a–16d), the authors recognized that the least square estimation being used would be insensitive to independent noises on the E field, but would bias the principal impedances to the low side for noises on the H-field data. It is the purpose of this paper to point out other least square estimators which will have the opposite effect, i.e., which will be insensitive to indepen dent noises on the H field and produce principal impedance values that are biased to the high side for noise on the E field. We suggest that the computation of several estimates will indicate when noise problems are severe and will serve as a guide for selection of estimates, which, under certain conditions, will have relatively less bias.

THE ESTIMATION PROBLEM

Consider the equation

$$E_x = Z_{xx}H_x + Z_{xy}H_y,$$

where E_x, H_x, and H_y may be considered to be Fourier transforms of measured electric and magnetic field data. If one has two independent measurements of E_x, H_x, and H_y at a given frequency, denoted by $E_{x1}, H_{x1}, H_{y1}, E_{x2}, H_{x2}$, and H_{y2} respectively,

$$Z_{xx} = \frac{\begin{vmatrix} E_{x1} & H_{y1} \\ E_{x2} & H_{y2} \end{vmatrix}}{\begin{vmatrix} H_{x1} & H_{y1} \\ H_{x2} & H_{y2} \end{vmatrix}} \tag{1a}$$

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*Shell Development Company, Houston, Texas 77001.

‡The University of Texas at Austin, Austin, Texas 78712.