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Research note

Discrete polar motion equations

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Summary. A digital filter equation is derived which is appropriate for predicting polar motion from excitation axis displacements, or for inferring the excitation axis changes from observed polar motion. The result differs from previously published equations in its phase response. Two additional equations are presented which are useful if samples of the excitation and polar motion functions are required to be at the same time values.

This discussion is concerned with the problem of designing equations which may be used in the study of the Earth's polar motion. The objectives are to predict polar motion from time samples of variations in the excitation axis (equivalent to the principal axis when there is no motion relative to the Earth); or to estimate variations in the excitation axis from samples of the pole position. Let the motion of the pole be represented by $M(t)$, a continuous complex function of time t, with real part denoting displacements along the Greenwich Meridian, and imaginary part displacements along 90° east longitude. Similarly, $X(t)$ is a complex function describing the motion of the excitation axis. Finally, F_c is Chandler's frequency of the free nutation (about 0.843 cycle yr^{-1}), $1/Q_c$ is the dissipation factor (Q_c is the quality factor) of the free nutation, and $\sigma_c = 2\pi F_c(1 + i/2Q_c)$ is a complex frequency which succinctly describes the Chandler frequency and damping.

A differential equation describing the relationship between $M(t)$ and $X(t)$ is derived from Euler's rigid body equations with corrections to account for dissipation and for the lengthening of the free period due to the non-rigid nature of the Earth. Letting $i = \sqrt{-1}$, the equation is:

$$X(t) = \frac{i}{\sigma_c}\frac{dM(t)}{dt} + M(t). \tag{1a}$$

The transfer function of this equation, which is the ratio of the Fourier transform of $M(t)$ to that of $X(t)$ at frequency f, is

$$\frac{\sigma_c}{\sigma_c - 2\pi f}. \tag{1b}$$