VERTICAL DISPLACEMENT RATE FIELD OF TAIWAN FROM GEODETIC LEVELLING DATA 2000-2008

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ABSTRACT
We have developed the most precise vertical displacement rate field of Taiwan Island to date from four sets of precise geodetic levelling data between 2000 and 2008. More than 4000 km of levelling lines between solidly constructed benchmarks were repeatedly surveyed by 1st-order levelling. Strict specifications for field work procedures and comprehensive corrections for systemic errors were applied to the levelling data. Furthermore, in order to account for the vertical deformations caused by four major earthquakes during the data span, coseismic offset corrections were computed and applied. The adjusted orthometric heights and vertical displacement rates of the benchmarks were obtained by least-squares estimation (LSE). The estimated standard deviations of the vertical rates are in the range of ± 0.41 mm/yr and ± 2.47 mm/yr, with a mean value of ± 1.64 mm/yr. These highly accurate results reveal that the most significant subsidence rate is observed in SW Taiwan at -109.4 mm/yr. Land uplift is widely observed in the Central Range and the Coastal Range, with the largest uplift rate of +29.4 mm/yr in SE Taiwan, at the convergence boundary of the Philippine Sea and the Eurasian plates.


INTRODUCTION
Geodetic levelling has been regarded as the most precise method for height determination in geodesy. A relative accuracy level of 0.5-0.7 mm over a distance of 1 km has been consistently reported in 1st-order levelling networks [1]. Yet, the major disadvantage of geodetic levelling is the high cost and the large amount of time needed for collecting the data over long distances or over a large network.

Taiwan is situated at the convergence boundary of the Philippine Sea and the Eurasian plates. As a result, significant crustal deformation rates in the region derived from geodetic techniques such as the Global Positioning System (GPS) have been reported by many researchers over the last 15 years [3], [4], [12], [15], [21], [24], [25]. However, among the vast number of research projects carried out during this period, only a few estimated the vertical rates from levelling data in some limited areas due to the high cost and the time required [10], [23].

Although the geodetic levelling is time consuming and costly, a number of determinations of uplift rates from precise levelling were reported outside Taiwan [6], [7], [11], [14], [16], [17]. Compared with these, the terrain over Taiwan is complex. The highest point (Mt. Jade) is 3952 m high. In Eastern Taiwan, the heights of the mountains can be almost 2000 m only a few km from the coast. In such terrain, the vertical rate field plays an important role in describing the crustal movements in Taiwan.

From 2000 to 2001, an island-wide geodetic levelling-based vertical datum was officially established in Taiwan, namely the Taiwan Vertical Datum 2001.
(TWVD2001) [22], which initially consisted of 1010 1st-order 1st-class benchmarks in a network of about 2200 km of levelling lines (Figure 1). In 2002, the TWVD2001 was extended to include an additional 1055 1st-order 2nd-class benchmarks in a network of 2300 km of levelling lines. After that, the TWVD2001 benchmarks were re-surveyed two more times between 2005 and 2008. Altogether, these four levelling campaigns covering the whole island provide an unique and precious opportunity to derive a highly precise vertical displacement rate field for Taiwan.

**RESEARCH DATA AND METHODS**

The general information of the four levelling measurements of the TWVD2001 network is listed in Table 1. The average spacing between adjacent benchmarks in the network is about 2 km (Figure 1). The levelling data were mostly collected at night by Zeiss DiNi-11 and DiNi-12 digital levelling instruments with Nedo invar rods supported by corresponding struts. A number of stringent specifications were designed and applied to the levelling field work in order to reduce or eliminate measurement errors. First, the maximum permissible difference in sight lengths between forward and backward sights is 0.5 m per set-up, and the cumulative difference is limited to 1.5 m per section. Second, the maximum length of sight is restricted to just 30 m in order to greatly reduce the influence of atmospheric refraction. The minimum and maximum sight ground clearances (or staff readings) are 0.3 m and 2.7 m, respectively. Third, the maximum standard deviation of each levelling reading in a set-up is ± 0.2 mm. Fourth, the maximum difference of the two height differences from the double readings at a set-up, e.g. foresight (F)-backsight (B)-backsight (B)-foresight (F) (FBBF) or BFFB...
measurements, is limited to 0.4 mm. These procedures of alternate BFFB and FBBF
are also used to eliminate the obliquity of horizon in the TWVD2001 levelling network
[5]. Some strict error control measures were also enforced: the maximum misclosure
between forward and backward runs in a section is limited to ± 2.5 mm√k (k being
the section distance in km), and the maximum loop misclosure is limited to ± 2.0 mm
√F (F being the loop distance in km). In addition, systematic errors of various kinds
were calculated and removed from the measurements, as listed in Table 2. Theoretically, if the foresight (FS) and backsight (BS) distances are equal in each
set-up, the corrections of earth curvature and collimation errors are not necessary.
Actually, the FS and BS distances are not perfectly equal in each set-up in the field
even though the maximum permissible difference in the sight lengths between FS and
BS is specified. For these small differences between the FS and BS distances from
each set-up, we did apply corrections for the earth curvature and the collimation error
as shown in Table 2. The two-peg-method was used to obtain the collimation error
(unit: mm/m) for each digital levelling instrument before starting the field work on all
field days. Because the levelling was mostly carried out at night, the variations of the
temperature were small. Thus, the corrections for the collimation errors can be made
using the differences between FS and BS distances measured at each set-up.

Table 1. Precise levelling campaigns in Taiwan from 2000 to 2008

<table>
<thead>
<tr>
<th>Measurement Period</th>
<th>Observations</th>
<th>Benchmarks</th>
<th>Total Length of Lines (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec. 2000 - Aug. 2001</td>
<td>1033</td>
<td>1020</td>
<td>2052</td>
</tr>
<tr>
<td>Jun. 2002 - Dec. 2002</td>
<td>1152</td>
<td>1137</td>
<td>2201</td>
</tr>
<tr>
<td>Apr. 2007 - May 2008</td>
<td>1934</td>
<td>1872</td>
<td>4287</td>
</tr>
</tbody>
</table>

Table 2. Specifications and systematic error correction methods

<table>
<thead>
<tr>
<th>Field Procedures</th>
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<tbody>
<tr>
<td>Maximum permissible difference in sight lengths between foresight and backsight per set-up</td>
</tr>
<tr>
<td>Maximum cumulative difference in sight lengths per section</td>
</tr>
<tr>
<td>Maximum length of sight</td>
</tr>
<tr>
<td>Minimum and Maximum sight ground clearances (or staff readings)</td>
</tr>
<tr>
<td>Maximum standard deviation of each staff reading in a set-up</td>
</tr>
<tr>
<td>Maximum difference between two height differences from the double readings (FBBF or BFFB) at a set-up</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum Permissible Misclosures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section; forward and backward</td>
</tr>
<tr>
<td>Loop</td>
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<tr>
<th>Corrections of Systemic Errors</th>
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<tr>
<td>Invar rod correction from calibration against standard</td>
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<tr>
<td>Invar rod correction for thermal expansion (the coefficient of thermal expansion is from the certificate of Nedo GmbH &amp; Co. KG, Germany)</td>
</tr>
<tr>
<td>Curvature correction</td>
</tr>
<tr>
<td>Collimation error</td>
</tr>
<tr>
<td>Orthometric correction with surface gravity measurements [8]</td>
</tr>
<tr>
<td>Kukkamäki refraction error model [9], [18]</td>
</tr>
<tr>
<td>Obliquity of horizon (cancelled by the procedures of alternate BFFB and FBBF)</td>
</tr>
</tbody>
</table>
Between 2000 and 2008, four $M > 6.0$ intense earthquakes, generating significant coseismic displacements in Taiwan, were reported by previous studies \cite{2}, \cite{3}, \cite{20}. To correct the effects caused by those, we applied vertical coseismic offset corrections to the TWVD2001 benchmarks. The corrections were calculated by the coseismic source models of the four intense earthquakes, using the formulations of Okada \cite{13} for surface displacements due to the dislocations in elastic half-space. The coseismic source model of the 2002 $M_w 6.8$ Hualien earthquake was inverted by the horizontal displacements from \cite{2}. The fault geometry was established based on the focal mechanism solution from the Broadband Array in Taiwan for Seismology (BATS). We then inverted the coseismic displacements of the 2003 $M_w 6.6$ Chengkung earthquake from \cite{3} to estimate the optimal slip distribution on the fault. The fault geometry, used in this paper, was modified from the configurations of \cite{3}. Next, because the coseismic source model of the 2006 $M_w 6.2$ Taitung earthquake was supplied by \cite{20}, we directly evaluated the coseismic correction of each benchmark according to their source models. Finally, the coseismic source model of the 2006 $M_w 7.0$ offshore Pingtung earthquake was inverted by the surface displacements, evaluated in this study on the basis of continuous GPS observations. The fault geometry was established in terms of the focal mechanism solution from the BATS. (See Appendix for short explanations of the above geophysical terms.)

According to the coseismic source models, the largest value of vertical coseismic displacement correction of the 2002 $M_w 6.8$ Hualien earthquake is -25.45 mm and the mean is -1.6 mm, distributed in NE Taiwan. The mean of +3.55 mm and the largest value of +260.13 mm for the 2003 $M_w 6.6$ Chengkung earthquake are obtained mainly in SE Taiwan. For the 2006 $M_w 6.2$ Taitung earthquake, the largest and the mean value are -17.09 mm and -0.03 mm, respectively. Finally, the peak value of the correction is -38.78 mm with the mean value of -0.81 mm for the 2006 $M_w 7.0$ offshore Pingtung earthquake.

Because the actual time spans of the data collection for each benchmark during the four levelling campaigns are different (Table 1), we adopted a LSE model to estimate the orthometric heights and vertical rate parameters of the benchmarks in the network. The fundamental measurement used in the LSE model is the orthometric height difference between two adjacent benchmarks $i$ and $j$, as derived from the levelling data after applying corrections for the systematic errors listed in Table 2 and for the effect of the vertical coseismic displacements. The LSE model is given by

$$
\varepsilon_{ij} + \Delta H_{ij}^t - \Delta D_{ij}^t = H_i^t - H_j^t + V_i \Delta t - V_j \Delta t
$$

where:
- $\Delta H_{ij}^t$ the orthometric height difference between the benchmarks $i$ and $j$ at epoch $t$
- $\Delta D_{ij}^t$ the total difference of the vertical coseismic displacements between the benchmarks $i$ and $j$ at epoch $t$, the value is zero when $t$ is before the 2002 $M_w 6.8$ Hualien earthquake
- $H_i^t$ the orthometric height of the benchmarks $i$
- $H_j^t$ the orthometric height of the benchmarks $j$
- $V_i$ the vertical rates of the benchmarks $i$
- $V_j$ the vertical rates of the benchmarks $j$
- $\Delta t$ the time interval between the measurement epoch $t$ and the reference time which is defined as 1 January 2001, the starting epoch of TWVD2001
$\varepsilon_{ij}$ the residual of the measurement

The LSE model is minimally constrained to the known orthometric height value of the Keelung tide gauge (Fig. 1) located in northern Taiwan [22]. Each height difference observation is weighted inversely proportional to the corresponding section distance [1].

RESULTS

In the LSE computation, 4481 levelling observations were used. The number of unknowns is 3686, including 1843 height parameters and 1843 vertical displacement rate parameters. Thus, the redundancy of the model is 795. Standard deviations of the estimated vertical rates are in the range of $\pm 0.41$ mm/yr and $\pm 2.47$ mm/yr, with an average of $\pm 1.64$ mm/yr. The precision (relative to the reference tide gauge at Keelung as shown in Fig. 1b) of the orthometric heights in the 2000-2008 adjustment is $\pm 0.128\ mm/\sqrt{k}$ ($k$ being the distance from Keelung in km). The spatial variation of the vertical displacement rate field of the 1843 TWVD2001 benchmarks is shown in Fig. 2, as illustrated by using the minimum curvature method of the Generic Mapping Tools (GMT) [19] with an interpolation radius of 15 km.

Our precision of the vertical rates are consistent with those of other investigations [6], [7], [11], [14], [16], [17]. The determination of the vertical rate field in this paper could provide more information about the vertical deformation of the crust in Taiwan than previous research work that used the GPS technique.

In terms of the spatial variations of the vertical displacement rate field (Figure 2), significant land subsidence occurs in SW Taiwan, where the maximum subsidence rate reaches -109.4 mm/yr. In contrast, a significant uplift area is evident between the Central Range and the Coastal Range, with the largest rate of +29.4 mm/yr located in SE Taiwan at the plate collision boundary. The vertical displacement rate field obtained here could provide a significant contribution to the understanding of the vertical deformations in Taiwan. The interpretation of the spatial distribution of the displacement rate field is not within the central scope of this paper. This important issue will be analysed later.

CONCLUSIONS

The most precise vertical displacement rate field of Taiwan to date has been developed in this paper. By means of repeated precise geodetic levelling in conjunction with strict field procedures and comprehensive systematic error corrections, including corrections for vertical coseismic offsets of major earthquakes between 2000 and 2008, we have obtained vertical displacement rates of 1843 TWVD2001 benchmarks with formal errors in the range of $\pm 0.41$ mm/yr and $\pm 2.47$ mm/yr. These highly accurate results complement previous research that estimated high-precision horizontal displacement rates of Taiwan using the GPS technique whose intrinsic accuracy limitation is exactly in the vertical direction. Hence, it is believed that future studies can make maximum use of the vertical rates to provide detailed explanations of the complex crustal deformation activities in the region.

APPENDIX

Intense earthquakes usually generate significant permanent surface displacements, i.e., coseismic displacements (or coseismic offsets). If the levelling campaigns pass through the rupture epoch of the intense earthquake, we need to correct the
contamination by the coseismic displacements when we calculate the average vertical displacement rates. This process is called the **coseismic offset correction**. To get the coseismic offset correction, we usually estimate the magnitude of the coseismic displacement at each levelling benchmark according to the coseismic source model. In this model, we first assume a fault (i.e., dislocation) in an elastic half-space, which is a mathematical model used to approximate the earth; the elastic half-space is much simpler than the real earth. Then the surface displacements are assumed to be the result of the movement of the fault (dislocation) in an elastic half-space. In other words, the coseismic source model is used to describe the slip distribution along the fault plane.

Then we can invert the known permanent surface displacements to get the slip distribution along the fault plane (coseismic source model), using the formulations of Okada [13]. When we estimate the slip distribution, a fault geometry is required. Because the focal mechanism solution is a way of showing the fault and the direction of slip on it from an earthquake, using circles with two intersecting curves that look like beach balls, we usually obtain the parameters of the fault geometry from the focal mechanism solutions.

![Fig.2. Vertical displacement rate field of Taiwan, 2000-2008. (Dots show the benchmarks)](image-url)
ACKNOWLEDGEMENTS

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References