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A Stochastic Fault Model
1. Static Case

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The number-size distribution of earthquakes requires that irregularities exist on a fault at all length scales. The assumption of self-similar irregularity is used to formulate a stochastic description of the faulting process. A random irregularity is termed self similar if it remains statistically similar upon a change of length scale. Self-similar geometric irregularity of a fault surface is represented in this model by stress and friction functions that fluctuate self similarly on a plane. If the set of rupture areas of all earthquakes on the brittle portion of a fault plane is assumed to be self similar, then the number of ruptures with area greater than A is proportional to $1/A$. If stress drop is independent of earthquake size, then the number of earthquakes with moment greater than M_0 is proportional to $M_0^{-2/3}$. The size of an earthquake is determined by spatial fluctuation of the initial stress and sliding friction functions. The spectrum of the stress function is related to both the average stress drop as a function of earthquake size and the number-moment distribution. A model of the slip and stress change functions of an earthquake is constructed in the Fourier transform domain. While the stress function becomes smoother in an earthquake at the length scale of the rupture, it becomes rougher at shorter length scales to prepare the fault for future smaller earthquakes. Seismicity is a cascade of stored elastic energy from longer to shorter wavelengths.

INTRODUCTION

There has been increasing interest in recent years in the role that heterogeneity of stress, material properties, and fault geometry may play in fault mechanics. Heterogeneity is certainly important to the generation of damaging high-frequency ground motion, and it may be essential in determining the occurrence and size of earthquakes. Such concepts and relevant observations have been reviewed by *Aki* [1979] and *Nur* [1978]. *Bakun et al.* [1980] find correlations of seismicity with discontinuities of fault geometry. *Hanks* [1979] has proposed that the spectrum of stress fluctuations on a fault is related to (1) average stress drop as a function of earthquake size, (2) the number-size distribution of earthquakes, and (3) the high-frequency spectrum of dynamic ground motion. In this and a subsequent paper, Hanks' proposed relationships will be examined by applying concepts of self-similar irregularity and of fault energetics. The model proposed here differs from earlier stochastic models [*Aki,* 1967; *Haskell,* 1966] in that the random component of stress does not have a characteristic length. Some mechanical concepts that motivate the model will be reviewed first.

Aki [1979] discusses rupture in terms of fracture mechanics concepts. A propagating rupture is stopped by an increase in fracture surface energy. However, in contrast to a sharp-tipped crack model having a stress singularity, Aki imagines that the fracture energy barrier consists of a significant area of fault surface on which slip occurs and stress increases by a modest amount. Such a model is conceptually the same as a frictional model of faulting.

If a rupture is to stop naturally in a frictional model, it is necessary that the difference between initial shear stress and sliding friction stress vary on the length scale of the rupture, allowing a stress drop in the center of a slip patch to release energy and allowing a stress increase around the border of the patch to stop the rupture. A general feature of such frictional models [*Burridge and Halliday,* 1971; *Andrews,* 1975] is that the difference between stress and sliding friction decreases and becomes smoother in each event. This result must be accounted for in any model of recurring earthquakes.

Sliding friction cannot be a function of position alone. If it were, then after enough earthquakes had occurred to eliminate the difference between the stress and sliding friction functions, no more small earthquakes would occur. Further tectonic loading would increase stress over a broad area of the fault. A rupture initiated at any point would propagate over all of the loaded area. Somehow, the difference between stress and sliding friction must be maintained in a rough state. One might appeal to material heterogeneity to explain a single earthquake, but not recurring earthquakes.

Byerlee [1970] and *Nur* [1978] have proposed that fault instability is related to variation of sliding friction with displacement as well as with position. One might imagine a fault as two rough surfaces pressed together. The effective frictional stress will change as a function of displacement due to roughness on the scale of the displacement.

How is roughness of a fault surface produced and how is it maintained? If fresh fracture occurs on some portion of a fault, the surface is changed there. A locked patch that is loaded as a seismic gap, with slip occurring all around it, will experience a stress state in which the principal stress axes are at 45° on the average to the fault surface and to the slip direction. If fresh fractures occur in the seismic gap, they will have a different orientation to the principal stress axes, forming an en echelon pattern. Excavation of the fracture surface produced by a tremor in a deep gold mine revealed an en echelon break [*Gay and Ortlepp,* 1979; *Spottiswoode and McGarr,* 1975; *McGarr et al.,* 1979].

The mechanism of fracture misalignment is independent of length scale. Whatever the mechanisms are in general that maintain fault roughness, it is plausible that they have little, if any, length scale dependence. Patterns of faulting observed in the field and in laboratory models show similar irregularity [*Tchalenko,* 1970; *Segall and Pollard,* 1980].

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SELF SIMILARITY

The concept of self-similar irregularity is familiar in the field of turbulent fluid flow. It means that pictures of the flow field at different magnifications will be statistically similar. Turbulent flow is self similar at length scales between an outer scale, determined by the driving mechanism of the overall flow, and an inner scale, where viscosity becomes important. In a self-similar flow the distribution of eddy sizes is a power law, and a Fourier transform of any variable is also a power law.

In a different application, *Madden* [1976] appeals to a self-similar distribution of interconnecting fluid-filled cracks to explain Archie's Law of electrical conductivity in rocks.

In this paper the concept of self-similar irregularity will be applied to the faulting process. Characteristic length scales relevant to shallow faulting are, first, the depth of the brittle seismogenic region, of the order of 10 km, and, second, the grain size of the medium. It is assumed that in the broad band between these length scales there is no other length characterizing fault irregularity or the seismic process. Accordingly, earthquakes with rupture dimension of less than 10 km are assumed to be geometrically similar on the average and are determined by fault irregularities or initial stress fluctuations that are self similar.

Mandelbrot's [1977] provocative book *Fractals* is concerned with self-similar irregularity. Following his chapter 7 on the distribution of number of meteorite craters as a function of size, we will find that the number-size distribution of earthquakes is determined by the self-similarity assumption alone.

Consider only earthquakes having rupture dimension of less than 10 km occurring on a given planar fault surface. Consider the set of all earthquake rupture areas (they will overlap) having centroids within a region of the fault surface of area S. The area S is assumed to lie entirely within the brittle region of the fault surface. Assuming long-term uniformity of seismicity over the brittle region, the number of earthquakes is proportional to S. The distribution of number as a function of rupture area A must be a power law, for no other functional form is self similar. Then the number of earthquakes having rupture area greater than A and centroids within S is

$$N = KSA^{-\gamma} \tag{1}$$

where K and γ are constants. To have a self-similar distribution, we require further that the relative density of ruptures be independent of scale. Set $A = \alpha S$. The number of earthquakes with rupture area greater than the fraction α of S is

$$N = KS^{1-\gamma}\alpha^{-\gamma} \tag{2}$$

If this number is to be independent of S, then $\gamma = 1$, and the distribution becomes

$$N = KS/A \tag{3}$$

The seismic moment of an earthquake is

$$M_0 = \mu\langle D\rangle A \tag{4}$$

where μ is shear modulus and $\langle D\rangle$ is average slip over the rupture area A. Assuming strict geometric similarity, on the average, for earthquakes of different sizes, slip is proportional to rupture dimension

$$\langle D\rangle \propto A^{1/2} \tag{5}$$

and stress drop, on the average, is independent of earthquake size. Then

$$M_0 \propto A^{3/2} \tag{6}$$

and the number of earthquakes with moment greater than M_0 is

$$N = CM_0^{-2/3} \tag{7}$$

In the logarithmic form,

$$\log N = a' - b' \log M_0 \tag{8}$$

written in analogy to the number-magnitude law, the slope predicted by strict similarity is $b' = 2/3$.

The rupture patches are assumed to be distributed randomly over the brittle fault surface. With each patch is associated a slip function, having nonzero values only at points within the patch, and a stress change function, having values decreasing with distance away from the patch. The sum of the stress change functions over all patches will be an irregular function with random fluctuations at all length scales. In the next section this stress sum will be assumed to reflect the initial stress state that determined the set of earthquakes, and it will be of interest to characterize this random function by its spectrum.

Assume that on the average each patch is circular and its stress change function is radially symmetric. For a patch with radius a, let the stress change function be

$$\tau(\mathbf{r}; a) = \sigma(a)g(r/a) \tag{9}$$

where g is a shape function applicable for all radii and σ is a characteristic stress drop, which will be taken to have a power law dependence on rupture radius,

$$\sigma(a) \propto a^{\delta} \tag{10}$$

Note that for strict similarity, $\delta = 0$.

The two-dimensional Fourier transform of the stress change function has the form

$$\tau(\mathbf{k}; a) = \sigma(a)a^2 f(ka) \tag{11}$$

where k is the magnitude of the wave number vector $\mathbf{k}$. A particular example of such a function will be seen in the next section.

Because the patches are distributed at random, the transforms have random phases, and their squares are additive. The square of the total stress spectrum is

$$|\tau^2(\mathbf{k})| = \int |\tau^2(\mathbf{k}; a)| \, dN \tag{12}$$

From (3) the distribution of number as a function of patch radius is

$$\begin{aligned} N &\propto a^{-2} \\ dN &\propto a^{-3}\, da \end{aligned} \tag{13}$$

so the total square stress transform is proportional to

$$\begin{aligned} |\tau^2(\mathbf{k})| &\propto \int a^{2\delta}a^4 \, |f^2(ka)| \, a^{-3}\, da \\ &\propto k^{-2\delta-2} \int x^{2\delta+1} \, |f^2(x)| \, dx \\ &\propto k^{-2\delta-2} \end{aligned} \tag{14}$$

Therefore if stress changes in earthquakes reflect spatial fluctuations of the initial stress state, then stress spectral amplitudes proportional to $k^{-\delta-1}$ will produce earthquakes with stress drops varying as a^{δ}.

Mean square stress is found by integrating the square of the stress transform over the two-dimensional wave number plane. Because of assumed isotropy the contribution to stress variance from wave number magnitudes between k and $k + dk$ is proportional to

$$|\tau^2(\mathbf{k})|\ k\ dk \propto k^{-2\delta-1}\ dk \qquad (15)$$

For strict similarity, $\delta = 0$, (15) becomes dk/k, and equal logarithmic intervals of wave number contribute equally to stress variance. I found by numerical experiment that if the two-dimensional stress function is sampled along a line, its one-dimensional transform will have amplitude proportional to $k^{-1/2}$ so that again the squared spectral amplitude is proportional to $1/k$. Discussions of this random function, with spectrum midway between white noise and Brownian motion, may be found in the work of *Mandelbrot* [1967], *Mandelbrot and Wallis* [1968, 1969*a*, *b*], and *Gardner* [1978].

A STOCHASTIC MODEL OF FAULT ENERGETICS

To one, such as I, who is interested in physical mechanisms, the preceding discussion may be frustrating, in that it does not go beyond geometric description. In the remainder of this paper a more physical model of faulting is discussed. Although discussion of deterministic mechanisms is still avoided, the requirement that fault roughness must be regnerated will be accounted for, and the energy released in an earthquake and the energy balance of the overall seismic process will be examined in the Fourier transform domain. It will be seen that the seismic process is a cascade of stored elastic energy from longer to shorter wavelengths, analogous to the cascade of energy from larger to smaller eddies in turbulent flow. Thus Yan Kagan's aphorism, 'seismicity is the turbulence of solids,' takes on added significance.

In turbulent flow, no energy is lost in the cascade; it is all dissipated by viscosity in the smallest eddies. In the case of seismicity, however, stored elastic energy is lost in each earthquake to frictional heat and seismic radiation, so that there is negligible energy left for earthquakes at the bottom of the cascade at the scale of the grain size.

Some general physical principles of faulting will be reviewed before the stochastic model is presented.

The stress function on a fault surface arises from both local and distant sources. One source is irregular slip on the given fault surface. Stress arising from this source is termed self stress. The remainder, termed tectonic stress, is due to slip on distant faults, fault creep at depth, and viscous drag at the base of the lithosphere. Because the source of tectonic stress is distant, it will vary smoothly over the fault surface. Tectonic stress will have significant components at wavelengths of the order of 10 km, the depth of the brittle region, but not at shorter wavelengths. Spatial fluctuations of stress that determine smaller earthquakes cannot arise from tectonic processes but must be self-stress fluctuations. An increase in the level of tectonic stress may trigger a small earthquake, but the size of the small earthquake is determined by irregularity of previous slip on the fault. Energy supplied to the brittle seismogenic region by tectonic processes will appear as stored elastic energy at long wavelengths only.

A complete mechanical model of an earthquake would have to consider the coupled response of the nonelastic fault zone and the elastic medium in which it is contained. The nonelastic response at each point is determined by the initial stress and material properties, the detailed variations of which are essentially unknowable, as well as by stress coupled through the elastic medium arising from slip at other points.

The latter effect complicates interpretation of fault response in terms of laboratory experiments. One is not dealing with single numbers representing force and displacement on a sample. Rather, one is dealing with a function space in which the value of the stress function at a particular point on the fault surface is related to the slip function at all points of the surface. For instance, a localized increase of slip is associated with a stress drop at the center of the slip patch and stress increase around the border of the patch. Static stress change integrated over an infinite fault surface is exactly zero. Stiffness, the ratio of static stress change to slip at the center of the slip patch, depends on the size of the patch [*Walsh*, 1971].

For a fault in an infinite elastic medium the stress function in the space domain is the convolution of the slip function with second derivatives of the point-force Green's function. In this paper we will be concerned with Fourier transforms of slip and stress over the two space dimensions on the fault surface. In the transform domain the stress function is the product of the slip function and a stiffness function. It is in the transform domain that the stiffness concept has a precise definition. An advantage of working in the transform domain is that multiplication is simpler than convolution.

Energy stored in the elastic medium is calculable from either the slip or the stress function via the stiffness function. Another advantage to considering energetics in the transform domain is that energy of different Fourier components of slip is additive, while energy of different slip patches is not additive owing to interference.

Different Fourier components of the slip function are not independent, for they are coupled by the nonlinear response of the fault zone. Deterministic modeling of faulting must be done in the space-time domain. In a deterministic continuum model, stress history at each point in the fault zone determines plastic strain at that point by some nonelastic constitutive relation. The integral of plastic strain through the thickness of the slip zone (another unknown and essential variable) is the slip function, which must be matched to the response of the elastic medium. If the fault zone is idealized to a surface, one deals with frictional stress that varies with slip.

Stochastic modeling need not be restricted to the space-time domain. A stochastic process may be specified by either an autocorrelation function in space-time or a spectrum in the transform domain with prescribed amplitude and random phase. The two specifications are related by the Wiener-Khintchine theorem. Particular realizations may be readily constructed from the spectral representation.

The stochastic model of this paper is formulated in the transform domain, and the possibility of dealing with any detailed mechanism in the fault zone is abandoned.

The seismogenic fault surface is assumed to be purely brittle, with no fault creep, so that self stress is due to the cumulative slip of all past earthquakes. However, fault creep that has the same spectrum of roughness as the slip function of earthquakes may be admitted without any essential change in the model.

In this model it is assumed that initial stress and sliding friction vary randomly over the fault plane. (Variation of sliding friction on a plane is intended to represent the effect of random geometric irregularity of the fault surface.) Then it is reasonable to assume that different small elements of the fault surface that slip in an earthquake will go through stress changes that are, to a large extent, random and uncorrelated. However, this cannot be entirely true, for different fault elements must behave cooperatively on the average, in order that energy be released. Initiation and stopping of slip on each element of fault surface is influenced (but not completely determined) by the collective stress change propagated from other elements. The collective effect can be expected to vary smoothly over the entire extent of rupture. Therefore it is assumed that the functions representing slip and stress changes in an earthquake consist of a smooth coherent component plus a random component. In the transform domain the coherent component is important at wavelengths comparable to the extent of rupture, and the random component becomes dominant at shorter wavelengths.

The model is concerned only with earthquakes having rupture dimensions smaller than the depth interval over which the fault is brittle. From the point of view of this model all such earthquakes are aftershocks. The occurrence of an earthquake of a particular size implies not only that stress decreases as rupture nucleates and propagates but also that stress increases in the region where rupture slows and stops. Although friction may change in the course of an earthquake, one may expect that on the average a spatial fluctuation of the difference between stress and sliding friction must be established by earlier earthquakes.

In models of earthquakes that stop naturally [*Burridge and Halliday*, 1971; *Andrews*, 1975] the difference between stress and friction becomes smoother in each event. These were smooth models with significant variations only at the length scale of the rupture. It is assumed that the three-dimensional generalization of the Burridge and Halliday model [*Dahlen*, 1974] is a reasonable representation of the coherent part of the stochastic model. Accordingly, it is assumed that the difference between stress and sliding friction becomes smoother at the length scale of the rupture, but in order to have a model of recurring earthquakes, it is assumed that the difference between stress and friction becomes rougher at smaller length scales.

Although it is the difference between initial stress and sliding friction that is relevant in determining an earthquake, it is assumed that on the average an earthquake will tend to reduce the spatial fluctuation of initial stress alone at the length scale of the rupture. One might imagine one limiting case in which initial stress is uniform and an earthquake is determined by a spatial fluctuation of friction. In that case there is no correlation between the initial stress and stress change functions. In the other limiting case of uniform friction there must be a correlation (with a negative coefficient) between the stress change function and a local fluctuation of the initial stress function. It is assumed that such a correlation exists on the average.

THE MODEL, STATIC CASE

Let the fault plane be normal to the x_2 axis in a Cartesian coordinate system. Slip $D(x_1, x_3)$ is assumed to occur in the x_1 direction. The complementary component of stress, which enters into energy considerations, is σ_{12}. Let this shear stress component before an earthquake be denoted by $\tau^0(x_1, x_3)$, and let its change in an earthquake be $\tau(x_1, x_3)$. Slip and stress changes (final minus initial static states) in an infinite homogeneous medium are related by a convolution integral,

$$\tau(\mathbf{x}) = \frac{\mu}{4\pi}\iint \frac{1}{r}\left[\frac{2(\lambda+\mu)}{\lambda+2\mu} D_{,11}(\mathbf{x}') + D_{,33}(\mathbf{x}')\right] dx_1 dx_3 \quad (16)$$

where λ and μ are Lamé constants and r is distance from a source point to a field point on the fault plane. Subscripts following a comma denote differentiation.

The two-dimensional Fourier transform over the fault plane of the slip function is

$$D(\mathbf{k}) = \frac{1}{2\pi}\iint D(\mathbf{x}) \exp\left[-i(k_1x_1 + k_3x_3)\right] dx_1 dx_3 \quad (17)$$

The transform of a function is denoted here only by its arguments. With the normalization used here, no numerical factor appears in the relation between an inner product in the wave number domain and an inner product in the space domain,

$$\iint f^*(\mathbf{x})g(\mathbf{x})\, dx_1 dx_3 = \iint f^*(\mathbf{k})g(\mathbf{k})\, dk_1 dk_3 \quad (18)$$

In the transform domain the convolution (16) becomes a multiplicative relation,

$$\tau(\mathbf{k}) = K(\mathbf{k})D(\mathbf{k}) \quad (19)$$

where the static stiffness function is

$$K(\mathbf{k}) = -\frac{1}{2}\frac{\mu}{k}\left[\frac{2(\lambda+\mu)}{\lambda+2\mu}k_1^2 + k_3^2\right] \quad (20)$$

and

$$k = (k_1^2 + k_3^2)^{1/2} \quad (21)$$

The stiffness function is derived by *Andrews* [1978], but with an error of 2π, which is corrected here. For $\lambda = \mu$, stiffness is within 33% of being isotropic in wave number space,

$$K(\mathbf{k}) \approx -\tfrac{1}{2}\mu k \quad (22)$$

We now proceed to construct a model that will be used to illustrate the overall energy balance in a cycle of earthquakes of all sizes. The model is meant to be only illustrative and is not to be taken rigorously in its specifics.

Consider an ensemble of earthquakes with given rupture area, and plot the slip function of each with its centroid translated to the origin. At each point of the rupture surface, find the mean value of slip over the ensemble of functions. This ensemble mean is represented in the model by the slip function of a smooth coherent event that stops gradually, a three-dimensional generalization of the *Burridge and Halliday* [1971] model, as discussed by *Dahlen* [1974]. This mean slip function in the final static state is an azimuthally symmetric version of the function found by Burridge and Halliday in two dimensions,

$$\begin{aligned} D(\mathbf{x}) &= D_0(a)[1-(r/a)^2]^{3/2} \qquad r<a \\ D(\mathbf{x}) &= 0 \qquad r>a \end{aligned} \quad (23)$$

where $r = (x_1^2 + x_3^2)^{1/2}$ and D_0 is expected slip amplitude for rupture radius a. The function approaches zero gradually with a continuous first derivative at the edge of rupture, so that its associated stress function is finite. This function is plotted in Figure 1. Because the function is azimuthally symmetric, its transform is

$$D(\mathbf{k}) = \frac{1}{2\pi}\iint D(\mathbf{x}) \cos \mathbf{k}\cdot\mathbf{x}\, dx_1 dx_3$$

$$= \int_0^a D(r) J_0(kr) r \, dr$$

$$= D_0 a^2 2^{3/2} \Gamma(\tfrac{5}{2}) (ka)^{-5/2} J_{5/2}(ka) \tag{24}$$

An integration formula from *Watson* [1944, p. 373] is used to get the third line of (24). The slip transform approaches a constant at $k = 0$ and asymptotically is proportional to k^{-3}.

Multiplication of (24) by stiffness (20) gives the stress transform, which goes to zero at $k = 0$ (indicating that self stress has zero mean in the space domain) and which is asymptotically proportional to k^{-2}. Inverse transformation yields the stress function plotted in Figure 2. It is finite, continuous, negative at the center of the slip patch and positive in the neighborhood of $r = a$. For clarity of plotting, a cusp at $r = a$ has been removed by applying a high-cut filter.

In the stochastic model a random component is added to the expected slip and stress functions. The spectrum of the random component is limited by the rupture dimension a at long wavelengths and by the grain size of the medium at short wavelengths. Within this broad band of wavelengths we assume that there is no characteristic length scale. Mathematically, the assumption is that a change of length scale by a multiplicative constant changes the spectrum by only a multiplicative constant. Then the random spectrum is a power law. Using the isotropic approximation for stiffness (22), the random slip and stress spectra are proportional to

$$|D(\mathbf{k})| \propto k^{-\nu-1} \tag{25}$$

$$|\tau(\mathbf{k})| \propto k^{-\nu} \tag{26}$$

From a strict point of view, both amplitude and phase are random functions, and the expected values of the power spectra are proportional to the squares of (25) and (26). Random fluctuation of amplitude is unimportant in the following analysis and is ignored. In order that the random component decrease less rapidly than the coherent component with increasing wave number, the exponent must satisfy the inequality

$$\nu < 2 \tag{27}$$

The following procedure was followed to construct a realization of the stochastic model for plotting. A random slip spectrum was constructed on a discrete wave number grid. Amplitude was chosen according to (25) with $\nu = 1$, except that amplitude was set to zero at $k = 0$. Phase was chosen randomly, but it was subject to the constraint

$$D(\mathbf{k}) = D^*(-\mathbf{k}) \tag{28}$$

where the asterisk denotes complex conjugation, so that the inverse transform would be real. The inverse transformation was performed, yielding a function extending over all the fault plane. To limit the random function to the space domain on which the coherent slip function is nonzero, multiplication by a window function is required. In order that the spectrum not be changed at high wave numbers, the window function must be at least as smooth as the coherent slip function. The window function was chosen to be the coherent slip function itself. After checking that the windowed random slip function had a mean value near zero, it was added to the coherent slip function. The result for the case $\nu = 1$ is plotted in Figure 3. Transformation back to the wave number domain, multiplication by stiffness, and inverse transformation yield the stress function plotted in Figure 4. The amplitude of the random component was chosen such that the short wavelength fluctuations have amplitudes comparable to the coherent stress drop. For clarity in plotting, a high-cut filter was applied at four mesh points per wavelength.

To construct an approximate analytic model, we want to cut off the random spectrum (25) in an approximate and reasonable way near $ka = 1$. In the work to follow, we need not be concerned that the spectrum correspond to a wave packet limited in space. A simple solution is to multiply the coherent spectrum (24) by $k^{2-\nu}$ to get the desired asymptotic dependence (25).

Because the stochastic model will be only a rough approximation, we might as well approximate the coherent displacement spectrum (24). At small k it approaches

$$\tfrac{1}{5} D_0 a^2 \tag{29}$$

and at large k it oscillates with the root mean square value

$$3(2)^{-1/2} D_0 a^2 (ka)^{-3} \tag{30}$$

The principal conclusions to follow will depend only on these asymptotic dependencies. A simple composite function approximating the magnitude of the coherent spectrum is

$$D(\mathbf{k}) = \frac{1}{5} D_0 a^2 \frac{1}{1 + (ka/2.20)^3} \tag{31}$$

The total (coherent plus random) slip spectrum chosen for

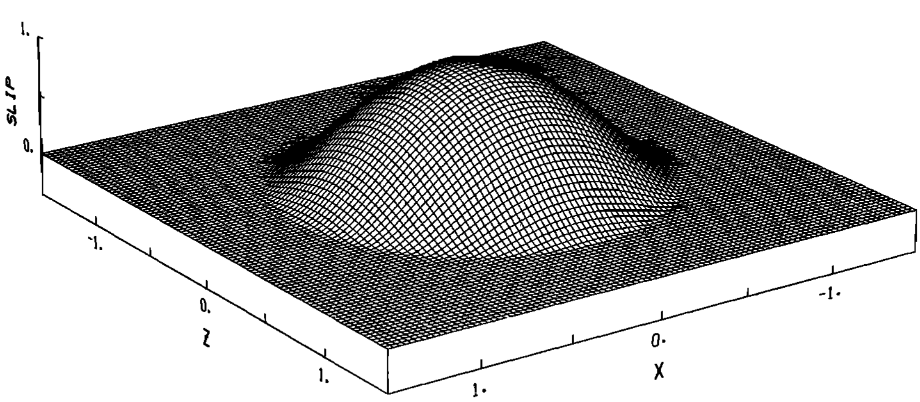

Fig. 1. Coherent static slip function plotted on the fault plane for unit slip amplitude, $D_0 = 1$, and unit radius, $a = 1$.

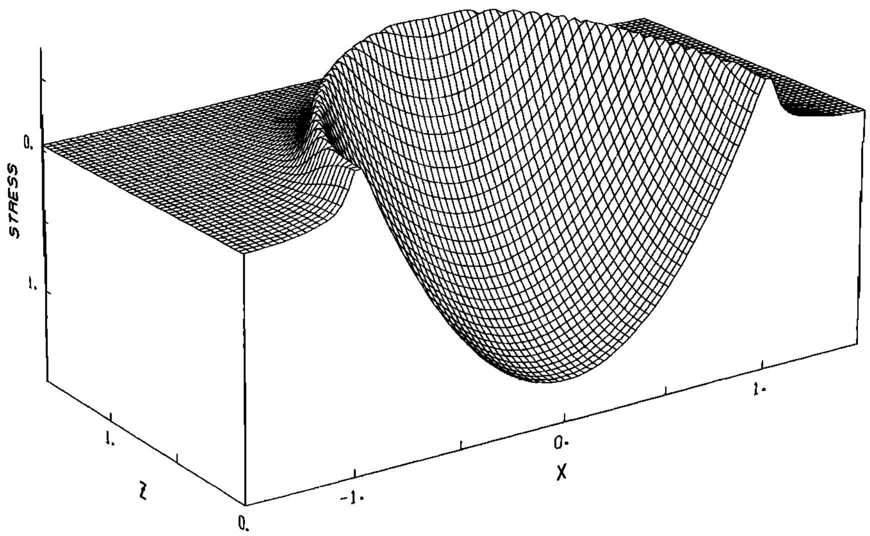

Fig. 2. Coherent static stress change function plotted on the fault plane for $\mu D_0/a = 1$ and $a = 1$.

the stochastic model is

$$D(\mathbf{k}) = \frac{1}{5} D_0 a^2 \frac{1 + pe^{i\psi}(ka)^{2-\nu}}{1 + (ka/2.2)^3} \tag{32}$$

where p is the coefficient of the random component (and is undetermined at this point) and ψ is a random function of $\mathbf{k}$. Multiplying by the isotropic approximation to stiffness (22), the spectrum of static stress change in an earthquake is

$$\tau(\mathbf{k}) = -\frac{1}{10} \sigma a^2 \frac{ka + pe^{i\psi}(ka)^{3-\nu}}{1 + (ka/2.2)^3} \tag{33}$$

where

$$\sigma(a) = \mu D_0(a)/a \tag{34}$$

is a characteristic value of the coherent stress drop. It is proportional to 'the' stress drop, which is determined from observations of moment and rupture dimensions. It is termed 'average stress drop' here to distinguish it from the stress change function. The dependence of σ, or equivalently of D_0, on rupture radius a is not yet specified.

The dependence of average stress drop on rupture radius is related to the exponent ν of the random spectrum. This relationship follows from the assumption that the stress change function in an earthquake will generally be correlated (but with the opposite sign) with the spatial fluctuation of the initial stress function and that the degree of this correlation on the average is independent of earthquake size. In other words, a rupture will tend to start where initial stress is relatively high and will tend to stop where initial stress is relatively low. This local fluctuation of initial stress is assumed to be, on the average, a constant times the coherent stress change in the earthquake.

The fault will tend to be in a relatively smooth state just before a large earthquake and will be in a relatively rough state just afterward. The random stress component of the large earthquake establishes the initial stress fluctuations that determine stress drops of smaller earthquakes. For future small earthquakes the dependence of D_0 and σ on rupture radius a will be found from the spectrum of stress fluctuations of a past large earthquake. This derivation will not appeal to a self-similar set of rupture areas, as was done in the preceding section.

Given the random initial stress spectrum

$$|\tau^0(\mathbf{k})| \propto k^{-\nu} \tag{35}$$

we want to find the expected amplitude of local stress fluctuations at a length scale a. The first step is to filter the spectrum (35) by multiplying by the coherent stress spectrum of an earthquake of size a and unit stress drop, proportional to $a^2 f(ka)$, and multiplying by a^{-2} to correct the filter to be ap-

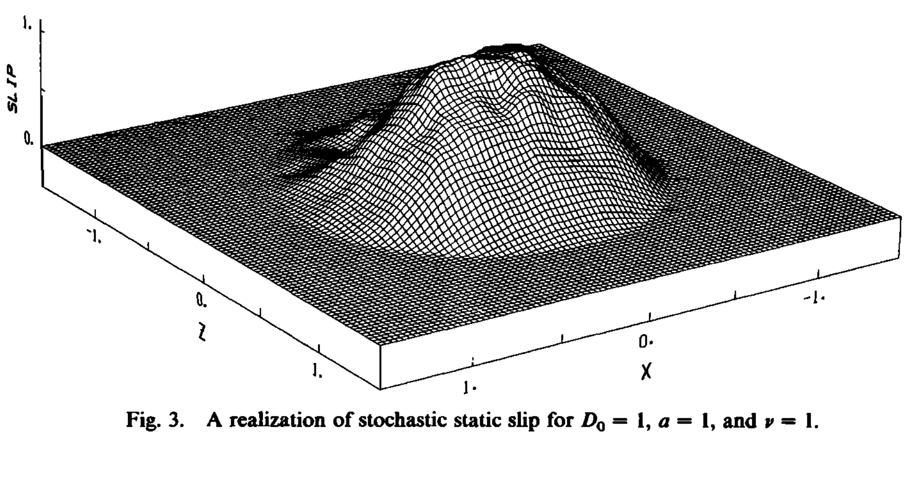

Fig. 3. A realization of stochastic static slip for $D_0 = 1$, $a = 1$, and $\nu = 1$.

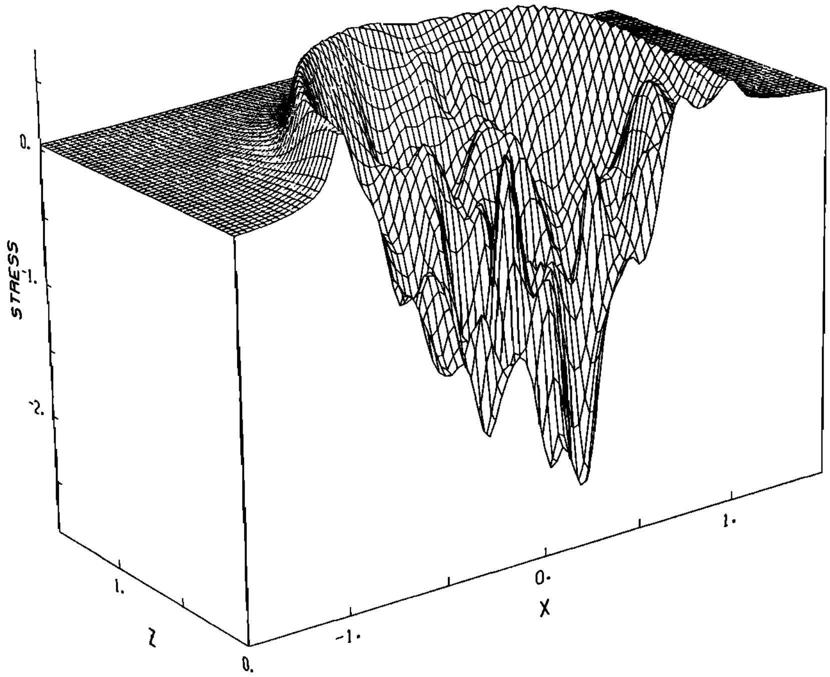

Fig. 4. A realization of stochastic static stress change for $\mu D_0/a = 1$, $a = 1$, and $\nu = 1$.

propriate to a fixed larger area. The filtered stress spectrum is proportional to

$$k^{-\nu}f(ka) \quad (36)$$

The mean square filtered stress is proportional to

$$\int [k^{-\nu}f(ka)]^2 k\, dk = a^{-2+2\nu} \int x^{1-2\nu} f^2(x)\, dx \quad (37)$$

With the change of variable of integration, the integral on the right-hand side in (37) is independent of a. The expected stress drop of an earthquake of size a is taken to be proportional to the square root of this mean square filtered initial stress

$$\sigma(a) \propto a^{\nu-1} \quad (38)$$

This result agrees with (14).

If earthquake stress drops are independent of earthquake size, as indicated by data of *Aki* [1972], *Thatcher and Hanks* [1973], *Kanamori and Anderson* [1975], and *Hanks* [1977], then $\nu = 1$.

The mean square stress fluctuation depends on the bandwidth of wave numbers considered. For $\nu = 1$ the contribution to mean square stress from wave number magnitudes in the range $k_{min} < k < k_{max}$ is proportion to

$$\log (k_{max}/k_{min}) \quad (39)$$

This means that for stress drop independent of earthquake size, equal logarithmic intervals of wave number contribute equally to stress fluctuation.

In the limit as the bandwidth is extended toward infinite wave number, stress will not converge to a regular function if $\nu \leq 1$. Stress defined by such a spectrum will be a generalized function, however, and the average value of stress over any finite area will exist. As the size of a test area shrinks toward zero, average stress on the test area will not converge, but will exhibit fluctuations increasing without limit, until the grain size is reached. This result is not unphysical, for individual silicate grains may support shear stress of several kilobars, while average stress drop in earthquakes is of the order of 100 bars.

An example of a random stress function, having a spectrum with $\nu = 1$ and a broad bandwidth, sampled along a line on the fault plane, is shown in Figure 5. One can see that fluctuations at different length scales have about the same amplitude on the average. Filtered versions of this same stress function in three different bands having equal logarithmic bandwidths are shown in Figure 6. Here fluctuations at different length scales are clearly separated. The three plots have approximately equal mean square values. (Lack of exact equality arises from discreteness of the numerical spectrum and sampling along a line.)

The elastic strain energy stored in the medium around the fault may be divided into two parts, tectonic energy and the fault's self energy, which arises from irregularity of past slip [*Andrews,* 1978]. Self energy is independent of tectonic stress.

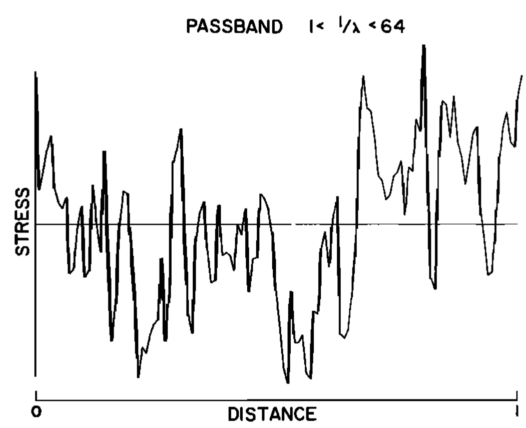

Fig. 5. A random stress function with spectrum proportional to 1/k on a plane sampled on a line of unit length. The spectrum is filtered with a passband $1 < k/2\pi < 64$.

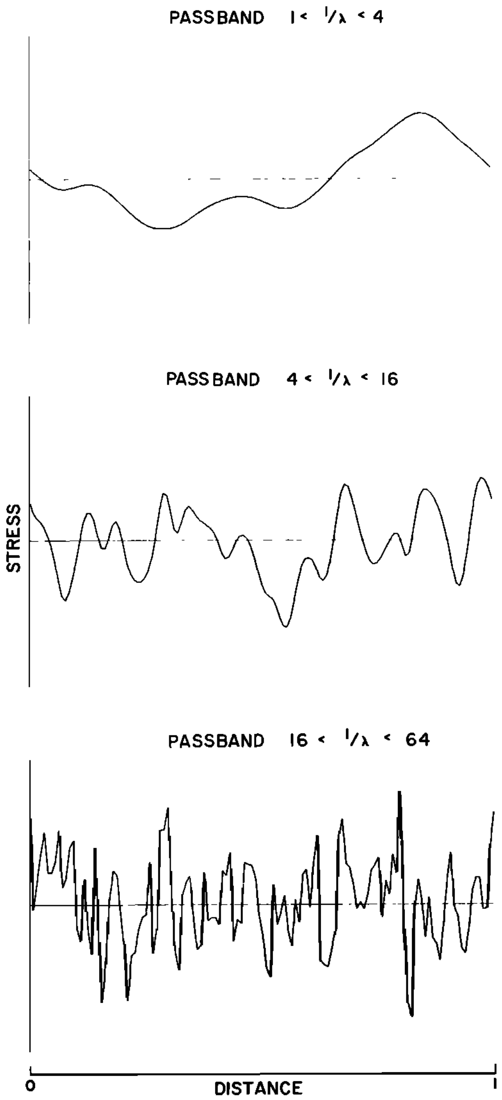

Fig. 6. The same function as in Figure 5 filtered with three different passbands: (top) $1 < k/2\pi < 4$, (center) $4 < k/2\pi < 16$, (bottom) $16 < k/2\pi < 64$.

The fault's self energy is to be distinguished from the self energy of a single slip event. The fault's self energy is considered here as a measure of the irregularity of stress on the fault surface, and its spectral decomposition must be stationary over a cycle of earthquakes of all sizes. In order that the subsequent energy analysis exclude tectonic energy, τ^0 is to represent initial self stress only, and its mean value over the entire fault surface is zero.

Let us now consider the change of static self energy stored in the elastic medium owing to the occurrence of an earthquake. This energy change is equal to the work done by tractions on the fault plane in a quasi-static process going from the initial to the final state,

$$E = -\iint [\tau^0(\mathbf{x}) + \tfrac{1}{2}\tau(\mathbf{x})]D(\mathbf{x})\, dx_1 dx_3 \qquad (40)$$

The integral may be written in the transform domain

$$E = -\iint [\tau^0(\mathbf{k}) + \tfrac{1}{2}\tau(\mathbf{k})]D^*(\mathbf{k})\, dk_1 dk_3 \qquad (41)$$

The integrand is identified as the spectral density of static energy $e(\mathbf{k})$.

Consideration is now restricted to the portion of the fault surface that is to slip in an earthquake, taken to be a circle of radius a. On this restricted area the initial stress does not have the spectrum (35), appropriate to a larger area, for we have purposely chosen an area containing a stress peak surrounded by a stress trough. Assume that the mean value of self stress over this area, averaged over all earthquakes, is zero. (This may be the worst assumption of the model and could be a point of departure for further work.) Then the initial stress spectrum is taken to be the negative of (33) multiplied by a coefficient f, but with an uncorrelated random part,

$$\tau^0(\mathbf{k}) = f\frac{1}{10}\sigma a^2 \frac{ka + pe^{i\phi}(ka)^{3-\nu}}{1 + (ka/2.2)^3} \qquad (42)$$

where ϕ is a random function of $\mathbf{k}$, independent of ψ.

In numerical models with uniform friction [*Andrews,* 1975] it was found that stress drop never exceeded the initial stress fluctuation, implying $1 \le f < \infty$. In another class of models one might imagine a uniform initial stress with the earthquake being determined by nonuniform friction. In that case, $f = 0$. The average value of f for all cases is highly uncertain, but if stress and friction vary independently, then a value of the order of 1 is not unreasonable. Initial and final stress states are illustrated in Figure 7 for a case in which $f = 2/3$.

Substitute the spectra (42), (32), and (33) into the energy integral (41), and perform an ensemble average. A term containing a single random function or product of independent random functions will average to zero, but the square of a random function will remain. The result for the integrand of (41) is

$$e(\mathbf{k}) = \frac{1}{50}\frac{\sigma^2}{\mu}a^5 \frac{[(-f + \frac{1}{2})ka + \frac{1}{2}p^2(ka)^{5-2\nu}]}{[1 + (ka/2.2)^3]^2} \qquad (43)$$

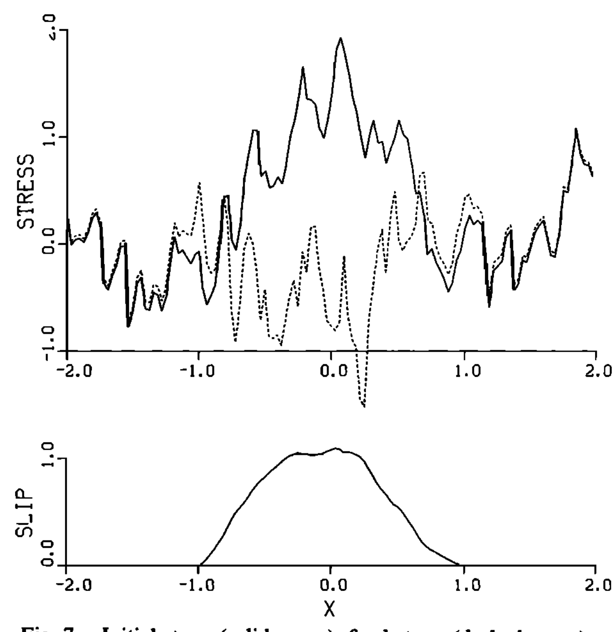

Fig. 7. Initial stress (solid curve), final stress (dashed curve), and slip function illustrated for a case in which static energy, ignoring interaction with tectonic stress, decreases in an earthquake. The initial stress function must be more concave downward than the final stress function is concave upward at the length scale of the rupture, as in this example, in order that stored elastic energy decreases at wavelengths comparable to the rupture dimension. Stored energy increases at shorter wavelengths.

and the integral over wave number is

$$E = 2\pi \int_0^\infty e(k)k\,dk \tag{44}$$

The integral converges at large k, meaning that processes at the scale of the grain size are unimportant to the energetics, if $\nu > 1/2$.

The integral may be evaluated by using a formula from *Gradshteyn and Ryzhik* [1965, p. 293],

$$E = \frac{\pi}{25}\frac{\sigma^2}{\mu}a^3\frac{(2.2)^3}{3}\left[-f + \frac{1}{2} + \frac{1}{2}p^2(2.2)^{4-2\nu} \cdot \frac{(4-2\nu)\pi/3}{\sin(4-2\nu)\pi/3}\right] \tag{45}$$

The three terms of (45) are, first, the interaction energy with the initial state, second, the self energy of the coherent component of the earthquake, and, third, the self energy of the random component. The self-energy terms are always positive, and it may be seen by comparing terms in the numerator of (43) that the random self energy is distributed at higher wave numbers. If $f > 1/2$, then the coherent terms produce a static energy decrease at wavelengths comparable to the rupture dimension. The ratio of random energy increase to coherent energy decrease is

$$R = \frac{\frac{1}{2}p^2(2.2)^{4-2\nu}}{f-\frac{1}{2}}\,\frac{(4-2\nu)\pi/3}{\sin(4-2\nu)\pi/3} \tag{46}$$

If stored elastic energy is to decrease in an earthquake, this ratio must lie in the range $0 < R < 1$. For the example plotted in Figures 3 and 4 the ratio of random self energy to coherent self energy is 0.11. Assuming $f = 1$, then $R = 0.11$ in the example.

The spectral distribution of energy change $ke(k)$ is plotted in Figure 8 for $\nu = 1$ and two different values of R. Note that spectral energy decreases for ka of the order of 1 and increases for large ka. We see that each earthquake contributes to a cascade of stored elastic energy from longer to shorter wavelengths.

The energy spectral density must be statistically stationary over a cycle of earthquakes of all sizes. We will see that this requirement relates the number-moment distribution of earthquakes to the amplitude and slope of the random spectrum.

The moment of an earthquake of size a is proportional to

$$M_0(a) \propto \sigma(a)a^3 \tag{47}$$

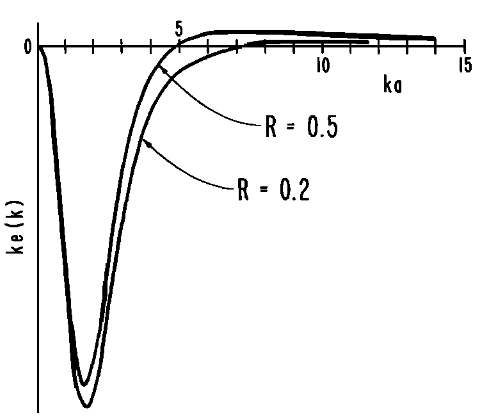

Fig. 8. The spectral distribution of static energy change, $ke(k)$, plotted for two different ratios of incoherent energy, R.

and substituting the size dependence of stress drop from (38),

$$M_0(a) \propto a^{\nu+2} \tag{48}$$

The distribution of number of earthquakes with moment greater than M_0 is taken to be

$$\log N = a' - b' \log M_0 \tag{49}$$

where primes on the coefficients are to emphasize that this is not the more familiar number-magnitude distribution. This number-moment distribution has been discussed by *Wyss* [1973]. The differential distribution is

$$dN \propto M_0^{-b'-1}dM_0 \tag{50}$$

The integral for cumulative moment

$$\int M_0\,dN \propto \int M_0^{-b'}\,dM_0 \tag{51}$$

converges at small earthquake size if $b' < 1$.

Substituting (48) into (50), one finds the differential distribution of rupture radii,

$$dN \propto a^{-\nu b'-2b'-1}\,da \tag{52}$$

Substituting the size dependence of stress drop (38) into (43), the spectral distribution of static energy change in an earthquake of size a is proportional to

$$e(k, a) \propto a^{2\nu+3}\,\frac{(-f+\frac{1}{2})ka + \frac{1}{2}p^2(ka)^{5-2\nu}}{[1+(ka/2.2)^3]^2} \tag{53}$$

The spectral energy change in a cycle of earthquakes of all sizes is

$$E(k) = \int e(k, a)\,dN \propto \int_0^{a_{\max}} e(k, a)a^{-\nu b'-2b'-1}\,da \tag{54}$$

where the number-size distribution (52) is assumed to extend to $a = 0$ but is truncated at a maximum size. Substituting (53) into (54) and changing the variable of integration,

$$E(k) \propto k^{\nu b'+2b'-2\nu-3} \cdot \int_0^{ka_{\max}} x^{-\nu b'-2b'+2\nu+2}\,\frac{(-f+\frac{1}{2})x + \frac{1}{2}p^2x^{5-2\nu}}{[1+(x/2.2)^3]^2}\,dx \tag{55}$$

Stationarity of the energy distribution over a cycle of earthquakes of all sizes requires that this integral be zero. For wavelengths much shorter than $a_{\max}$ the upper limit may be set to infinity. The integral converges for

$$-\nu b' - 2b' + 1 < -1 \tag{56}$$

or, for $\nu = 1$,

$$b' > 2/3 \tag{57}$$

For smaller values of b' there are not enough small earthquakes to remove the roughness established by larger earthquakes.

Evaluating the integral (55) for $ka_{\max} = \infty$ and substituting the expression (46) for ratio of random to coherent energy, the stationarity requirement is

$$R = \frac{A\pi/3}{\sin A\pi/3}\,\frac{\sin B\pi/3}{B\pi/3}\,\frac{C\pi/3}{\sin C\pi/3} \tag{58}$$

where

$$A = -\nu b' - 2b' + 2\nu + 1$$
$$B = -\nu b' - 2b' + 5 \quad (59)$$
$$C = 4 - 2\nu$$

The ratio R is plotted on the allowed portion of the ν, b' plane in Figure 9. For $\nu = 1$, R increases nearly linearly from 0 at $b' = 2/3$ to 1 at $b' = 1$. A small value of R, as suggested by the plotted example (assuming $f = 1$), predicts a b' value slightly larger than 2/3.

DISCUSSION

The spectrum of the stress function on a fault surface has been related to average stress drop as a function of earthquake size and to the number-size distribution of earthquakes by two different analyses in this paper. The first analysis was a straightforward application of concepts of self-similar irregularity from *Mandelbrot* [1977]. The second analysis, while still avoiding any discussion of deterministic mechanisms, attempted to construct a physical picture of an earthquake consistent with the stress state produced by all earthquakes and examined the spectral decomposition of the self energy of the fault. The assumption of self similarity was implicit in the second analysis.

The two analyses agree on the stress spectrum that is consistent with stress drop being independent of earthquake size. They disagree on the number-size distribution that arises from that spectrum. For strict self similarity the first analysis predicts the slope of the number-moment distribution to be $b' = 2/3$, while the second analysis allows the range $2/3 < b' < 1$, the lower end of the range being favored. The discrepancy will surely be a stimulus for further work. The second approach suggests greater physical insight into the seismic process, but because it is more complicated, its predictions are less convincing.

Let us now relate b' to the b value of the number-magnitude distribution. For the purpose of illustration, assume that moment and magnitude are related as

$$\log M_0 = cM + d \quad (60)$$

(A single relation of this type cannot hold for earthquakes having corner frequencies both above and below the resonant frequency of the instrument for which a particular magnitude scale is defined.) Then the number-moment distribution (49) becomes the number-magnitude distribution

$$\log N = a - bM \quad (61)$$

where

$$b = cb' \quad (62)$$

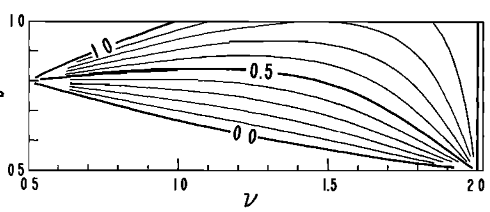

Fig. 9. Incoherent energy ratio, R, required by stationarity plotted on the ν, b' plane. Allowed values lie in the intersection of the regions $0 < R < 1$ and $b' < 1$.

Synthesizing different magnitude scales, *Hanks and Kanamori* [1979] find $c = 1.5$. Then the second model of this paper with $\nu = 1$ predicts that b values lie in the range

$$1 < b < 1.5 \quad (63)$$

At magnitudes less than 3, *Bakun and Lindh* [1977] find $c = 1.2$. Then this model predicts

$$0.8 < b < 1.2 \quad (64)$$

For microearthquakes on the San Andreas Fault, b values lie roughly in the range 0.8–1.2 with smaller values on locked portions and larger values on creeping portions [*Pfluke and Steppe*, 1973; *Grosenbaugh and Lindh*, 1978]. *Wyss* [1973] cites data supporting higher b values in regions where creep may be expected, such as midocean rifts and the Gulf of California, and lower b values in fracture zones and Baja California.

Perhaps $b' \cong 1$ on creeping faults, so that the cumulative moment is dominated by very small earthquakes. The value $b' = 2/3$, predicted by strict self similarity, may be appropriate to locked faults. *Grosenbaugh and Lindh* [1978] also find that hypocenters are distributed rather uniformly where the San Andreas Fault is creeping, but hypocenters are clustered where the fault is locked and b values are lower. Perhaps self-similar clustering, another fractal phenomenon discussed in chapters 4, 5, and 6 of *Mandelbrot* [1977], is applicable to earthquakes on locked faults.

If average stress drops are independent of earthquake size ($\nu = 1$), the slip spectrum goes as k^{-2}. This is the same spectrum of roughness as proposed by *Kamb* [1970] for the topography of a glacier bed. If two surfaces with the spectral roughness of a glacier bed are pressed together elastically and sheared with frictional sliding, the spectra of fluctuations of both normal stress and shear stress will go as k^{-1}, in agreement with this model with $\nu = 1$.

The model has some limitations. The distribution of average stress drops at fixed earthquake size is not considered. Fracture of asperities on the fault surface will produce local stress drops of kilobars. Perhaps average stress drops, observed to lie in the range of 10–100 bars, are determined by a stochastic process in which fracture occurs on a small fraction of the sliding surface, and the expected value of this fraction is the same for any size earthquake.

The predictions of the model are testable. However, a rigorous comparison with data would be a formidable undertaking. Broadband high-dynamic-range recordings would be required to determine moment and stress drop for an unbiased sample of earthquakes over several orders of magnitude of moment and over a time interval greater than the recurrence time of magnitude 6 earthquakes.

In regard to earthquake prediction, the model is pessimistic about the relevance of a point measurement of stress. The model is concerned with statistical averages over a time interval much longer than the recurrence time of large earthquakes, in order to obtain stationarity. Over shorter time intervals the seismic process need not be stationary. It is possible that stress drops, b' values, and high-frequency ground motion spectra are different before and after a large earthquake.

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