

# Sea Surface Elevation and Bottom Pressure Anomalies due to Thermohaline Forcing. Part I: Isolated Perturbations\*

RUI XIN HUANG

*Department of Physical Oceanography, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts*

XIANGZE JIN

*Department of Physical Oceanography, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts, and LASG, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing, China*

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## ABSTRACT

Sea surface elevation and bottom pressure anomalies due to thermohaline forcing are examined through analytical and numerical models, including Boussinesq and non-Boussinesq models. It is shown that Boussinesq approximations can introduce noticeable errors, depending on the spatial and temporal scales of the perturbations. According to the theory of geostrophic adjustment, when the initial perturbations have horizontal scales comparable to the barotropic radius of deformation, the initial pressure perturbations will be basically retained through the adjustment. On the other hand, if the initial perturbations have horizontal scales much smaller than the barotropic radius of deformation, the initial pressure perturbations will be largely lost. Precipitation has horizontal scales on the order of 10–100 km, much smaller than the barotropic radius of deformation. Thus, for timescales longer than days, the contribution from individual precipitation events to the local free surface elevation and bottom pressure is small and is difficult to identify from satellite data. On the other hand, thermal forcing has horizontal scales comparable to the barotropic radius of deformation, so its long-term contribution to sea surface height anomaly is noticeable and is easily identified from satellite data. Because Boussinesq models induce faulty sea surface height and bottom pressure signals, the errors introduced by these models are noticeable for anomalies in large-scale [ $O(1000\text{ km})$ ] thermohaline forcing.

## 1. Introduction

Boussinesq (1903) first introduced certain approximations that have been widely used in oceanic circulation models. The Boussinesq approximations can be summarized by two points. 1) The fluctuations of density are primarily due to thermohaline effects (Boussinesq's original study was focused on the contribution due to thermal forcing, as opposed to pressure; however, this assumption has been extended to include also the dynamic effect of salinity). 2) In the momentum and mass conservation equations, density variations may be neglected, except in the terms associated with buoyancy force. Thus, most existing numerical circulation models use volume conservation to replace mass conservation

and use a fixed-value reference density to replace in situ density in the horizontal momentum equations.

The applicability of the Boussinesq approximations to fluid dynamics has been discussed in many papers. Spiegel and Veronis (1960) discussed the case for polytropic gases, and its application to liquids has been discussed by many other authors, such as Mihaljan (1962), Zeytounian (1989), and Bois (1991). Most of these studies have been based on nonrotating fluid; the possible dynamic effect of rotation has not been studied thoroughly.

In this study, we will emphasize that the errors in the oceanic circulation induced by Boussinesq approximations vary greatly, depending on the spatial and temporal scales of perturbations. In actuality, in the ocean both the sea level and bottom pressure may go through geostrophic adjustment processes that are very different in comparison with a Boussinesq ocean.

In the ocean, for example, the sea level increases in direct response to local heating, but there is no initial perturbation in bottom pressure. On the other hand, in a Boussinesq ocean, surface heating induces a loss of mass (thus, a negative bottom pressure signal), with no immediate increase in sea level. Because most existing

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*Corresponding author address:* Rui Xin Huang, Dept. of Physical Oceanography, Woods Hole Oceanographic Institution, Woods Hole, MA 02543.  
E-mail: rhuang@whoi.edu