Coupling of the random properties of the source and the ground motion for the 1999 Chi Chi earthquake

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[1] We show that both the slip distribution and the peak ground acceleration (PGA) for the 1999 Chi Chi earthquake can be described by Lévy laws. Furthermore, we found that the tails of the probability density functions (PDF) characterizing the slip and the |PGA| are governed by a parameter, the Lévy index, with almost the same values as predicted by the Central Limit theorem. The PDF tail controls the frequency at which extreme large events can occur. These events are the large stress drops–asperities–distributed over the fault surface and the large |PGA| observed in the ground motion. Our results suggest that the frequency of these events is coupled; the PDF of the |PGA| is a direct consequence of the PDF of the asperities. The physical rationale responsible for the coupling is the principle of superposition of wave signals characterized by random properties that are governed by the Central Limit theorem. **Citation:** Lavallée, D., and R. J. Archuleta (2005), Coupling of the random properties of the source and the ground motion for the 1999 Chi Chi earthquake, *Geophys. Res. Lett.*, *32*, L08311, doi:10.1029/2004GL022202.

1. Introduction

[2] Two of the most fundamental principles in physics and mathematics are founded on the similitude in properties between a single event and a sum of these events. In seismology, the principle of superposition stipulates that during an earthquake, the waveform observed at a given distance of the fault is essentially the sum of waves emitted by point sources distributed over the fault surface. On the other hand, the Central Limit theorem postulates that the sum of Lévy random variables is also a Lévy random variable [*Uchaikin and Zolotarev*, 1999]. *Kagan* [1994] used the term "self-replication" to characterize this property in a study that showed that the stress increments caused by a fractal set of earthquakes on subsequent earthquakes is also distributed according to a Lévy law.

[3] In studies of source models for several earthquakes—the 1979 Imperial Valley, the 1989 Loma Prieta, the 1994 Northridge and 1995 Hyogo-ken Nanbu (Kobe)—we have found that the spatial distributions of slip are characterized by a Lévy law [see *Lavallée and Archuleta*, 2003; D. Lavallée et al., Stochastic model of heterogeneity in earthquake slip spatial distributions, submitted to *Geophysical Journal International*, 2004, hereinafter referred to as Lavallée et al., submitted manuscript, 2004]. This result can be used to deduce statistical properties of the radiated field. During an earthquake, the rupture front propagates over the fault surface; as the rupture front reaches different points on the fault, each point source emits a wave with an amplitude proportional to the stress released or the "stress drop" [*Tumarkin and Archuleta*, 1994]. We assume a linear relationship between the stress released and the slip [*Andrews*, 1980]. Because the slip is distributed according to a Lévy law, so will the stress released and the point source wave amplitude. The signal observed at a given distance from the source will be the sum of the signals emitted by the point sources. Because the point source wave amplitudes are distributed according to a Lévy law, the sum of these signal amplitudes observed at a given distance from the sources will also be distributed according to a Lévy law. Using these concepts, we investigate the statistical properties of both the slip distribution and the peak ground acceleration (PGA) for the 1999 Chi Chi earthquake.

2. Stochastic Modeling of the Source Model

[4] *Lavallée and Archuleta* [2003] and Lavallée et al. (submitted manuscript, 2004) proposed and tested a model that includes one-point and two-point statistics for the slip distribution of several earthquakes. The stochastic model is similar to the fractional Brownian motion fBm [see *Peitgen and Saupe*, 1988; *Falconer*, 1990]. In fBm, white noise is Gaussian distributed then filtered in the Fourier space to generate a signal characterized by a spectrum with a power law behavior. In these papers, we relaxed the constraint that the random variables be distributed according to a Gaussian law and assumed the most general case of the Lévy, or stable law [*Uchaikin and Zolotarev*, 1999]. The Lévy law, which includes as special cases the Gauss and Cauchy laws, is the most general law for which the Central Limit theorem applies (see next Section). In this stochastic model, the probability law is fixed by the data.

[5] The stochastic model proposed here consists of a convolution in the Fourier space between the Fourier transform of random variables (white noise) X and some function with a power law dependence $k_x^{-\nu/2}$ where k_x is the horizontal wave number. The scaling exponent ν measures the departure from the non-correlated random variable (white noise when $\nu = 0$). In one dimension, the stochastic model Y_x is given by the following relationship:

$$Y_x \propto \sum_{s=2-N/2}^{1+N/2} |k_x|^{-\nu/2} F_s[X_x] \exp[-2\pi i(x-1)(s-1)/N], \quad (1)$$

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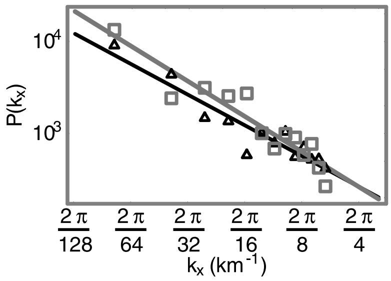

Figure 1. The mean power spectrum $P(k_x)$ as a function of the wave number k_x and the best straight line that fits the log-log curve are reported for the dip slip (hollow black triangle), and the strike slip (hollow gray box). These results suggest that the scaling behavior is observed for scale lengths that range from 3 to 72 km.

for a set of random variables X_x distributed over a one–dimensional lattice of length N (an even number), where x is the integer spatial variable on the one-dimensional lattice. The discrete spatial frequency s is related to k_x by $k_x = 2\pi(s - 1)$; $F_s[X_x]$ is the discrete Fourier transform of X_x (for $s \leq 0$, the index $s = N + s$ in $F_s[X_x]$). We assume that $k_x^{-\nu/2} F_s[X_x] \rightarrow 0$ at $s = 1$. The power spectrum $P(k_x)$ associated with Y_x is then given by the following relation:

$$P(k_x) = |F_s[Y_x]|^2 \propto k_x^{-\nu} \tag{2}$$

[6] This equation can be used to compute the values of ν associated with Y_x. Using this scaling exponent, the underlying random variables X_x associated with a stochastic model Y_x can be computed by using the following relationship:

$$X_x \propto F_x^{-1}\left[F_s[Y_x] \times k_x^{\nu/2}\right], \tag{3}$$

where F_x^{-1} is the Fourier inverse. The one point statistical properties of the stochastic model are completely specified when the probability law and parameters governing X_x are identified.

[7] The model outlined above is applied to the slip (along both dip and strike) spatial distribution of the 1999 Chi Chi earthquake [*Zhang et al.*, 2003]. The power spectrum of the slip, $P(k_x)$ is computed for each of the horizontal layers (along strike). For slip along dip and strike the mean power spectrum of all the horizontal layers has been computed (Figure 1). For each slip distribution, the spectrum shows that there are no dominating wave numbers. The curves illustrated in Figure 1 show not only that all the wave numbers contribute to the slip variability but also that the weight of the wave numbers follows approximately a decaying power law. Values of the scaling exponents ν are reported in Table 1.

Table 1. Parameters of the Stochastic Model for the Dip and Strike Slip of the Chi-Chi Earthquake

	ν	α	β	γ	μ
Dip slip	1.11	0.95	−0.3	14.6	79.
Strike slip	1.27	1.0	0.3	12.3	9.7

[8] After estimating ν, the spatial distribution of slip for each layer is filtered in the Fourier space in such a way that the resulting field has a flat mean power spectrum (white noise). We assume that the filtered slip can be approximated by random variables of magnitude X. We compute the probability density function (PDF) of X (see Figure 2).

[9] Next we proceed to determine the probability law that provides the best fit to the estimated PDF of X. Three laws are considered: Gauss, Cauchy and the more general Lévy law [*Uchaikin and Zolotarev*, 1999]. The Lévy law is characterized by four parameters α, β, γ and μ. The parameter α, with $0 < \alpha \leq 2$, controls the rate of falloff of the PDF tails. The larger the value of α, the less likely it is to find a random variable far from the central location. The case $\alpha = 2$ corresponds to the Gaussian law, while $\alpha = 1$ (with $\beta = 0$) corresponds to the Cauchy law. The parameter β, with $-1 \leq \beta \leq 1$, controls the departure from symmetry of the PDF curve. When $\beta = 0$, the PDF is symmetric and centered about μ. The (scale) parameter γ, $\gamma > 0$, is mainly responsible for the PDF width whereas μ is the translation, or location parameter of the PDF.

[10] For both the dip and strike slip distribution, we computed the probability law parameters that minimize the following expression:

$$\sum_t |PDF(X_t) - p_{th}(X_t)| \tag{4}$$

where $p_{th}(X)$ corresponds to the theoretical values of the PDF associated with either the Gauss, Cauchy or Lévy law computed for X [see *Grigoriu*, 1995; Lavallée et al., submitted manuscript, 2004]. The PDF of X (the filtered slip) and the PDF curves corresponding to the best-fitting Gaussian, Cauchy and Lévy laws are illustrated in Figure 2. For each slip distribution, the parameters of the best-fitting Lévy laws are reported in Table 1. For both samples used in this study, the Lévy law provided the best fit to the PDF. The results presented in Table 1, in particular the values taken by α, are in good agreement with the stochastic parameters computed for several other earthquakes [see

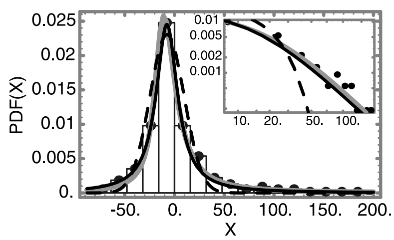

Figure 2. The (discrete) probability density function PDF (dots and bars) associated with the filtered strike slip X is compared to the curves of the three probability laws that best fit the PDF: the Cauchy law (black curve), the Gaussian law (dashed curve) and the Lévy law (gray curve). The magnitude of the random variables, i.e., the filtered slip, is given by X. In the inset, the positive tails of the curves are on a log-log plot to emphasize the fit for values far in the tail.

Lavallée and Archuleta, 2003; Lavallée et al., submitted manuscript, 2004]. In the second paper, we compare stochastic models computed for different source models and discuss the causes of uncertainties in computing the parameters ν, α, β, γ and μ.

3. The Central Limit Theorem, the Principle of Superposition and the Consequences for the Statistical Properties of the Ground Motion

[11] According to the Central Limit theorem, a combination of Lévy random variables $X(\alpha, \beta, \gamma, \mu)$ will result in a random variable that also belongs to a Lévy law:

$$A_1X_1(\alpha,\beta,\gamma,\mu) + A_2X_2(\alpha,\beta,\gamma,\mu) + \ldots \stackrel{d}{=} AX(\alpha,\beta,\gamma,\mu) + B \stackrel{d}{=} X(\alpha,\beta,\gamma',\mu') \qquad (5)$$

where A_i, A and B are real constants, and $\stackrel{d}{=}$ stands for equal in probability distribution [*Uchaikin and Zolotarev*, 1999]. Equation (5) indicates that the random variable on the right hand side will be characterized by the same α and β, but different parameters γ' and μ'. It should be noted that since any pair (or combination of terms) on the left hand side of equation (5) can be rewritten as $A_iX_i(\alpha, \beta, \gamma, \mu) + A_jX_j(\alpha, \beta, \gamma, \mu) + \ldots \stackrel{d}{=} X(\alpha, \beta, \gamma'', \mu'')$, the Central Limit theorem remains valid even if the random variables on the left hand side of equation (5) are characterized by different γ and μ values.

[12] According to the Central Limit theorem, the stochastic model Y_x in equation (1) will have its amplitude distributed according to a Lévy law. Consequently, the slip, or stress drop, spatial distribution is also distributed according to a Lévy law, although the parameters of the Lévy law —γ and μ— will be a function of position. Here, we assume that the stress released is also related to the same random variables that characterized the slip distribution through a filtering similar to the one given in equation (1) but with a different scaling exponent [see *Andrews*, 1980; *Mai and Beroza*, 2002].

[13] On the other hand, the principle of superposition stipulates that during an earthquake, the waveform $\psi(\mathbf{r}_j, t)$ observed at a given distance of the fault is essentially the sum of waves $C_i\psi_i(\mathbf{r}_{ij}, t)$ emitted by point sources distributed over the fault surface.

$$\psi(\mathbf{r}_j, t) = \sum_{i=1}^{N_{ps}} C_i \psi_i(\mathbf{r}_{ij}, t) \qquad (6)$$

[14] The variable $\mathbf{r}_j$ is the distance to the j^{th} observation site, and t is time. The parameter C_i corresponds to the random point source amplitude, and the function $\psi_i(\mathbf{r}_{ij}, t)$ includes the periodic signal, phase, directivity effect and other non-random contributions to the signal. The variable $\mathbf{r}_{ij}$ is the distance between the point source and the observation. The sum is performed over all the N_{ps} point sources over the fault surface. Comparing equation (6) to equation (5), there is a correspondence between C_i and X_i as well as between ψ_i and A_i. Thus, the random contribution to $\psi(\mathbf{r}_j, t)$ can be understood as a sum of random variables weighted by constants. In such a case, the Central Limit theorem applies.

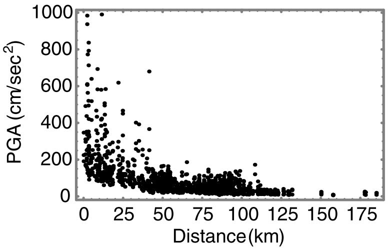

Figure 3. The |PGA| amplitude of the three components of recordings estimated at different stations as a function of the closest distance between the station and the rupture surface.

[15] It should be noted that these considerations are quite general and will apply independently of the probability law that governs the slip, the stress drop spatial distribution or the wave amplitude C_i. If C_i randomness is characterized by a Lévy distribution, so will be the random properties associated with $\psi(\mathbf{r}, t)$; if C_i is characterized by a non-Lévy law, for instance a Uniform law [*Oglesby and Day*, 2003], the Central Limit theorem stipulates that the sum of such random variables will converge to a Gaussian law.

[16] We assume that as an earthquake rupture propagates over the fault surface each point at the rupture front emits a wave with an amplitude proportional to the stress released. Because the magnitude of the stress released is distributed according to a Lévy law, so is the point source wave amplitude. We also assume a linear relationship between $\psi(\mathbf{r}, t)$ and the ground motion displacement and acceleration at a given site. Thus, the statistical properties of the ground motion are coupled to the statistical properties of the stress released. Both should be described by a Lévy law. Note that according to equation (6), the functions $\psi_i(\mathbf{r}_{ij}, t)$ will vary from one position $\mathbf{r}_j$ to another. This is equivalent to changing the constants A_i in equation (5) which will affect the values of the parameters γ' and μ' associated with the random variables on the right side of equation (6)—see also *Gusev* [1996]. However, the parameters α must be the same and independent of the constant A_i. We assume that the same property holds for $\psi(\mathbf{r}_j, t)$, i.e., it is independent of position $\mathbf{r}_j$ (with the qualification that $\mathbf{r}_j$ must be close to the fault, see below) and defined by the same parameter α.

[17] This theory is tested by computing the PDF of the peak ground acceleration PGA and fitting it to a Lévy law. Here, we consider the PDF of the absolute value of PGA as is traditionally done in seismology. By doing so, the random variable associated with |PGA| is bounded between a minimum and infinity. For this reason, the PDF of the |PGA| is often assumed to be characterized by a log-normal law [*Abrahamson*, 1988; and references therein]. Nevertheless other probability laws can be considered with an appropriate truncation of the PDF.

[18] The |PGA| amplitude estimated at different stations as a function of the closest distance between the station and the rupture surface is illustrated in Figure 3. This figure suggests that the probability density function of the |PGA| is a function of the distance (as it should be for reasons already discussed above). However, we can assume that variations

in the statistical properties are not too large if we only consider the stations located within distance intervals with limited range. To test the effect of the location of the stations on the computed PDF, we computed the PDF of the |PGA| for stations located within distance intervals of different sizes. The stations located between 0 and 30 km of the fault trace are divided into three groups with a distance interval of 10 km, four groups with a distance interval of 7.5 km and six groups with a distance interval of 5 km. We computed the PDF of the |PGA| for each group. Assuming that the PDF of the |PGA| can be approximated by a Lévy law, we compute the parameters of the Lévy law that fit the PDF curves by minimizing equation (4). The results are reported in Table 2 and illustrated in Figure 4 for the stations located between 0 and 10 km.

[19] The values of α are close to 1 for almost all of the PDF's reported in Table 2 except for two cases —stations located between 15 to 20 km and stations located between 25 to 30 km. (The PDF computed for stations located between 15 to 20 km has only seven values. Therefore results obtained for this group are not significant.) It should be noted that convergence to a value close to 1 increases with the size of the distance interval. There is a very small deviation from $\alpha = 1$ for the three PDFs computed with a distance interval of 10 km. The number of stations included within a larger distance interval is larger than the number of stations within a smaller distance interval. Accordingly the PDF of the former are computed with more accuracy — especially the PDF tails— since more PGA values are included in groups with a larger distance interval.

[20] These results show that the Lévy law provides an accurate description of the PDF associated with the |PGA|. Furthermore, these results suggest that the rate of decrease of the PDF tails of |PGA| —controlled by α— is (almost) invariant for stations located between 0 to 30 km, and (almost) independent of the size of the distance interval used to compute the PDF.

[21] In this paper we ignore site amplification effect either due to nonlinear soil [*Field et al.*, 1997] or scattering of waves in the upper kilometers of the earth's crust [*O'Connell*, 1999]. We also limit our study to sites located within 30 km. The typical shape illustrated in Figure 4 is gradually lost for the PDF associated with sites located at larger distances [*Gusev*, 1996].

Table 2. Parameters of the Lévy Law That Best Fits the PDF of the |PGA| for Stations Within Distance Intervals With Size of 10, 7.5 and 5 km

Location of the Stations, km	α	β	γ	μ
0–10	0.95	0.99	46.2	−493.0
10–20	0.96	0.58	33.8	−259.5
20–30	1.03	1.00	28.4	754.3
0–7.5	0.87	0.99	42.1	−161.3
7.5–15	0.87	1.00	25.6	−74.5
15–22.5	1.11	0.22	53.3	168.2
22.5–30	1.21	1.00	45.3	161.1
0–5	0.85	0.98	42.1	−125.8
5–10	1.20	0.99	97.4	279.3
10–15	1.00	0.00	51.9	124.2
15–20	1.47	0.00	169.2	135.2
20–25	1.06	0.90	46.2	463.9
25–30	1.34	0.93	83.1	136.2

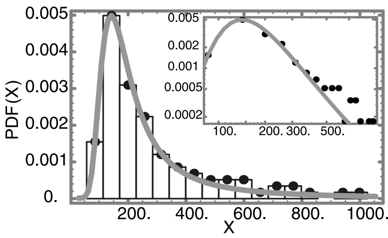

Figure 4. The PDF of the |PGA| for the stations located between a distance of 0 to 10 km is compared to the curves of the Lévy law that best fit the PDF (gray curve)—see Table 2. The variable X corresponds to the |PGA|. In the inset, the positive tails of the curves are plotted on a log-log scale.

4. Conclusion

[22] In this paper, we have shown that a stochastic model — based on a Lévy law— is best suited to reproduce the main features of the spatial variability embedded in the dip and strike slip distribution of the Chi Chi earthquake. We found that the tails of the probability density functions (PDF) characterizing the slip and the |PGA| are governed by a parameter α with almost the same values as predicted by the Central Limit theorem. These results suggest that the statistical properties of the slip spatial distribution and of the ground motions are coupled: the PDF of the |PGA| is a direct consequence of the PDF of the asperities [*Gusev*, 1989]. The coupling is physically based on the principle of superposition of wave signals characterized by random amplitudes that sum according to the Central Limit theorem.

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