It is clear, from (20), that for this case the multiplication number required is $[Q + (Q + 1)/2]\ (L - 1)/2$ instead of $Q(L - 1)$.

From the above discussions we see that, by suitably arranging arithmetic operations, the number of multiplications required can be reduced by about 25 percent. In case that a serial/parallel multiplier (such as Am25LS14 manufactured by Advanced Micro Devices, Inc.) is used for the filter implementation, a 25 percent reduction on the multiplication operations means a 25 percent increase on the filter speed because almost all nonmultiplication operations can be performed during the first half multiplication cycle of the multiplier while unwanted less significant bits are being generated.

IV. Conclusions

In this paper we examine some problems on the implementation of digital interpolator using linear-phase FIR filters. A procedure for selecting parameters L, N, and Q is presented. It is shown that, except for one interpolated sample in *Case A-2* where L is even and Q is odd, all $(L - 1)$ interpolated samples can be computed from the same set of Q original input samples. This fact can greatly simplify the design of the control section of interpolation filters.

The symmetry property of the impulse response of linear-phase FIR filters is exploited. It is shown that, by suitably arranging arithmetic operations, the number of multiplication operations required can be reduced about 25 percent. If a serial/parallel multiplier is used for the filter implementation, a 25 percent reduction on the multiplication operations means a 25 percent increase on the filter speed.

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Transformed Coherence Functions for Multivariate Studies

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***Abstract*—In this paper, transformed coherence function estimates are defined which display several desirable properties when compared with the conventional forms; 1) their probability distribution functions are more nearly normal, 2) their mean values are normalized to a value of unity for totally uncorrelated data, and 3) their variances are independent of the true values.**

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I. Introduction

The magnitude-squared coherence function (MSC) between two jointly stationary random processes, $x(t)$ and $y(t)$, is defined as

$$|\gamma(f)|^2 = \frac{|G_{xy}(f)|^2}{G_{xx}(f)\, G_{yy}(f)} \tag{1}$$

where $G_{xy}(f)$, $G_{xx}(f)$ and $G_{yy}(f)$ are the theoretical cross- and autospectral densities, respectively, at frequency f. The MSC can be estimated by ensemble averaging over various data segments, or by band averaging over adjoining frequency components by a suitable spectral window, of the sample spectra to yield estimate C^2 of γ^2. Both the MSC and its estimators are bounded by zero and unity. The necessity for determining smoothed estimators of (1) is described in detail in [1].

The MSC is a very useful indicator of various properties of the linear relationship between $x(t)$ and $y(t)$, that is, of the coherent common power between the two measured signals. A nonunity value infers either: 1) noise on $x(t)$ and/or $y(t)$, 2) the system relating $x(t)$ to $y(t)$ is nonlinear, or 3) that there are processes other than $x(t)$ and $y(t)$ involved.

However, it is relatively well known that the estimators of (1) are biased estimators. For example, for the case of smoothing by ensemble averaging, and assuming there to be no bias due to a misalignment [2], Nuttall and Carter [3] have shown that the bias of C^2 is given by

$$B(C^2) = E[C^2] - \gamma^2 \simeq \frac{1}{N}(1 - \gamma^2)^2 \left(1 + \frac{2\gamma^2}{N}\right) \tag{2}$$

where γ^2 is the theoretical MSC, C^2 is the estimated MSC, and N is the number of time data segments employed. The estimator C^2 of MSC γ^2 does not possess a probability distribution function (PDF) that has a normal (Gaussian) form, thus confidence limits and other statistical descriptors cannot be easily calculated (see [4] for graphs of the confidence bounds of the MSC at the 80 percent and 95 percent levels).

II. Normalized Transformed Magnitude Coherence Function (NTMCF)

It is suggested in [5] that application of R. A. Fisher's Z-transformation [6] to the positive square root of the estimate of the MSC, called the magnitude coherence (MC), yields a function that has a nearly normal PDF. This *transformed MC* function TMC is given by

$$T(f) = \operatorname{arctanh}\left(|\gamma(f)|\right). \tag{3}$$

Its estimate $\hat{T} = \operatorname{arctanh}(C)$, has a variance of

$$\sigma^2_{\hat{T}} = \operatorname{var}(\hat{T}(f)) \simeq \frac{1}{n-2} \tag{4}$$

[7] where n is the number of degrees of freedom associated with the estimate. For ensemble averaging with nonoverlapping data sets, $n = 2N$. Empirical studies by [8] have confirmed that this transformation is valid for $n > 20$ with $0.3 < \gamma^2 < 0.98$. The validity of the transformation may be extended to a larger range of γ^2 and for $n > 8$ if the estimate of the MC is first corrected for bias. Recent related work is reported in [9].

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