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Chapter 2 The physical state of the lithosphere

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2.1 Stress and Strain

2.1.1 Stresses in the lithosphere

Body forces: Body forces **act throughout the volume** of the solid and are directly proportional to its volume or mass. Density is important.

Surface forces: Surface forces **act only on the surface** area bounding a volume and arise from the inter-atomic stresses exerted from one side of the surface to the other. The magnitude of the force depends on the surface area over which the force acts and the orientation of the surface.

The surface force acting on a unit area at the base of a vertical column of rock is given by

$$\sigma_{xx} = \rho g y$$

ρ = density

g = gravity acceleration

y = height of the column

The normal force on horizontal planes due to the weight of the rock overburden is lithostatic stress or lithostatic pressure.

Body and surface forces are directly related to rock density.

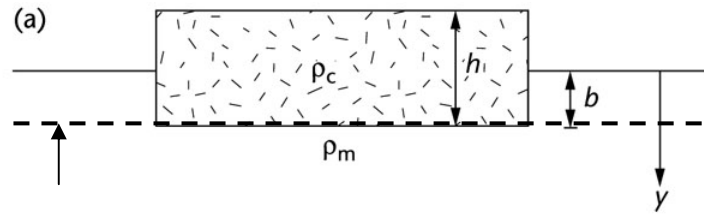
	Density kg m^{-3}	Young's modulus E 10^{11} Pa	Shear modulus G 10^{11} Pa	Poisson's ratio ν	Thermal conductivity $K \text{ W m}^{-1} \text{ K}^{-1}$	Coefficient of thermal expansion α , 10^{-5} K^{-1}
Sedimentary						
Shale	2100–2700	0.1–0.7	0.1–0.3	0.1–0.2	1.2–3	
Sandstone	1900–2500	0.1–0.6	0.04–0.2	0.1–0.3	1.5–4.2	3
Limestone	1600–2700	0.5–0.8	0.2–0.3	0.15–0.3	2–3.4	2.4
Dolomite	2700–2800	0.5–0.9	0.2–6.4	0.1–0.4	3.2–5	
Metamorphic						
Gneiss	2600–2850	0.4–0.6	0.2–0.3	0.15–0.25	2.1–4.2	
Amphibolite	2800–3150		0.5–1.0	0.4	2.1–3.8	
Marble	2670–2750	0.3–0.8	0.2–0.35	0.2–0.3	2.5–3	
Igneous						
Basalt	2950	0.6–0.8	0.25–0.35	0.2–0.25	1.3–2.9	
Granite	2650	0.4–0.7	0.2–0.3	0.2–0.25	2.4–3.8	2.4
Diabase	2900	0.8–1.1	0.3–0.45	0.25	2–4	
Gabbro	2950	0.6–1.0	0.2–0.35	0.15–0.2	1.9–4	1.6
Diorite	2800	0.6–0.8	0.3–0.35	0.25–0.3	2.8–3.6	
Pyroxenite	3250	1.0	0.4		4.1–5	
Anorthosite	2640–2920	0.83	0.35	0.25	1.7–2.1	
Granodiorite	2700	0.7	0.3	0.25	2.0–3.5	
Mantle						
Peridotite	3250				3–4.5	2.4
Dunite	3000–3700	1.4–1.6	0.6–0.7		3.7–4.6	
Miscellaneous						
Halite			0.3	0.15	5.4–7.2	13
Ice	917		0.092	0.31–0.36	2.2	5

Oceanic
Continental
crust

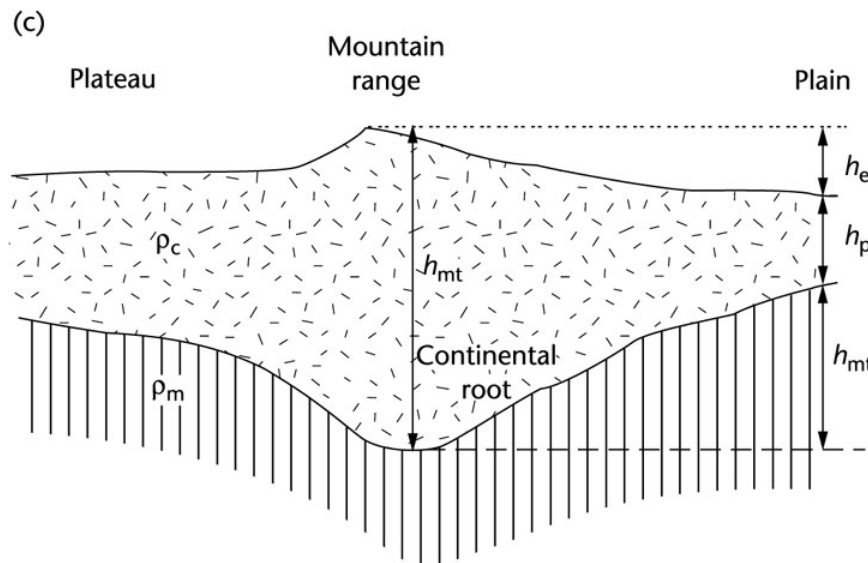
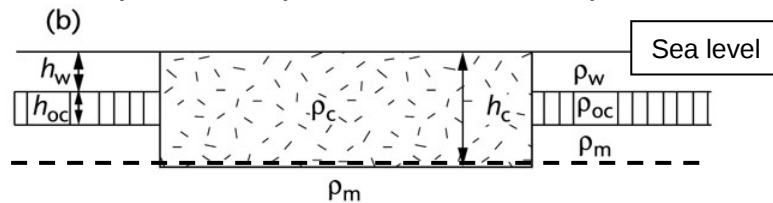


Mantle

Table. 2.1 Common physical properties of rocks (after Turcotte and Schubert 2002).



hydrostatic equilibrium (Archimedes Principle; local isostasy)



$$h\rho_c g = b\rho_m g$$

$$h - b = h \left(1 - \frac{\rho_c}{\rho_m} \right)$$

For $\rho_c = 2700 \text{ kgm}^{-3}$

$\rho_m = 2700 \text{ kgm}^{-3}$

$h = 35 \text{ km}$

$h - b = 6.4 \text{ km}$

$$h_c \rho_c g = h_w \rho_w g + h_{oc} \rho_{oc} g + (h_c - h_w - h_{oc}) \rho_m g$$

$$h_{oc} (\rho_m - \rho_{oc}) + h_w (\rho_m - \rho_w) = h_c (\rho_m - \rho_c)$$

For the above parameters and $\rho_{oc} = 2950 \text{ kgm}^{-3}$; $h_{oc} = 20 \text{ km}$. The water depth (h_w) is 6 km.

$$h_{mt} \rho_c g = h_p \rho_c g + (h_{mt} - h_p - h_e) \rho_m g$$

$$h_{mt} = h_p + h_c \left(\frac{\rho_m}{\rho_m - \rho_c} \right)$$

For the above parameters and $h_p = 35 \text{ km}$; $h_e = 5 \text{ km}$. The thickness of crust in the mountain belt h_{mt} is 62.5 km.

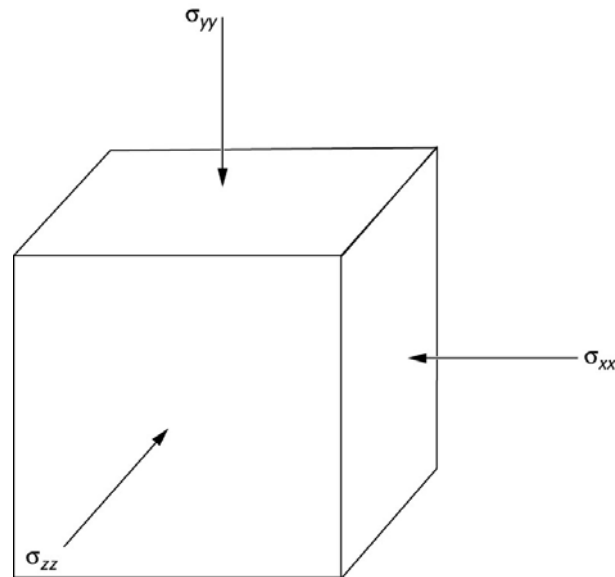
Fig. 2.1 Schematic diagrams illustrating the concepts of isostasy. (a) Continental block “floating” in a fluid mantle; (b) continental block flanked by oceanic crust and overlying water column; (c) continental mountain belt with a root surrounded by a plateau and low-lying plain.

If the normal surface forces σ_{xx} , σ_{yy} , σ_{zz} are all equal and they are also equal to the weight of overburden, the rock is in a lithostatic state of stress.

If a rock mass is being subjected to horizontal tectonic stress, then σ_{xx} is made of two terms:

$$\sigma_{xx} = \rho_c g y + \Delta \sigma_{xx}$$

Lithostatic term
↓
↑
Deviatoric stress
(tectonic contribution)



Shear stress: surface forces acting parallel to a surface. Examples: low angle thrust faults, gravitational sliding of a rock mass down an inclined plane.

σ_1 = maximum principle stress

σ_2 = intermediate principle stress

σ_3 = minimum principle stress

Fig. 2.2 Normal surface forces acting on vertical and horizontal planes.

At any point we can envisage three mutually perpendicular planes on which there are no shear stresses. Perpendicular to these planes are known as principal axes of stress.

σ_1 = maximum principal stress

σ_2 = intermediate principal stress

σ_3 = minimum principal stress

+ for compressional stress

- for extensional stress

States of stress:

- Uniaxial stress has a finite σ_1 , and $\sigma_2 = \sigma_3 = 0$
- biaxial or plane stress has $\sigma_1 > \sigma_2$ and $\sigma_3 = 0$
- triaxial stress is the general stress state $\sigma_1 > \sigma_2 > \sigma_3$

Hydrostatic state of stress: $\sigma_1 = \sigma_2 = \sigma_3$

Non-hydrostatic state of stress: $\longrightarrow p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$

Subtraction of the mean stress (or pressure) from the normal stress component reveals the deviatoric normal stresses.

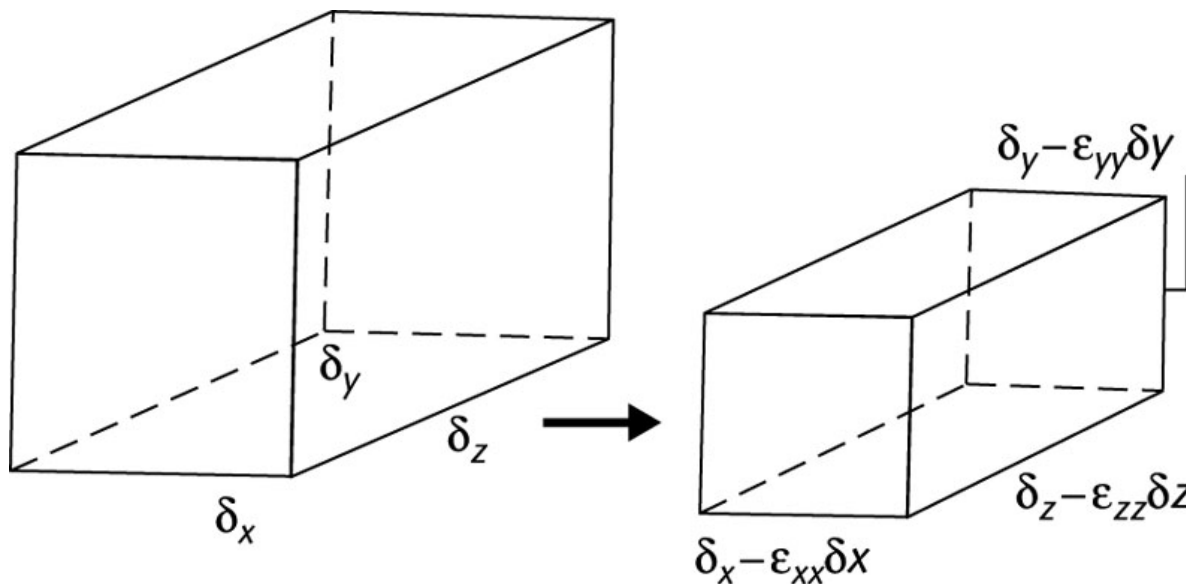
2.1.2 Strain in the lithosphere

Strain is the deformation of a solid caused by the application of stress.

Types of strain:

- Dilation: change of volume
- Displacement: change of position
- Shear strain: change of shape

ϵ_{xx} , ϵ_{yy} , ϵ_{zz} are the strains in the x, y, and z directions.



If the change of volume is small, the dilation is simply the sum of the strain component:

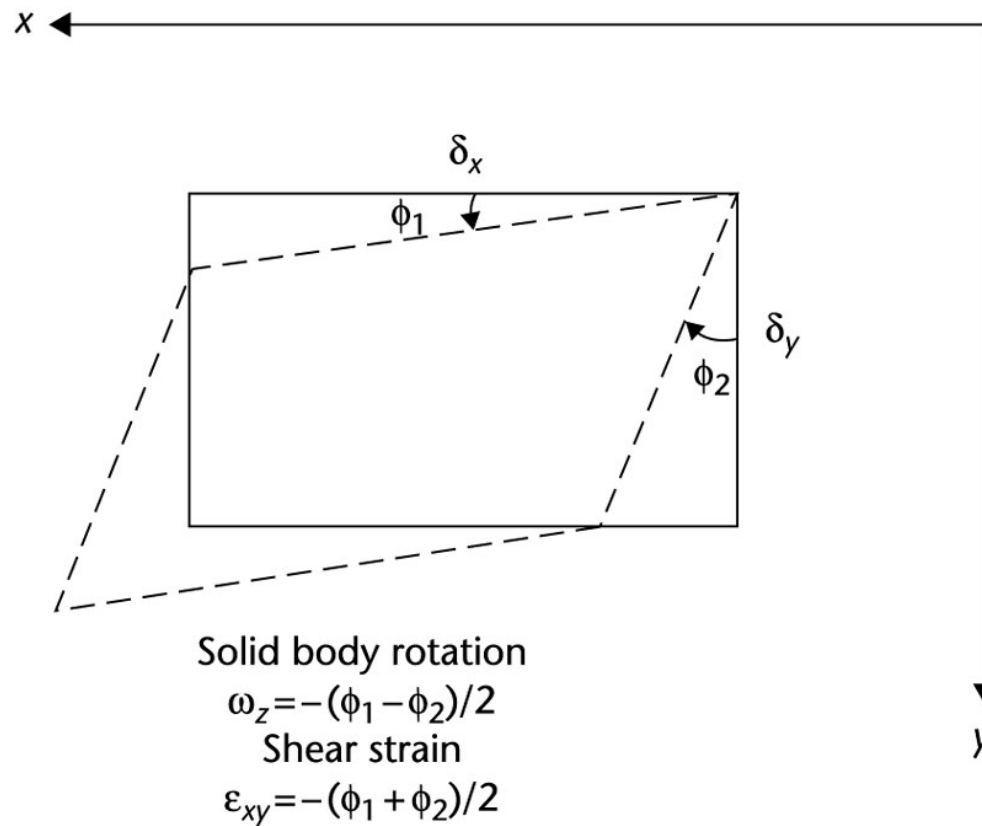
$$\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$$

Dilation: change of volume

Fig. 2.3 A rectangular block that **changes its dimensions** but not its shape: this is a deformation involving no shear.

Shear strain is dependent on the amount of rotation of the sides of the rectangular element. ϕ_1, ϕ_2

The deformation of an element can be described according to shear strain and solid body rotation.



Solid body rotation: ϕ_1, ϕ_2 not equal.

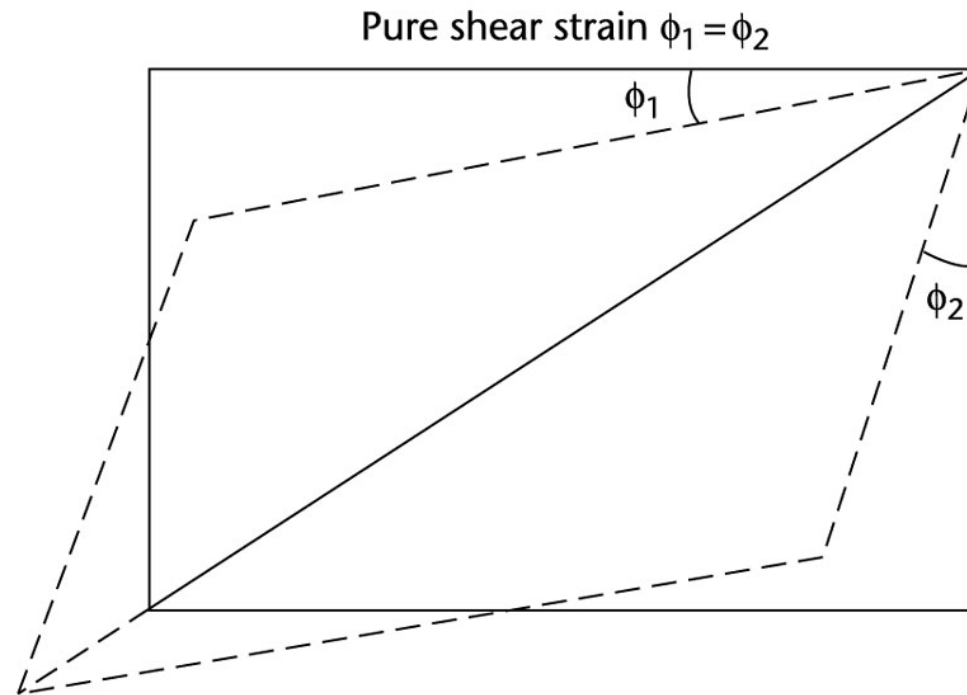
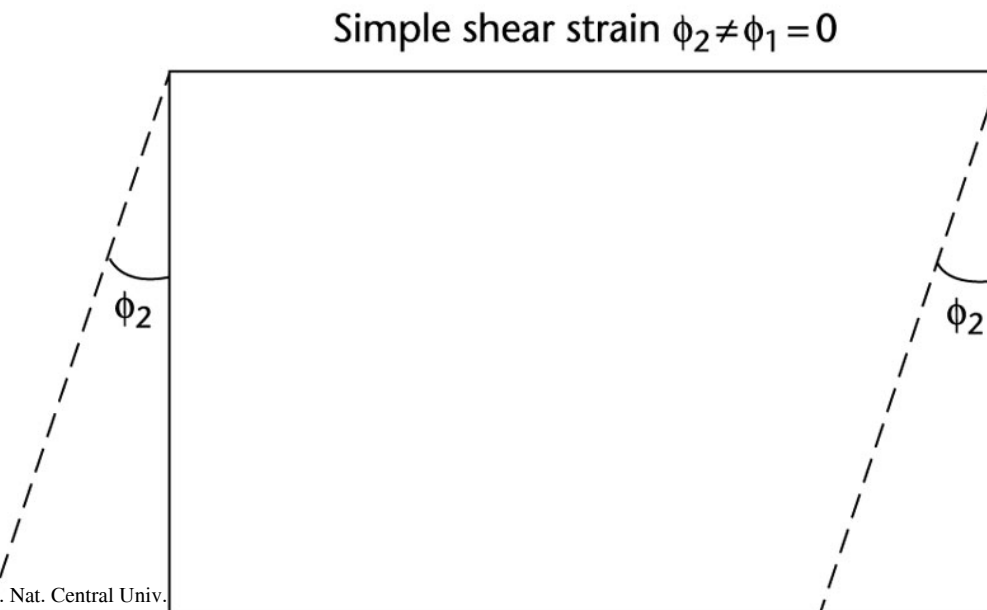


Fig. 2.5 Difference between pure shear strain (no solid body rotation) and simple shear strain (solid body rotation is $\phi_2/2$).

→ Uniform extension of lithosphere with depth



→ Asymmetrical extension associated with translithospheric shear zone

Principal axes of strain: As in the case of shear stresses, shear strains can be described with reference to a coordinate system that is orientated such that shear strain components are zero. Such a system contains the principal axes of strain.

Principal strain: The fractional changes in length along the directions of the principal axes. In the 3-D case the condition of isotropic strain is:

$$e = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) / 3 = \Delta / 3$$

where Δ is the dilation and e is the mean normal strain.

It is very rare, however, for strain to be homogeneous.

Deviatoric strain: Strains that are the difference between the actual strain and the mean normal strain.

States of strains:

- **uniaxial strain** is where there is only one nonzero component of principal strain, that is, assuming the nonzero axis to be ε_1 , and $\varepsilon_2 = \varepsilon_3 = 0$
- **plane strain** is where only one of the principal strain components is zero, for example, $\varepsilon_3 = 0$, and ε_1 and ε_2 are nonzero.

Plane strain is commonly assumed for the deformation of lithosphere since the strain in the direction of an infinite plate will be zero.

2.1.3 Linear elasticity

Elastic materials (for low T , P and stress conditions) deform when they are subjected to a force and regain their original shape and volume when the force is removed. The relation between the stress and strain is linear: $F=kx$ (Hooke's law).

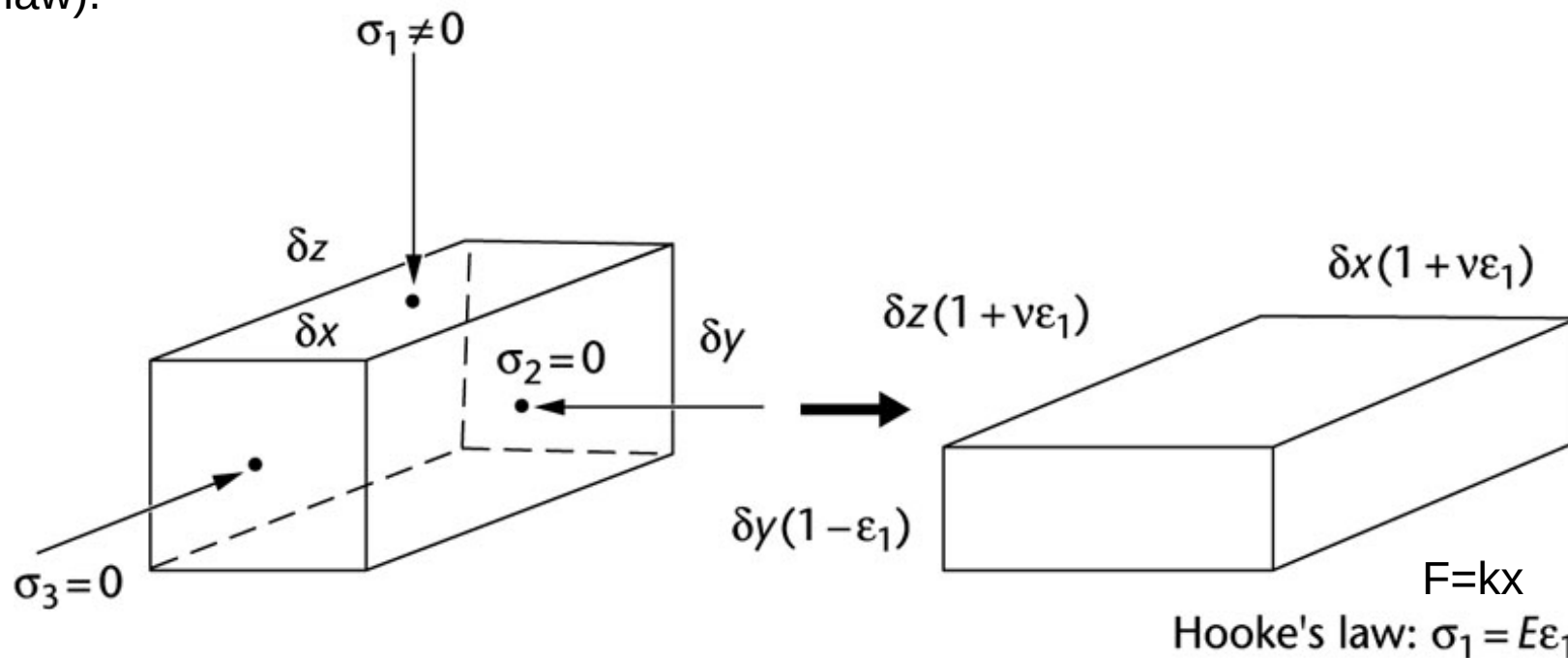


Fig. 2.6 Deformation under a **uniaxial stress**. Contraction in the direction of the compressive stress σ_1 is compensated by extension in the two orthogonal directions.

In a linear, isotropic, elastic solid the stresses are linearly proportional to strains. The relation between the principal strain and the components of principal stress is:

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E} - \frac{\nu\sigma_3}{E}$$

$$\varepsilon_2 = -\frac{\nu\sigma_1}{E} + \frac{\sigma_2}{E} - \frac{\nu\sigma_3}{E}$$

$$\varepsilon_3 = -\frac{\nu\sigma_1}{E} - \frac{\nu\sigma_2}{E} + \frac{\sigma_3}{E}$$

The strain is dependent on *Young's modulus* (E) and *Poisson's ratio* (ν).

A principal stress produces a strain component σ/E along the same axis and strain component $-\nu \sigma/E$ along the two other orthogonal axes.

For uniaxial stress:

$$\varepsilon_1 = \frac{\sigma_1}{E}$$

$$\varepsilon_2 = -\frac{\nu\sigma_1}{E}$$

$$\varepsilon_3 = -\frac{\nu\sigma_1}{E}$$

For isotropic state of stress:

Bulk modulus (k) and its reciprocal β , the *compressibility*. These parameters give the fractional volume change during isotropic compression under a given pressure.

$$K = \frac{1}{\beta} = \frac{E}{3(1-2\nu)}$$

If ν approaches 0.5 the bulk modulus tends to infinity, that is, the material becomes essentially incompressible.

2.1.4 Flexure in two dimensions

Concepts involved in the flexure of an elastic solid:

1. Flexure results from vertical forces, horizontal forces and torques (bending moments) in any combination. Horizontal loads are commonly neglected, perhaps unwisely, in geodynamical problems.
2. The bending moment is the integration of the fiber (normal) stresses on cross-sections of the plate acting over the distance to the midline of the plate. The bending moment is related to the local radius of curvature of the plate by a coefficient called the flexural rigidity. Flexural rigidity is proportional to the cube of the equivalent elastic thickness. When applied to the lithosphere the equivalent elastic thickness does not represent a real physical discontinuity.
3. A general flexural equation can be derived which expresses the deflection of the plate in terms of the vertical and horizontal loads, bending moment and flexural rigidity. This equation can readily be adapted for use in the study of geological problems.

Derivation of flexural rigidity and the general flexure equation

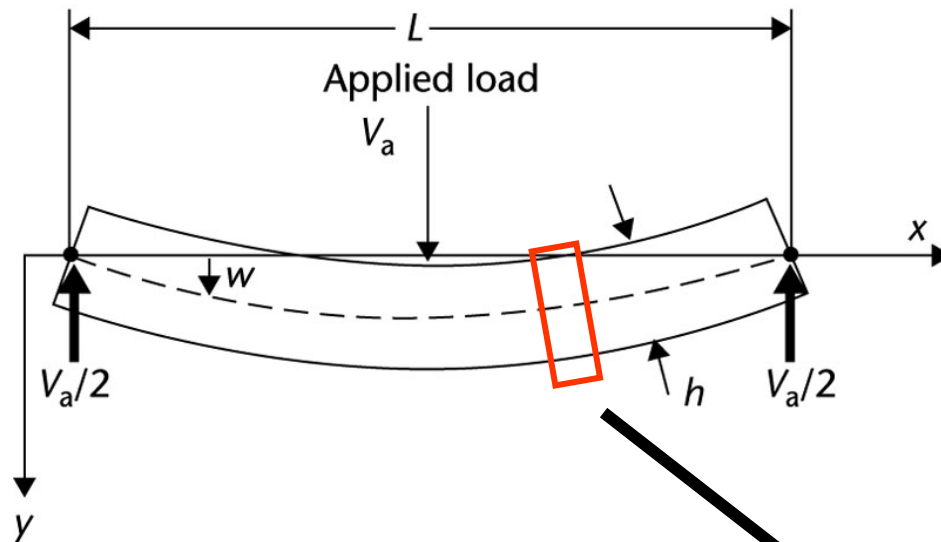


Fig. 2.8 Configuration and notation for a thin plate, pinned at both ends, bending under an applied load.

$q(x)$: downward force per unit area.

V : net shear force per unit length. V is a function of x .

P : horizontal force per unit length.

M : bending moment.

V_a : vertical line load
 h : elastic thickness
 w : amount of vertical deflection

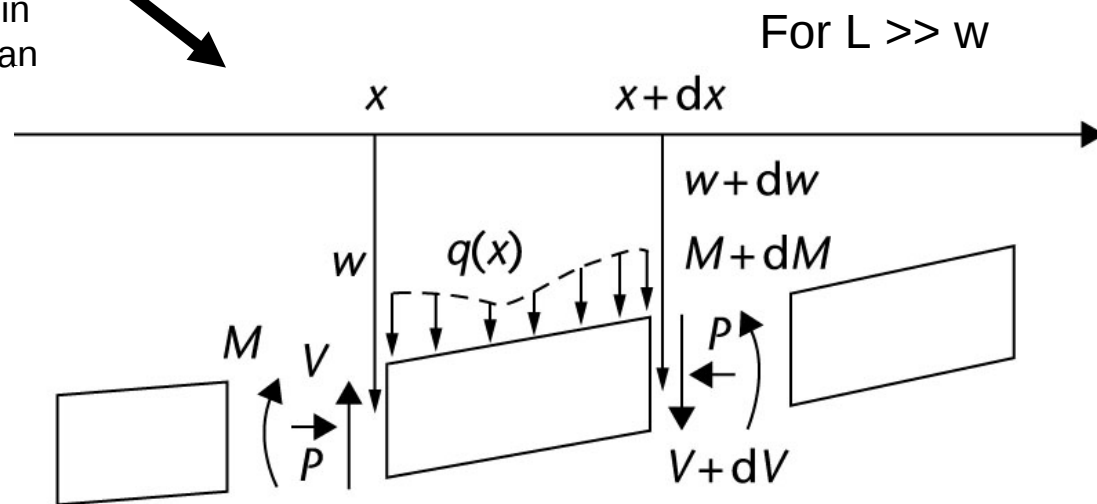


Fig. 2.8 Forces on a small element of a flexed plate. The forces can be balanced vertically and in terms of their tendency to rotate the element (moments or torques).

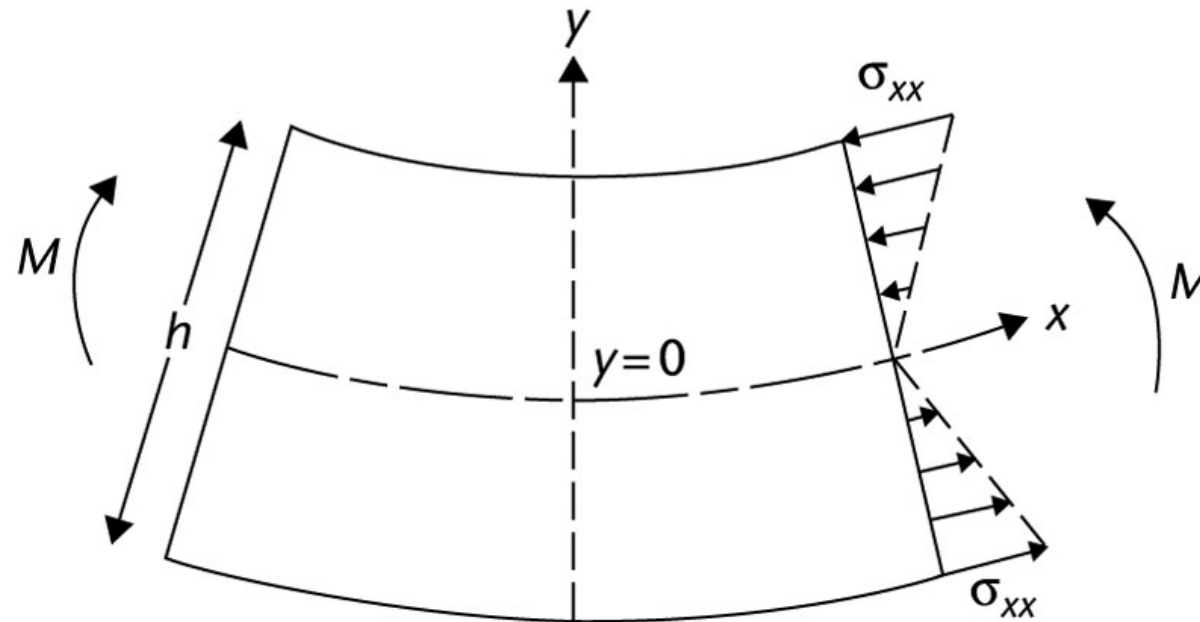


Fig. 2.9 Normal stresses (σ_{xx}) on an end section of a flexed plate. These normal or fiber stresses exert torques about the midpoint of the plate, which when integrated over the end section (from $-h/2$ to $+h/2$) gives the **bending moment (M)**.

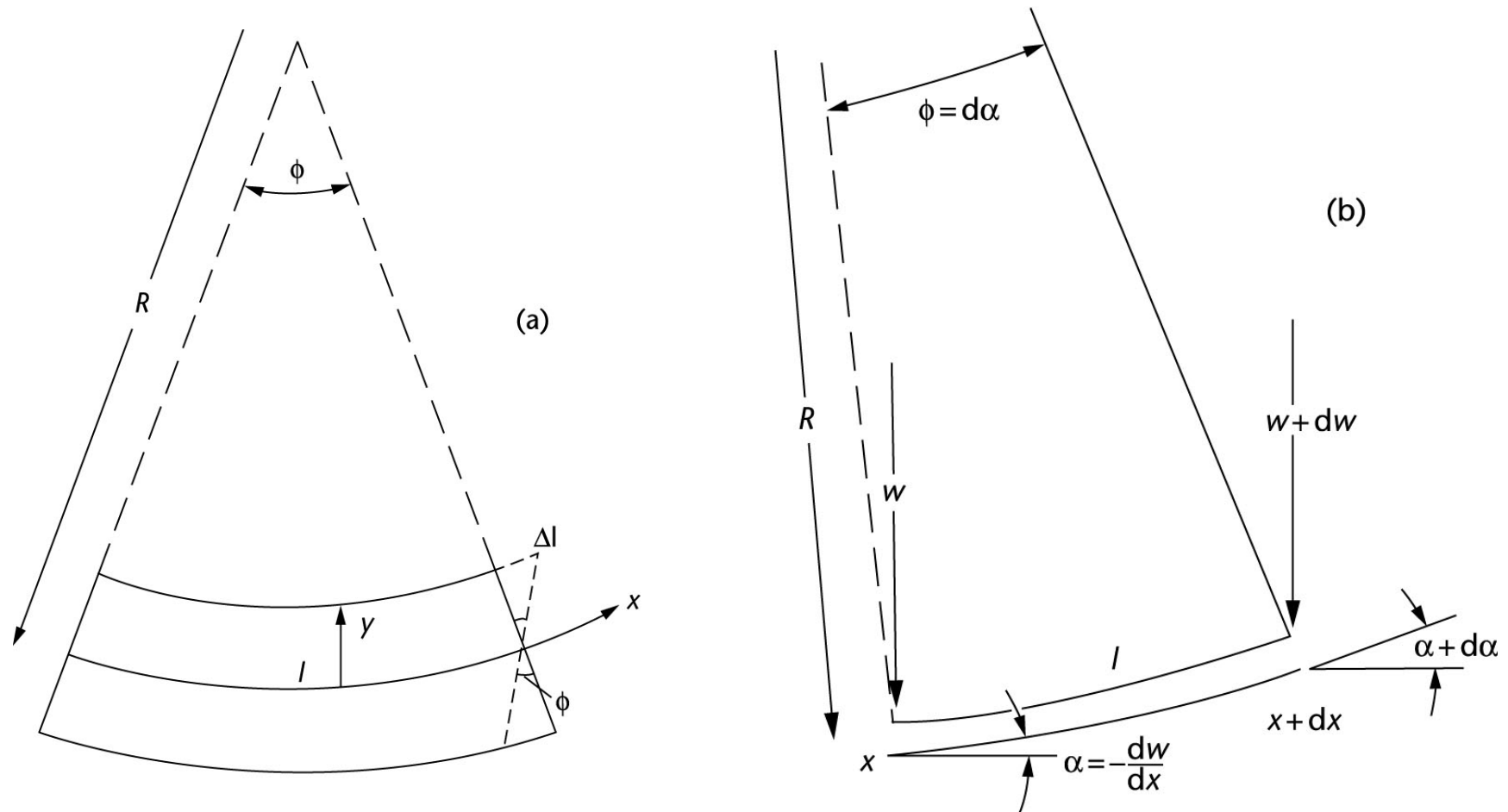


Fig. 2.10 Geometrical aspects of plate bending. (a) Longitudinal strain (extension below the midplane and contraction above the midplane of the plate) is a function of the distance from the midplane of the plate y , and the angle ϕ ; (b) Notation to show that the second derivative of the deflection (d^2w/dx^2) gives the rate of change of slope of the plate; this is inversely related to the local radius of curvature R of the plate.

General flexural equation for the deflection of a plate:

$$D \frac{d^4 w}{dx^4} = q(x) - P \frac{d^2 w}{dx^2}$$

D (flexural rigidity): $\longrightarrow$

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

h: elastic thickness

w: vertical deflection at location x

q(x): downward force per unit area

P: horizontal force per unit length

When applies the general flexural equation to deformation of the lithosphere the lithosphere is floating upon weak asthenosphere and the deflection may be filled with water or sediments. The general flexural equation then becomes:

$$q_h = \text{Upward hydrostatic force} = (\rho_m - \rho_w)gw$$

$$q = \text{Net force} = q_a - (\rho_m - \rho_w)gw$$

$$D \frac{d^4 w}{dx^4} + P \frac{d^2 w}{dx^2} + \Delta \rho g w = q(x)$$

Buoyancy force

$$\Delta \rho = \rho_m - \rho_w \text{ for water-filled deflection}$$

$$\Delta \rho = \rho_m - \rho_c \text{ for sediment-filled deflection}$$

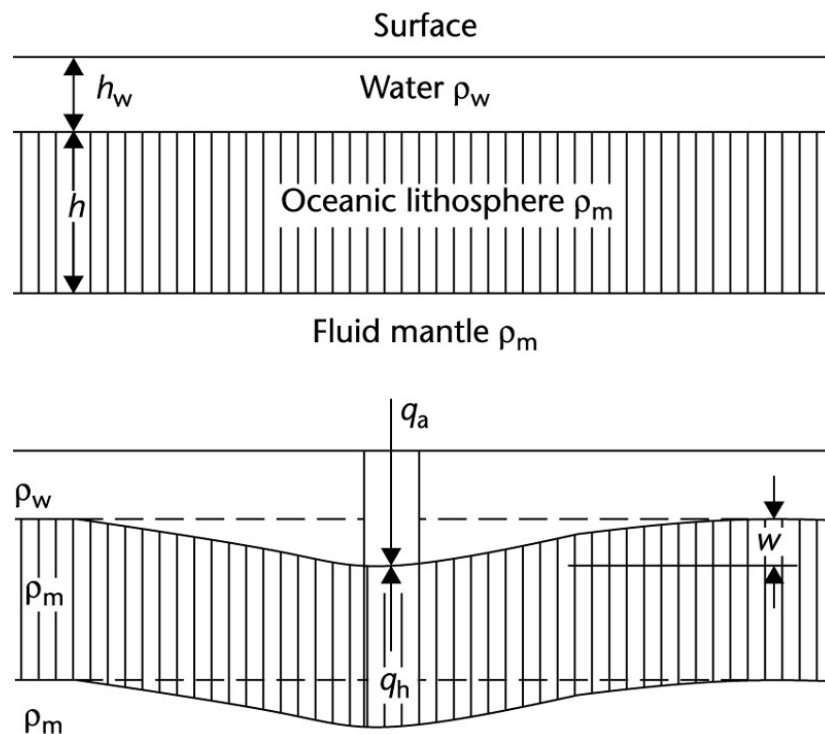
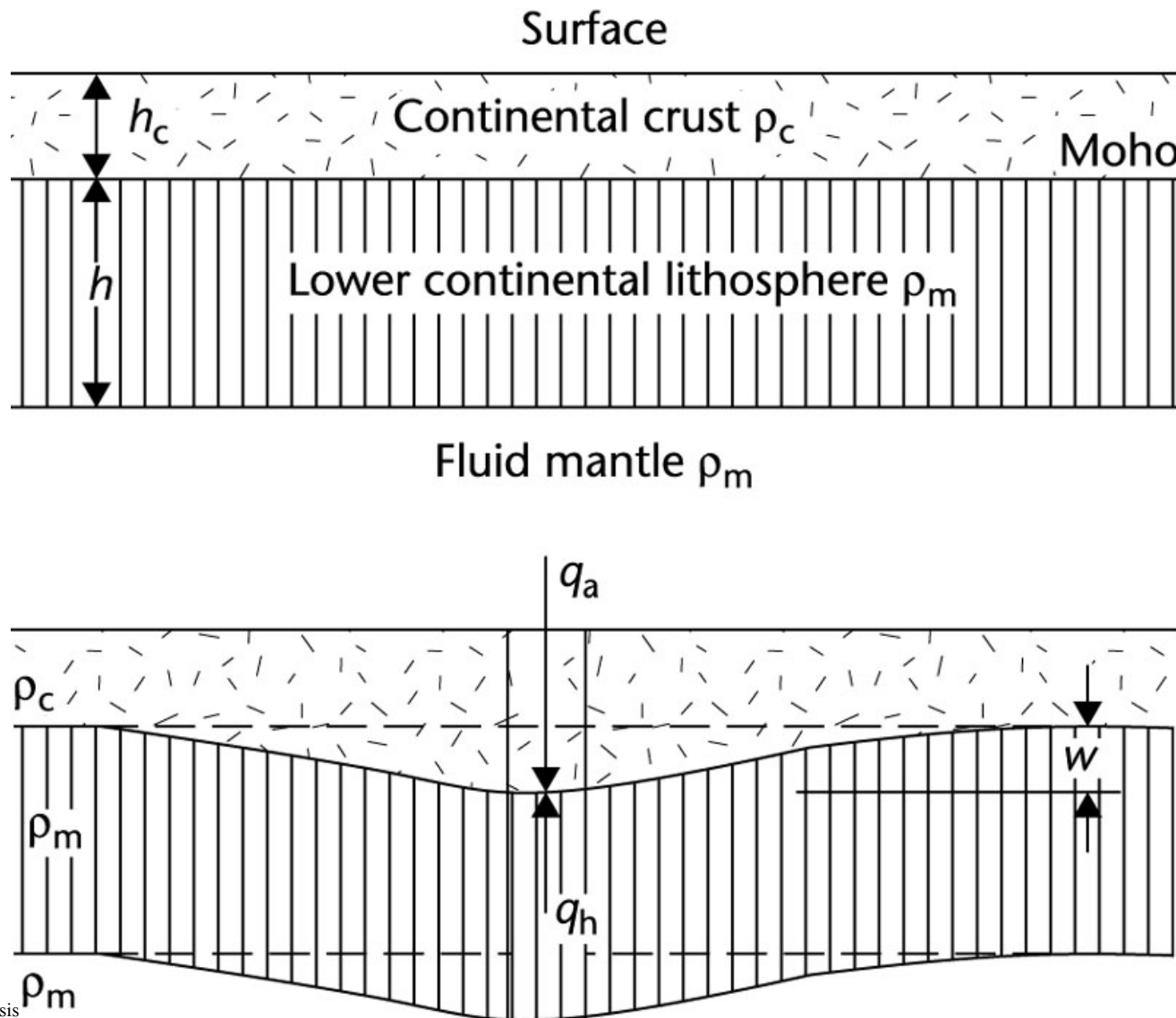


Fig. 2.11 Model for calculating the upward-acting hydrostatic restoring force on an oceanic plate, overlain by water, deflected by an applied force q_a .

$$q_h = \text{Upward hydrostatic force} = (\rho_m - \rho_w)gw$$

$$q = \text{Net force} = q_a - (\rho_m - \rho_c)gw$$

Fig. 2.12 Model for calculating the hydrostatic restoring force on the base of some continental crust where the deflection caused by the applied load q_a is assumed to be filled with material of the same density as the continental crust. This therefore approximates the case of a sediment-filled sedimentary basin on continental lithosphere.



2.2 Heat flow: conduction and convection

2.2.1 Fundamentals

Heat transfer in Earth due to processes of conduction and convection.

Conduction: operating mainly in the lithosphere. It is a diffusive process whereby kinetic energy is transferred by intermolecular collisions.

Convection: operating in the mantle and core. It requires motion of the medium to transmit heat.

Fourier's law of conductive heat transport:

$$Q = -K \frac{dT}{dy}$$

	Mean surface heat flow	
	mW m ⁻²	HFU
Continents:	56.6	1.35
Africa	49.8	1.19
North America	54.4	1.30
Australia	63.6	1.52
Oceans:	78.2	1.87
North Pacific	95.4	2.28
Indian Ocean	83.3	1.99
South Atlantic	59.0	1.41
Worldwide	69.9	1.67

Q: heat flow, unit: mW m⁻² or cal cm⁻² s⁻¹,
1 HFU (heat flow unit) = 41.84 mW m⁻²
or 10⁻⁶ cal cm⁻² s⁻¹

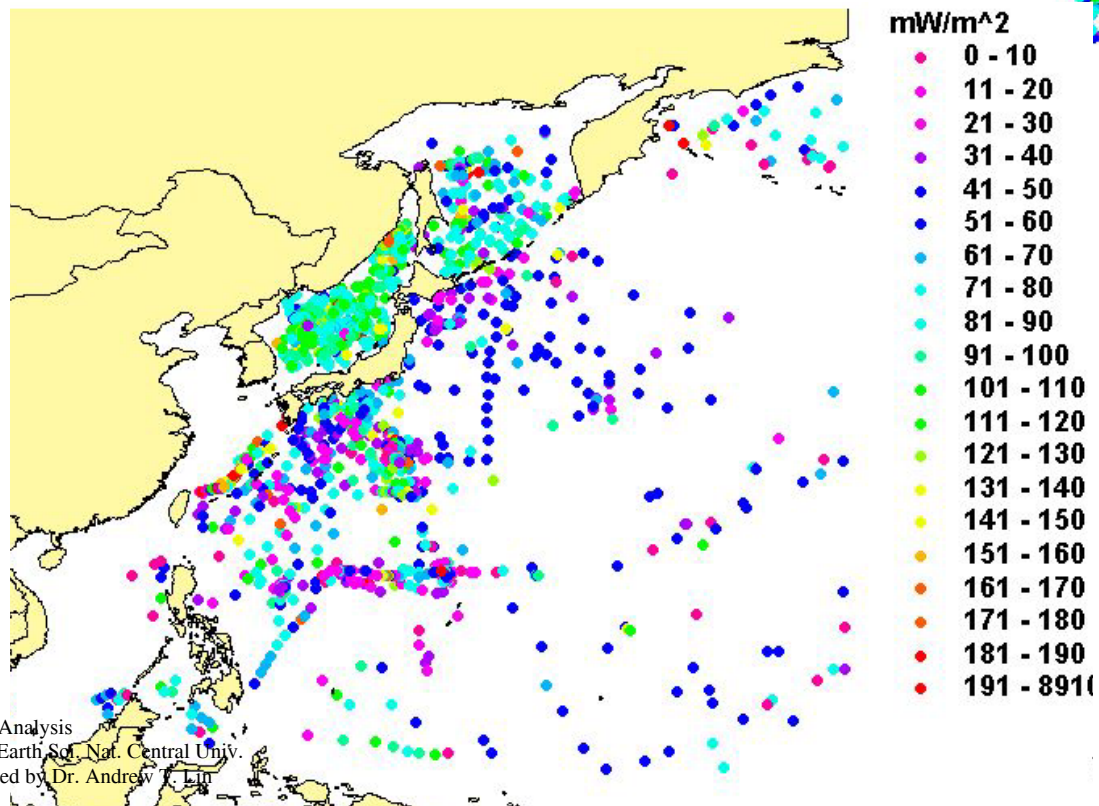
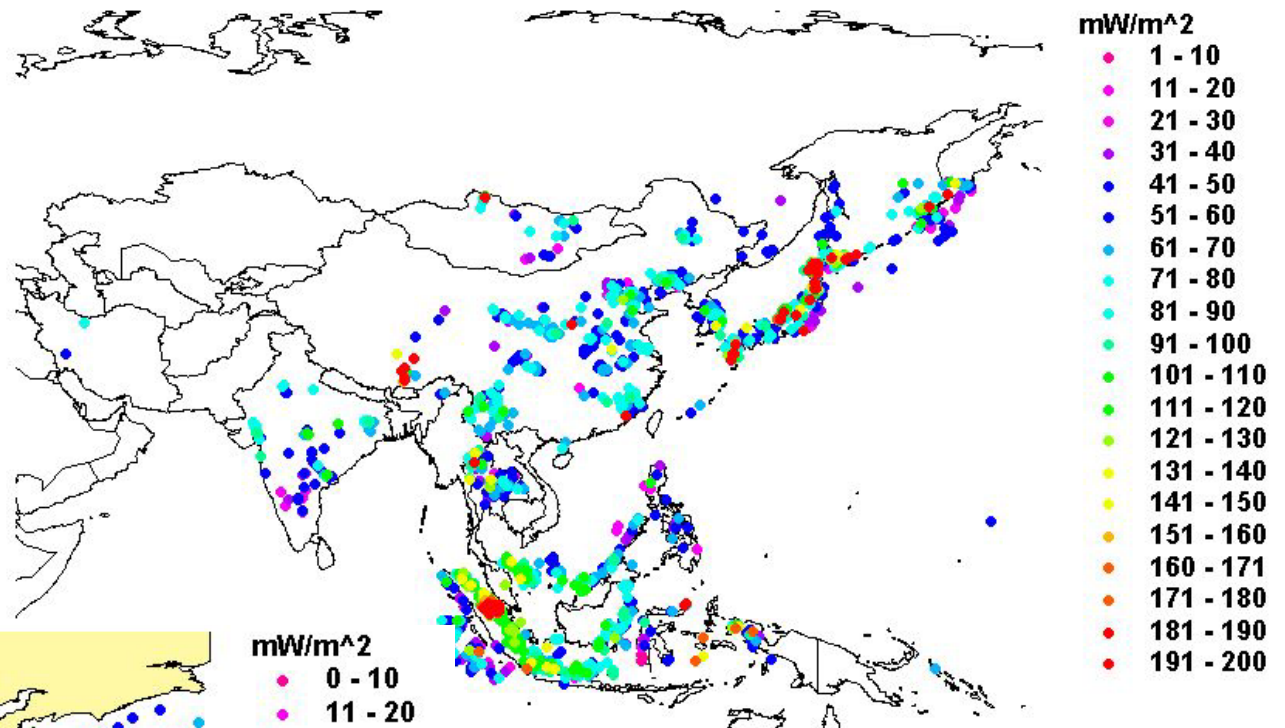
K: thermal conductivity, unit: Wm⁻¹°C⁻¹ or
cal cm⁻¹°C⁻¹

T: temperature

y: coordinate in the direction of T variation

Table 2.2 Regional variations in surface heat flow (data from Sclater et al. 1980a).

Heat flows in Asia



Heat flows in the Western Pacific

From global heat flow database at:

<http://www.heatflow.und.edu/index2.html>

Continents

Regions of high heat flow on the continent: active volcanic areas, stretched lithosphere.

Continental collision zone typically have low to normal surface heat flows.

In areas devoid of active tectonics and vulcanicity, the heat flow appears to be inversely correlated to the age of the rocks. This is because of the decreasing abundance with age of the radioactive heat-producing isotopes of uranium, thorium and potassium. Rock types (concentration of radioactive elements): granite, higher heat flow; basalt, lower heat flow.

Oceans

Surface heat flows are related not to the concentration of radioisotopes but to the **age** of the seafloor. Mean surface heat flows in ocean is slightly higher than their continental counterparts.

	Mean surface heat flow	
	mW m ⁻²	HFU
Continents:	56.6	1.35
Africa	49.8	1.19
North America	54.4	1.30
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Oceans:	78.2	1.87
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Worldwide	69.9	1.67

Table 2.2 Regional variations in surface heat flow (data from Sclater et al. 1980a).

Table. 2.3 Typical concentrations of radioactive elements and heat generation from typical rock types comprising the continental and oceanic crust and undepleted mantle (from Fowler 1990).

	Granite	Tholeiitic basalt	Alkali basalt	Peridotite	Average continental upper crust	Average oceanic crust	Undepleted mantle
<i>Concentration by weight</i>							
U (ppm)	4	0.1	0.8	0.006	1.6	0.9	0.02
Th (ppm)	15	0.4	2.5	0.04	5.8	2.7	0.10
K (%)	3.5	0.2	1.2	0.01	2.0	0.4	0.02
<i>Heat generation ($10^{-10} \text{ Wkg}^{-1}$)</i>							
U	3.9	0.1	0.8	0.006	1.6	0.9	0.02
Th	4.1	0.1	0.7	0.010	1.6	0.7	0.03
K	1.3	0.1	0.4	0.004	0.7	0.1	0.007
Total	9.3	0.3	1.9	0.020	3.9	1.7	0.057
Density (10^3 kg m^{-3})	2.7	2.8	2.7	3.2	2.7	2.9	3.2
Heat generation (10^{-6} Wm^{-3})	2.5	0.08	0.5	0.006	1.0	0.5	0.02

Some terminology:

Tholeiitic basalt: It is rich in silica and is dominated by clinopyroxene plus plagioclase, with minor iron-titanium oxides. Tholeiitic basalts are produced by submarine volcanism at mid-ocean ridges and make up much of the ocean crust. **MORB**, the acronym for typical **mid-ocean-ridge basalt**, is a type of tholeiitic basalt particularly low in incompatible elements. In contrast, alkali basalt is not typical at ocean ridges, but is erupted on some oceanic islands and on continents. Tholeiitic basalts also occur as plateau lavas on the continental crust.

Alkali basalt: Basalts with nepheline and/or acmite. Poor in silica (Si) and rich in sodium (Na).

Depleted mantle: A part of the mantle in which the concentrations on incompatible elements have been decreased, usually by partial melting. (Partial melting probably never exceeds 35%)

Incompatible element: During the fractional crystallization of magma, and magma generation by the partial melting of the Earth's mantle and crust, elements that have difficulty in entering cation sites of the minerals are concentrated in the melt phase of magma (liquid phase). An **incompatible element** is an element that is unsuitable in size and/or charge to the cation sites of the minerals, and is defined by the partition coefficient D between rock-forming minerals and melt being much smaller than 1.

Table. 2.4 Relative abundance of isotopes and crustal heat generation as a functions of age (from Jessop and Lewis 1978).

Shorter half-life

Age (Ma)	Relative abundance					Heat generation	
	²³⁸ U	²³⁵ U	U*	Th	K	Model A [†]	Model B [‡]
Present	1.00	1.00	1.00	1.00	1.00	1.00	1.00
500	1.08	1.62	1.10	1.03	1.31	1.13	1.17
1000	1.17	2.64	1.23	1.05	1.70	1.28	1.37
1500	1.26	4.30	1.39	1.08	2.34	1.48	1.64
2000	1.36	6.99	1.59	1.10	2.91	1.74	1.98
2500	1.47	11.4	1.88	1.13	3.79	2.08	2.43
3000	1.59	18.5	2.29	1.16	4.90	2.52	3.01
3500	1.71	29.9	2.88	1.19	6.42	3.13	3.81

Notes:

* Assuming a present isotopic composition of 99.2886% ²³⁸U and 0.7114% ²³⁵U.

† Model A based on Th/U = 4, K/U = 20,000.

‡ Model B based on Th/U = 4, K/U = 40,000.

Earth heat production is decreasing with increasing age. Heat production was twice or three times the present value 3000 Myr ago.

2.2.2 One-dimensional heat conduction

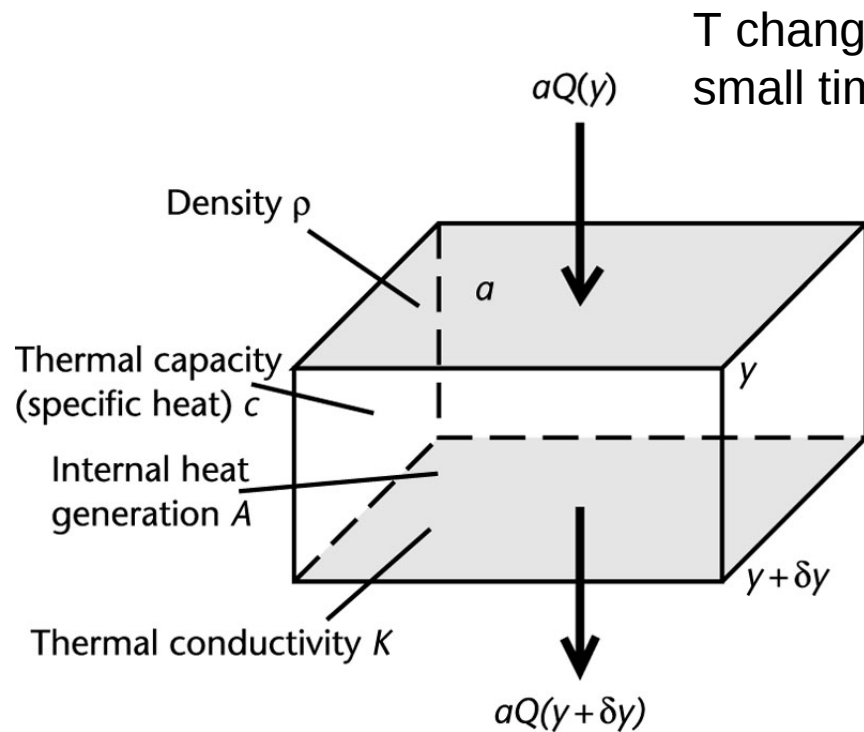


Fig. 2.13 Set-up and notation for conductive heat flow through a volume element of thickness δy , cross-sectional area a , density ρ , **thermal capacity (specific heat) c** , thermal conductivity K and internal heat generation A . heat is conducted across the shaded faces only (it is perfectly 1-D).

1-D heat conduction equation

$$\frac{\partial T}{\partial t} = \frac{A}{\rho c} + \frac{K}{\rho c} \frac{\partial^2 T}{\partial y^2}$$

internal heat generation term

basal heat flow term

κ = thermal diffusivity = $\frac{K}{\rho c}$, the ability of a material to gain or lose heat.
Unit: $\text{length}^2\text{time}^{-1}$.

t = duration of time.

A = heat generated internally per unit volume and unit time.

c = thermal capacity (specific heat), amount of heat needed to raise the T of 1 kg of material by 1°C .

Unit: $\text{Wkg}^{-1}\text{C}^{-1}$.

Some boundary conditions:

(a) Steady state solution: constant heat flux (i.e. constant basal heat flux and internal heat generation) there is no temperature change with time, i.e., $\frac{\partial T}{\partial t} = 0$

$$\frac{\partial^2 T}{\partial y^2} = -\frac{A}{K}$$

No advective term (such as the stable cratonic lithosphere).

(b) No internal heat generation, $A=0$ (such as in the cooling oceanic lithosphere)

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c} \frac{\partial^2 T}{\partial y^2} = \kappa \frac{\partial^2 T}{\partial y^2} \leftarrow \text{Diffusion equation}$$

(C) The crust may be in motion (e.g., a mountain belt subjected to uplift and erosion or a subducting slab). If the vertical motion is at a velocity of u_y in the y direction:

The advective term may be the movement towards the surface of a volume element of rock associated with the downcutting action of erosion. Or, with a different sign, it could be the velocity of deposition.

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c} \frac{\partial^2 T}{\partial y^2} + \frac{A}{\rho c} - u_y \frac{\partial T}{\partial y}$$

basal heat flow term

(e.g. heat from the mantle)

internal heat generation term

advective term

2.2.3 Geotherms

The variation of temperature with depth is called the geotherm.

For conductive lithosphere, using 1-D conduction and **assuming steady state conditions** (i.e., constant basal heat flux, heat generation, and crustal rocks do not move), the geotherm can be expressed in these two conditions:

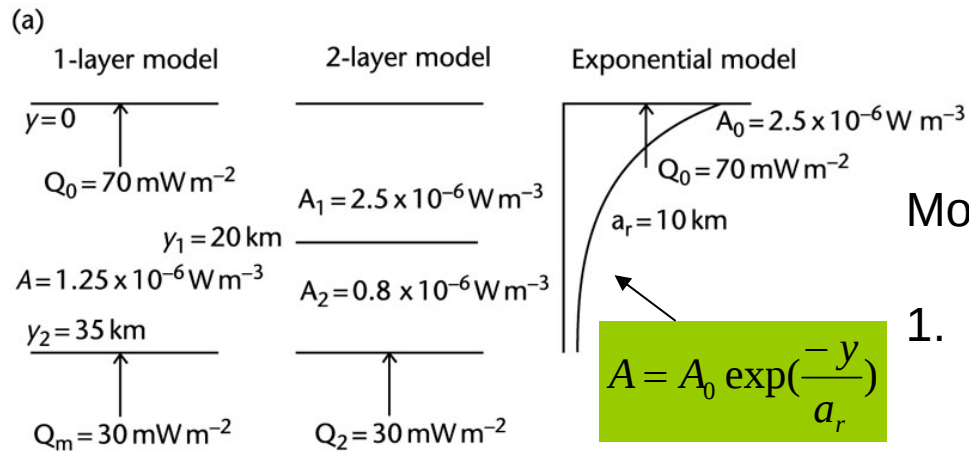
For temperature (T_0) and heat flow (Q_0) on earth surface are known

$$T = T_0 + \frac{Q_0}{K} y - \frac{A}{2K} y^2$$

For surface temperature (T_0) and basal heat flow (Q_m , heat from mantle) are known. y_c is the depth to the base of the crust (i.e. depth of Moho).

$$T = T_0 + \frac{(Q_m + Ay_c)}{K} y - \frac{A}{2K} y^2$$

Basal heat flow may be obtained from forward modeling of lithosphere stretching.



Radiogenic heat production (A)

Models for describing radiogenic heat production:

1. Slab model: The lithosphere and sedimentary cover can be dealt with as “slabs” of constant internal heat generation.
2. The internal heat generation can be modeled as decreasing exponentially with depth.

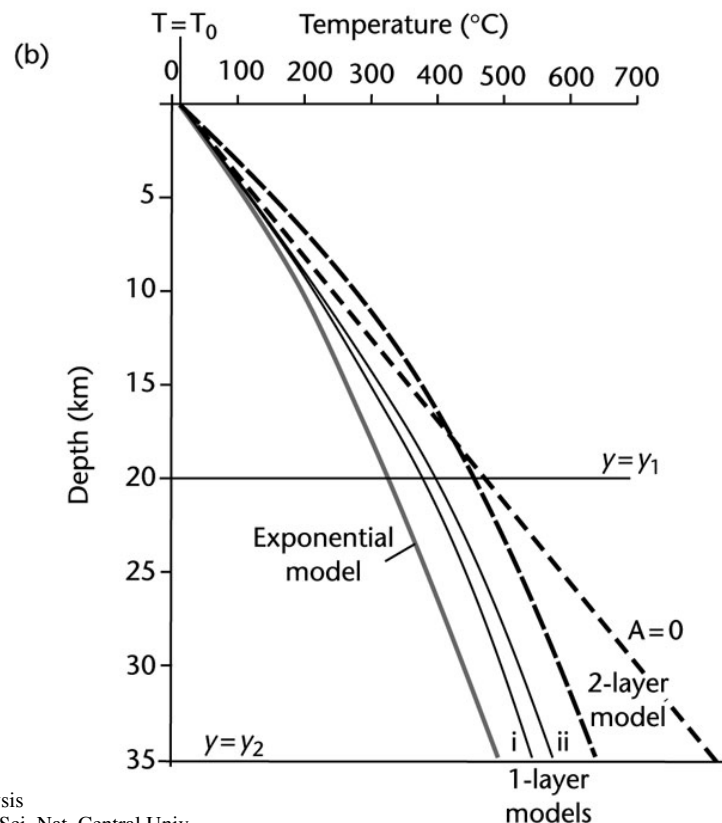


Fig. 2.14 Geotherms for the continental crust. (a) Model set-up; one-layer model with a constant internal heat generation for a 35 km-thick crust, with (i) a surface heat flow of 70 mWm^{-2} , and (ii) a basal heat flow of 30 mWm^{-2} ; a two-layer model with a highly radiogenic 20 km-thick upper crust and a weakly radiogenic 15 km-thick lower crust; and a model with an exponentially decreasing radiogenic heat production with depth; (b) Resultant geotherms, with a linear geotherm for zero radiogenic heat production ($A=0$) and surface heat flow of 70 mWm^{-2} shown for comparison.

Slab models:

One-layer:

$$\left\{ \begin{array}{l} T = T_0 + \frac{Q_0}{K} y - \frac{A}{2K} y^2 \quad \text{if } Q_0 \text{ is known, see "curve i" shown in Fig. 2.14} \\ T = T_0 + \frac{(Q_m + Ay_c)}{K} y - \frac{A}{2K} y^2 \quad \text{if } Q_m \text{ is known, see "curve ii" shown in Fig. 2.14} \end{array} \right.$$

Two layers:

$$\left\{ \begin{array}{l} T = T_0 + \left\{ \frac{Q_2}{K} + \frac{A_2(y_2 - y_1)}{K} + \frac{A_1 y_1}{K} \right\} y - \frac{A_1}{2K} y^2 \quad \text{For depth range between } y=0 \text{ and } y = y_1. \\ T = T_0 + \left\{ \frac{Q_2}{K} + \frac{A_2 y_2}{K} \right\} y + \left\{ \frac{A_1 - A_2}{2K} \right\} y_1^2 - \frac{A_2}{2K} y^2 \quad \text{For depth range between } y_1 \text{ and } y_2. \end{array} \right.$$

Exponential distribution of radiogenic heat generation: See the exponential model curve

$$T = T_0 + \frac{Q_m}{K} y + \frac{A_0 a_r^2}{K} \left\{ 1 - \exp\left(\frac{-y}{a_r}\right) \right\}$$

$$T = T_0 + \frac{Q_m}{K} y + \frac{(Q_0 - Q_m) a_r}{K} \left\{ 1 - \exp\left(\frac{-y}{a_r}\right) \right\}$$

a_r = length scale (between 4.5 km and 16 km) for the exponential decrease in radiogenic heat production. This parameter does not correspond to any physical boundary of the crust.

↑ if Q_0 is known

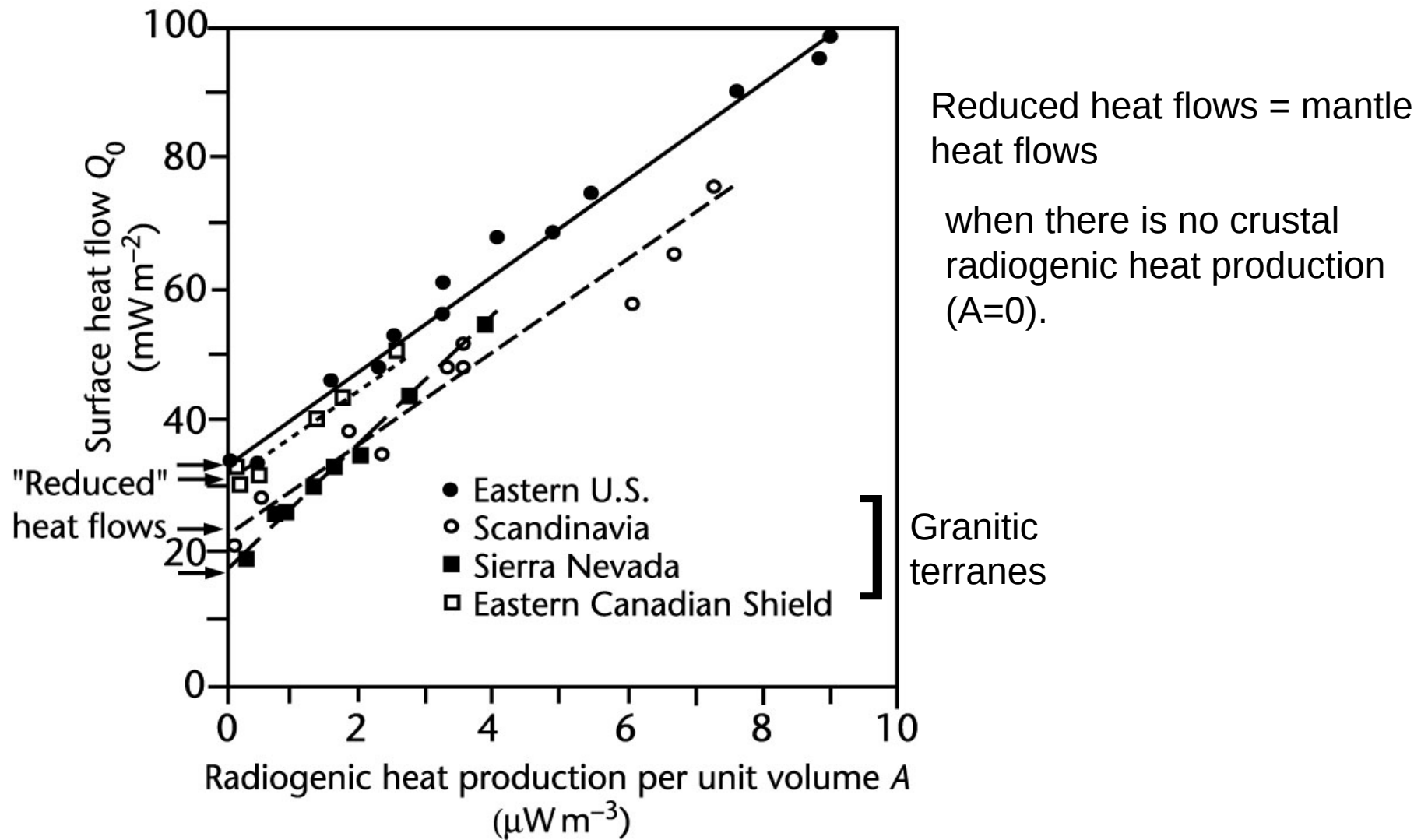


Fig. 2.15 The dependence of the surface heat flow Q_0 on the internal radiogenic heat production rate A in a number of geological provinces. The intercept on the y-axis gives the so-called “reduced” heat flow ($20\text{--}30 \text{ mW m}^{-2}$). Derived from Roy et al. (1968), Turcotte and Schubert (2002). Reproduced courtesy of Cambridge University Press.

2.2.4 Time-dependent heat conduction: the case of cooling oceanic lithosphere

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c} \frac{\partial^2 T}{\partial y^2} = \kappa \frac{\partial^2 T}{\partial y^2} \quad \leftarrow \text{Diffusion equation}$$

$$\tau = \frac{l^2}{\kappa}$$

τ : the time necessary for a temperature change to propagate a distance l in a material with a thermal diffusivity κ .

l : *thermal diffusion distance*: the distance that a temperature change propagates in time τ .

$$l = \sqrt{\kappa \tau}$$

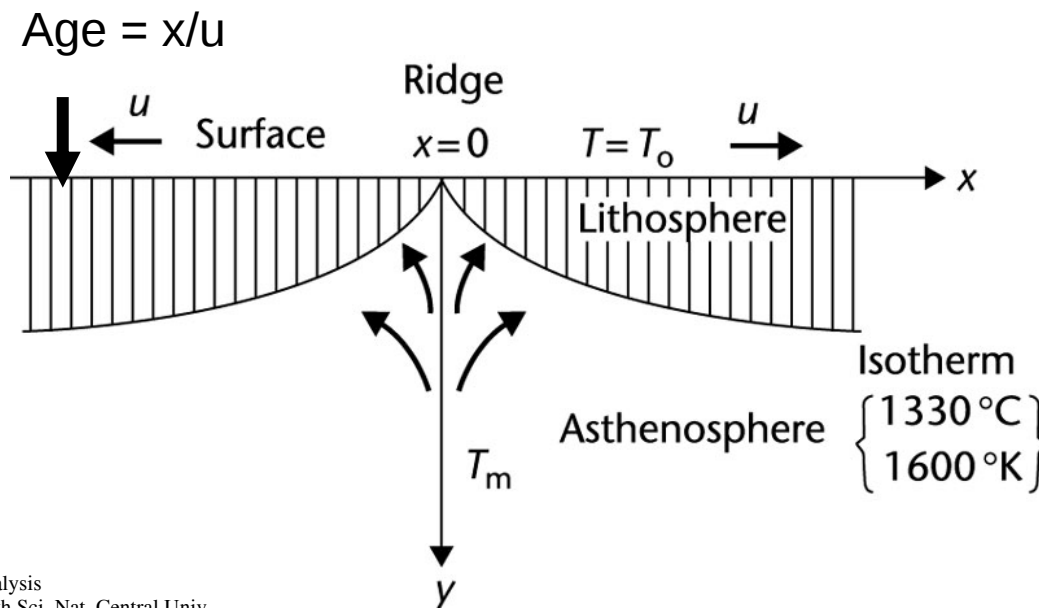


Fig. 2.16 Schematic diagram of the cooling oceanic lithosphere at a mid-ocean ridge. The oceanic plate moves away from the ridge at a velocity u . Its age is therefore determined by x/u , where x is the horizontal distance from the ridge crest.

Temperature as a function of age:

$$\theta = \operatorname{erfc}\left(\frac{x}{2\sqrt{\kappa t}}\right)$$

erfc: complementary error function.

x: horizontal distance from the mid-ocean ridge crest

t: age of oceanic crust

θ : dimensionless temperature ratio given by:

$$\theta = \frac{T - T_a}{T_s - T_a}$$

T_a : initial temperature (the asthenospheric temp.)

T_s : constant temp. of the space into which the ocean lithosphere is emplaced (the sea water temp.)

T: temp. at time t.

In the form of a thermal

diffusion distance $l = \sqrt{\kappa \tau}$

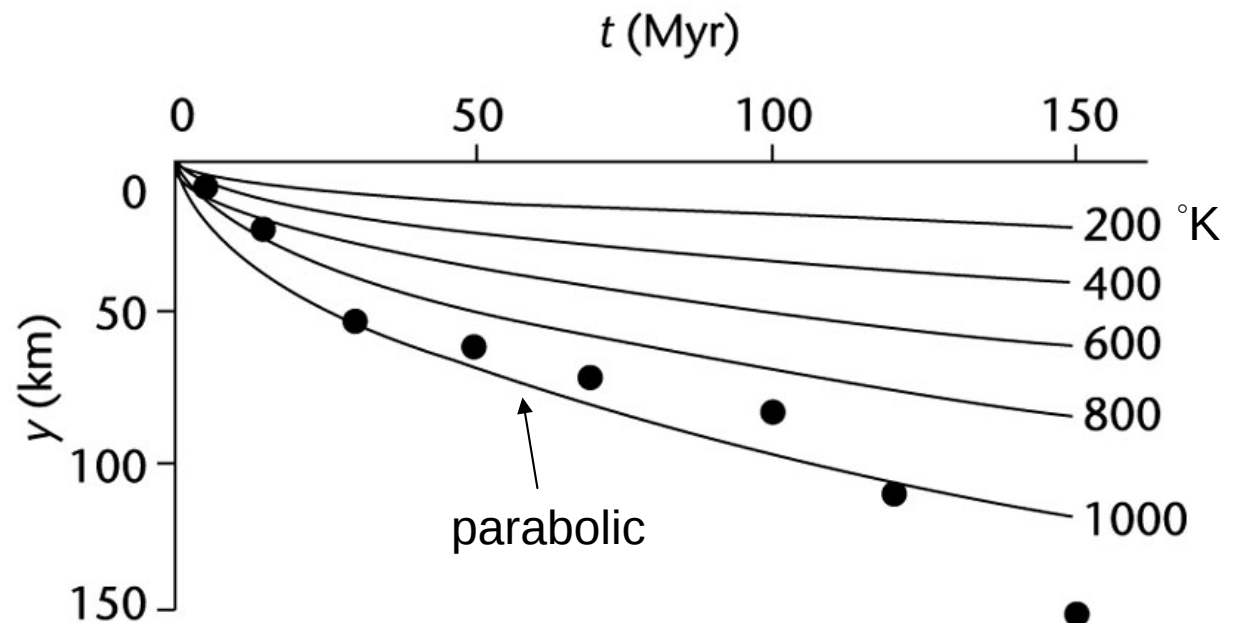


Fig. 2.17 Calculated isotherms for an oceanic lithosphere that is instantaneously cooled. The values of the isotherms are $T - T_s$ °K. The dots are the estimated thicknesses of the oceanic lithosphere in the Pacific, from Leeds et al. (1974).

The cooled oceanic material forms a thermal boundary layer equivalent to the thickness of the new oceanic lithosphere. If we choose $\theta = 0.1$ to define the thickness of the thermal boundary layer.

$$\theta = \frac{T - T_a}{T_s - T_a} \longrightarrow 0.1 = \frac{T - 1600}{273 - 1600} \longrightarrow T = 1467^\circ\text{K} \longrightarrow \text{The temperature at base of the thermal boundary layer}$$

The thickness of the thermal boundary layer (e.g., for 50 Myr old lithosphere, it is 92 km) is 2.32 times the thermal diffusion distance ($\sqrt{\kappa t}$ or $\sqrt{(\kappa x / u)}$).

For 50 Myr old lithosphere, assuming $\kappa = 10^{-6} \text{ m}^2\text{s}^{-1}$, the base of the oceanic lithosphere (i.e. $T = 1467^\circ\text{K}$) is at a depth of 92 km. (using diffusion equation)

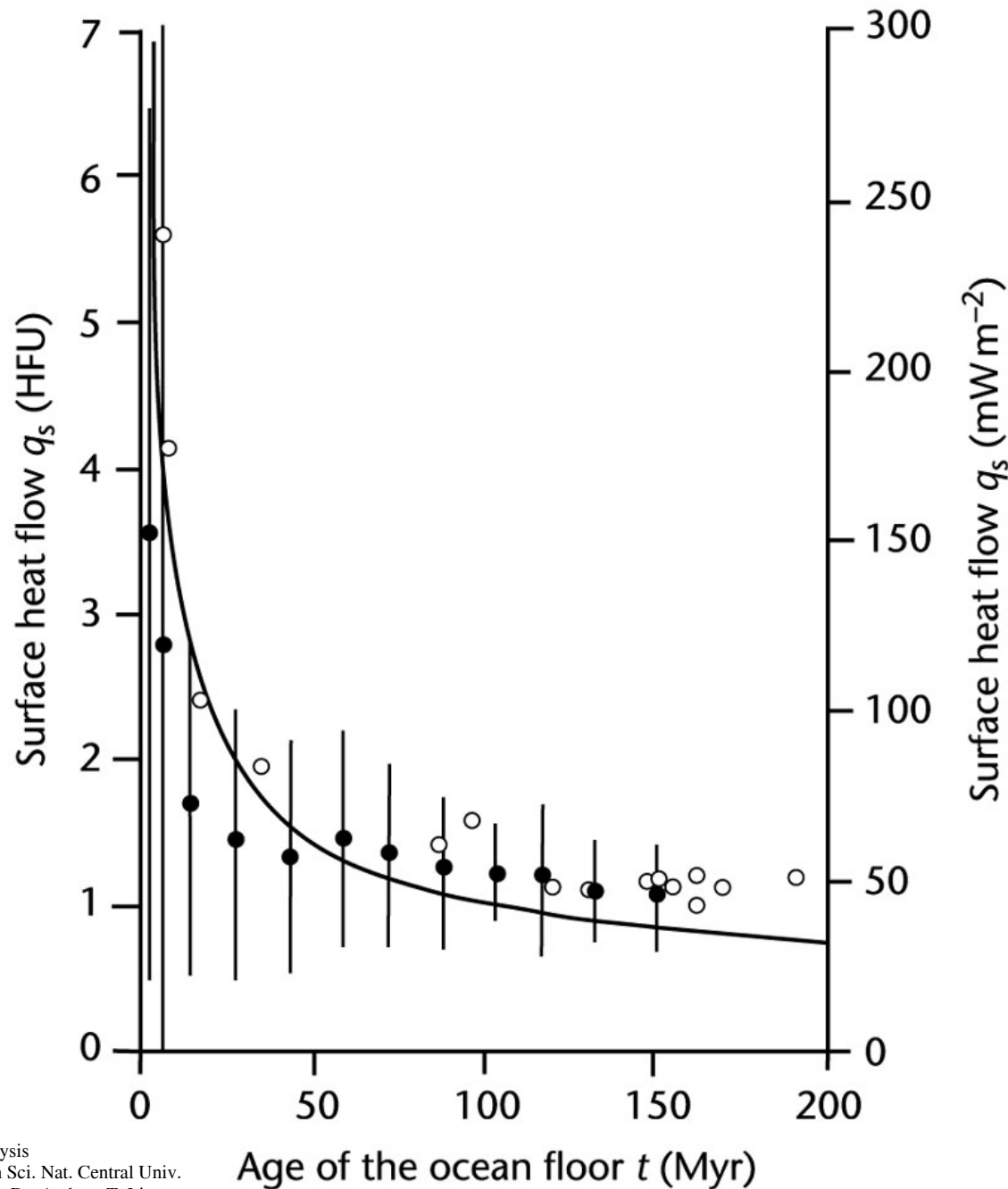


Fig. 2.18 Comparison of measured ocean heat flows (mean and standard deviation) and those predicted using the instantaneous cooling model, as a function of age. Black circles, data from Sclater et al. (1980a). Open circles, data from sediment covered regions of the Atlantic and Pacific Oceans, from Lister et al.(1990).

Difference between theoretical and observed values:

Young (< 50 Myr): due to hydrothermal circulation.
Older (>80 Myr): due to additional heat source from mantle convection.

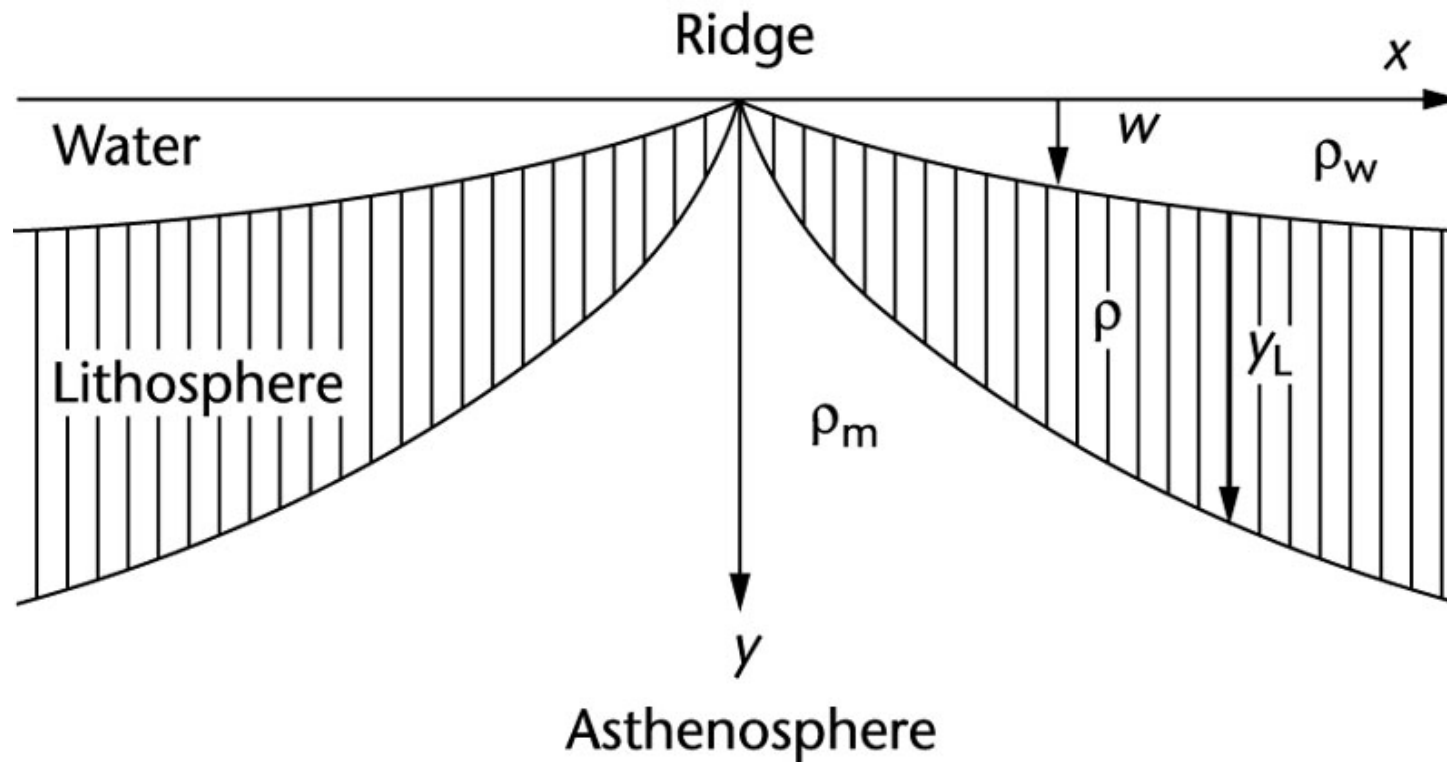


Fig. 2.19 The principle of isostasy requires the ocean to deepen with age to offset the effects of thermal contraction of the oceanic lithosphere. The water depth below the level of the ridge crest is w , the thickness of the oceanic lithosphere is y_L and ρ_m , ρ_w , ρ are the mantle, water, and lithospheric densities respectively.

Relationship between the bathymetry of the ocean floor and its thermal age:
root age relationship

For $t < 70$ Myr old

$$h = h_{\text{ridgecrest}} + Ct^{1/2}$$

h : depth of the ocean floor,

$h_{\text{ridgecrest}}$: usually 2.5 km,

t : age of the oceanic lithosphere

C : a coefficient about 0.35.

If $t=20$ Myr, $h=4$ km;

$t=50$ Myr, $h=5$ km.

Relationship between surface heat flows and thermal age
(**inverse root age relationship**).

For $t < 120$ Myr old

$$Q = 473t^{-1/2}$$

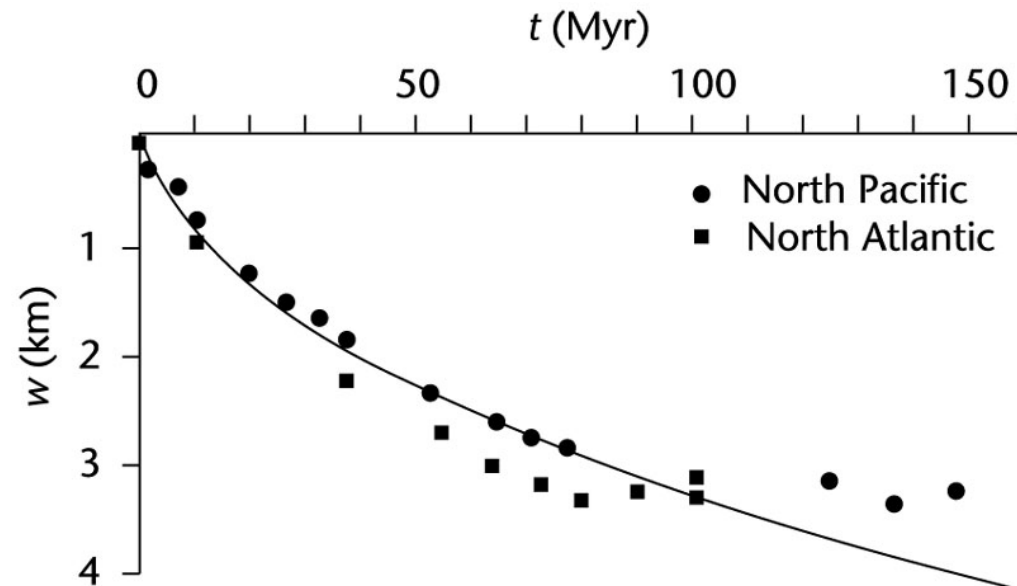


Fig. 2.20 Depth of the ocean floor below the level of the ridge crest as a function of age of the seafloor (Parsons and Sclater 1977). The solid line shows the theoretical result for an instantaneous cooling model. It is in close agreement with observations from the North Pacific and North Atlantic. The oceanic bathymetry follows a root-age relationship.

2.2.5 Thermal expansion

If a material is subjected to a change in pressure with temperature held constant, its volume will change. The change in volume for a certain pressure change is determined by the **isothermal compressibility** β .

The fractional change in specific volume with pressure at constant temperature:

$$\beta = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T = \rho \left(\frac{\partial v}{\partial p} \right)_T \quad \begin{array}{l} \nu = \text{specific volume } (\nu = 1/\rho): \\ \text{volume per unit mass.} \end{array}$$

—————> Volume change = $-V\beta dp$

Typical $\beta = 10^{-11} \text{ Pa}^{-1}$

If pressure is held constant and temperature is varied, there will also be a fractional change in specific volume. The factor determining the volume change is **volumetric coefficient of thermal expansion** α_v .

The fractional change in specific volume with temperature at constant pressure:

$$\alpha_v = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_P = \rho \left(\frac{\partial v}{\partial T} \right)_P \quad (2.64) \longrightarrow \text{Volume change} = +V\alpha_v dT$$

The total volume change is the net change resulting from the pressure change and the temperature:

$$\text{Total volume change} = -V\beta dp + V\alpha_v dT$$

Temperature changes cause large changes in stress but relatively small changes in density. For example, a 100 °K change in temperature results in a pressure change of 3 kbar (300 MPa) and 0.3% change in density for typical α_v and β values. This is an important result since sedimentary basins are commonly associated with thermal disturbances.

Rock density at a new temperature

$$\rho = \rho^* (1 - \alpha_v dT) \quad (2.66) \quad dT = T - T^*$$

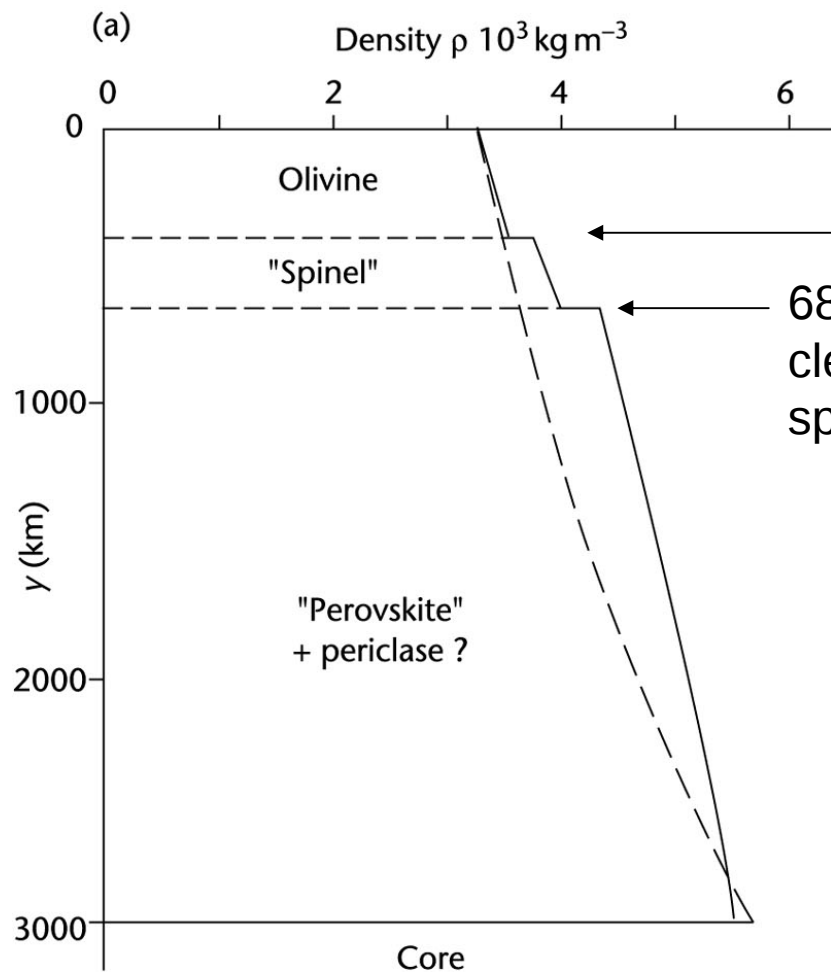
where ρ^* is a reference density at a reference T^* (usually 0°C)

2.2.6 Thermal structure of the upper mantle: effects of convection

In the interior of a vigorously convecting fluid, the mean temperature increases with depth along an *adiabat*, so that the *adiabatic temperature gradient* is the rate of temperature increase with depth caused by compression due to the overlying rock column.

Adiabatic process: An isocaloric process in which there is no heat transferred to or from the working fluid. The term “adiabatic” literally means impassable.

The increasing compressional pressure with depth cause a decrease in volume and therefore an increase in density. The relationship between the density and pressure changes is given by the adiabatic compressibility, β_a (a little smaller than isothermal compressibility, β). Eqs. 2.14 and 2.63



400 km density discontinuity: due to phase change of olivine at 13.5 GPa. The phase change is exothermic, causing heating of the rock by c. 160°K.

680 km density discontinuity: the origin is less clear, but is likely due to the phase change of spinel to perovskite at 23.1 GPa.

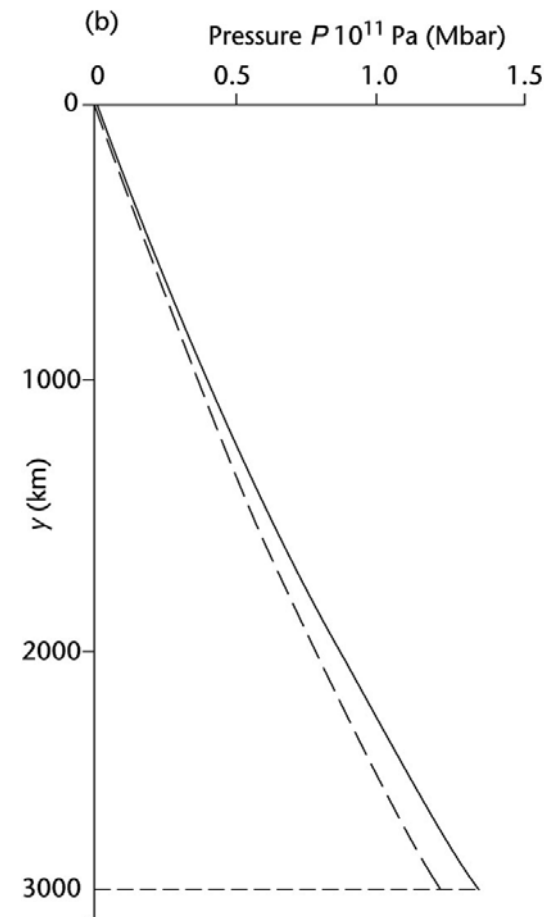


Fig. 2.21 Depth profiles of density (a) and pressure (b) in the mantle. Observed values are shown in solid lines; values calculated for a purely adiabatic behavior are shown in dashed lines.

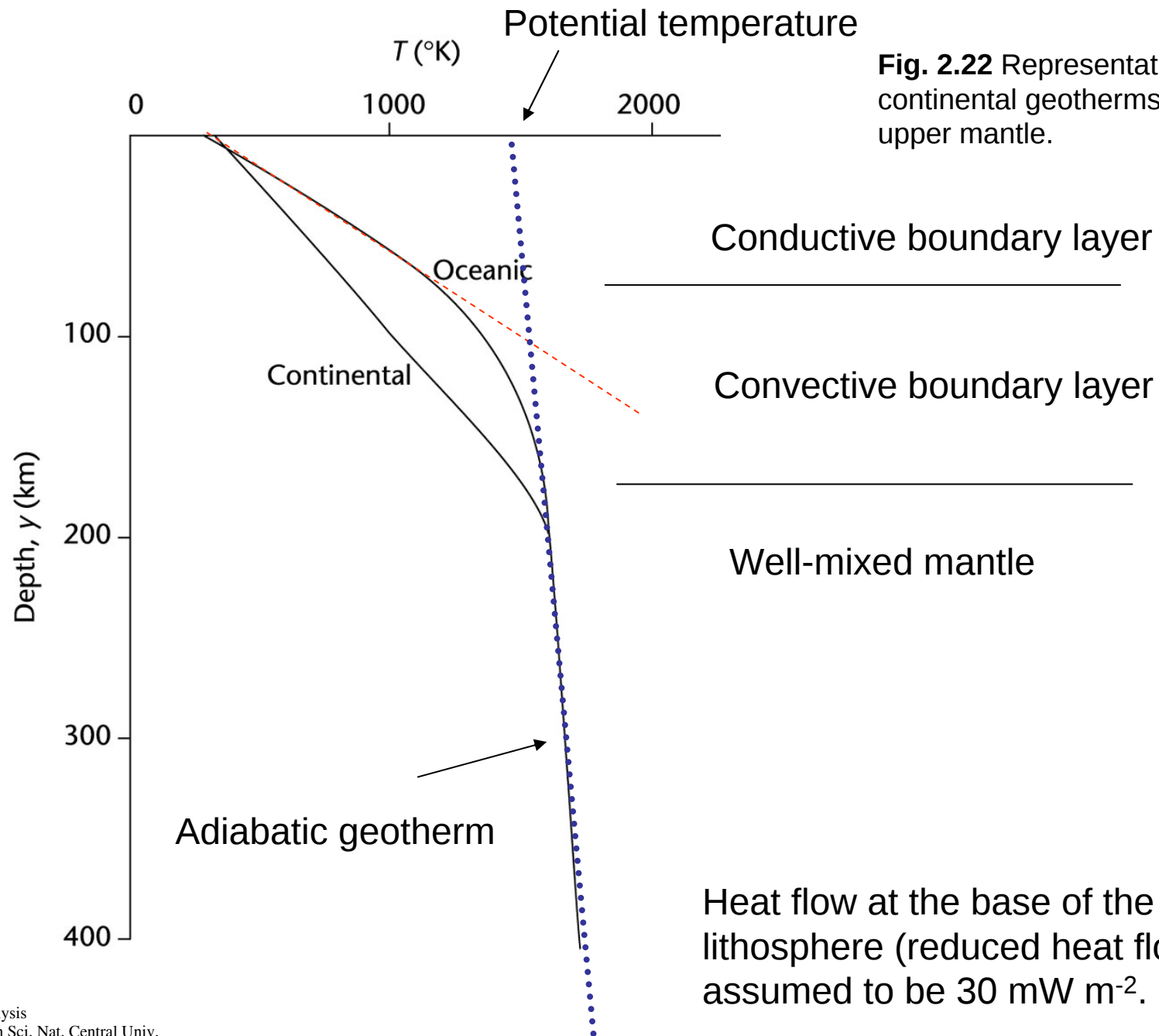


Fig. 2.22 Representative oceanic and continental geotherms in the shallow upper mantle.

Heat flow at the base of the lithosphere (reduced heat flow) is assumed to be 30 mW m^{-2} .

A Rayleigh number analysis indicates that the mantle must be fully convecting.

Lateral temperature heterogeneities in the mantle, such as where cold lithosphere plates are subducted at ocean trenches. The lateral temperature variations are extremely important in providing a driving force for mantle convection.

Mantle convection models

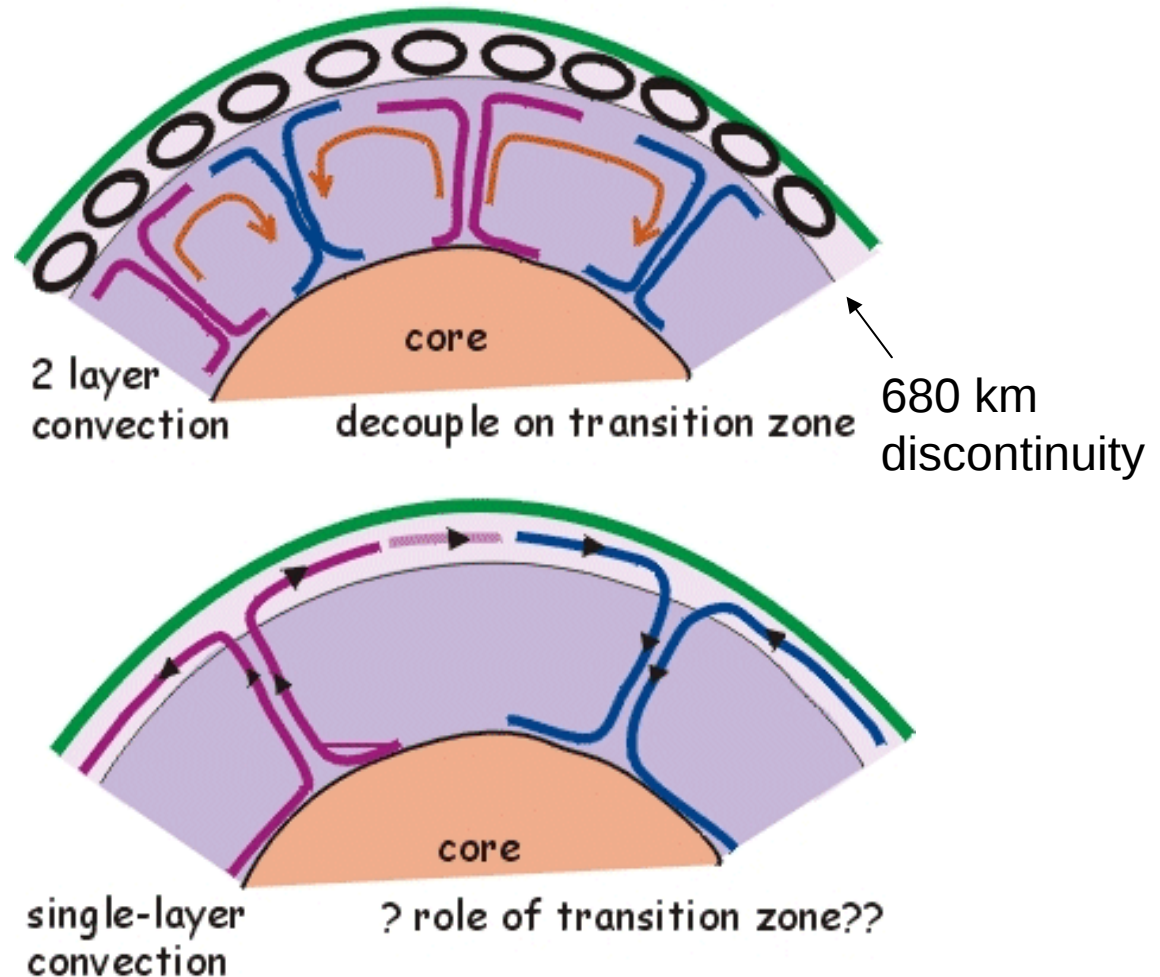
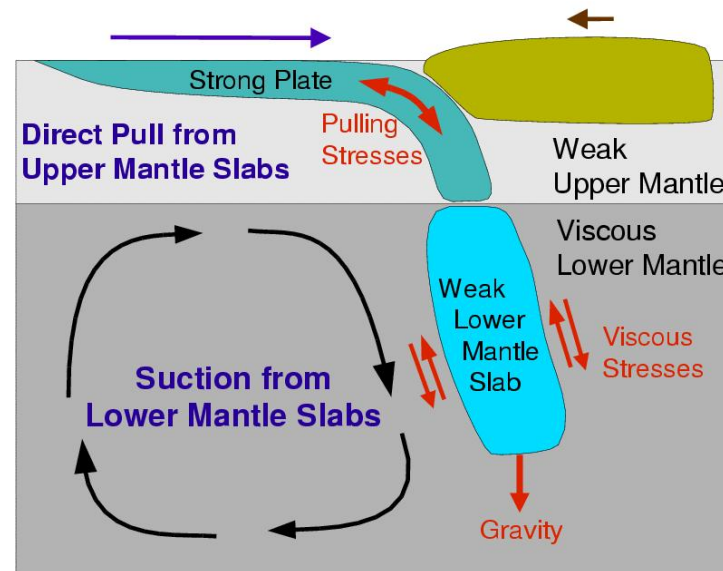


Plate tectonics driving forces:

1. Slab pull: origins for downward forces: (1) subduction of cold oceanic lithosphere, (2) upward displacement of the olivine-spinel phase-change boundary. The force is countered by frictional resistance along the base of the lithosphere.
2. Ridge push: Arising from the difference in gravitational potential energy, higher in ridge crests and lower in trenches. (an order of magnitude less than slab pull).
3. Traction by a convecting mantle. (less important)



How Mantle Slabs Drive Plate Motions



2.2.7 Mantle viscosity

Solid: response to an applied stress is to **acquire a finite strain**

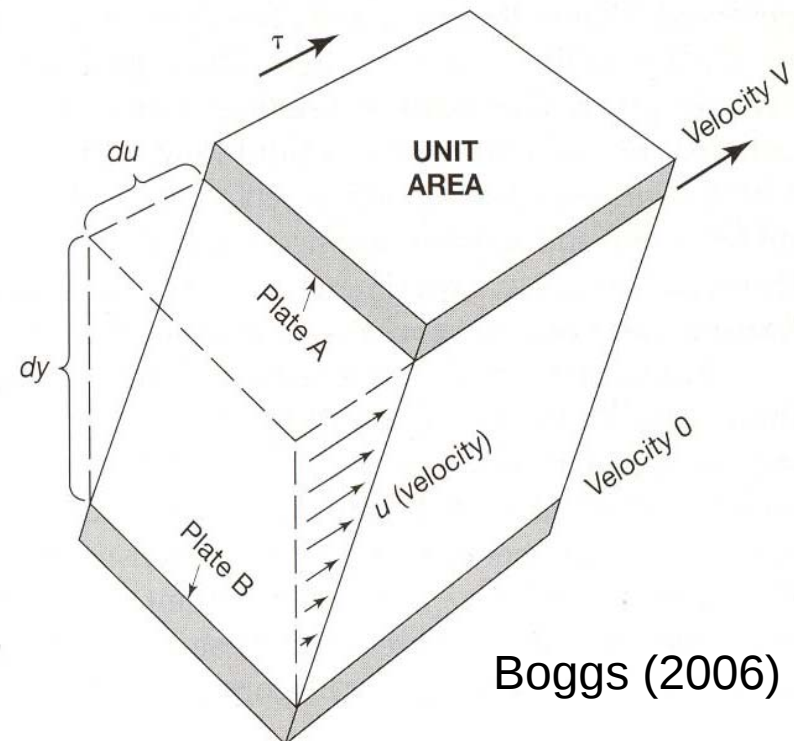
Fluid: response to an applied stress is to **deform continuously**, and rates of strain (e.g., velocity gradient) is related to the magnitude of applied stress.

In Newtonian fluids: the ratio of shear stress (τ) and strain rate (or velocity gradient) is viscosity (μ).

$$\mu = \frac{\tau}{du / dy}$$

$$\tau = \mu \frac{du}{dy}$$

Newton's law of viscosity



Boggs (2006)

Newtonian fluid

The mantle is treated as a Newtonian fluid with the diffusion creep as the main deformation mechanism. Viscous flows in the mantle are important in maintaining the hydrostatic equilibrium with the changing near surface events.

How do we know the viscosity of the mantle?

If w is the displacement at any time and w_m is the initial displacement of the surface,

$$w = w_m \exp(-t/\tau_r) \quad (2.67)$$

where τ_r is the characteristic time for the exponential relaxation of the initial displacement.

$$\text{Relaxation time: } \tau_r = (4\pi\mu)/(\rho g \lambda) \quad (2.68)$$

where μ is the viscosity and λ is the wavelength of the initial displacement.

What is a relaxation time?

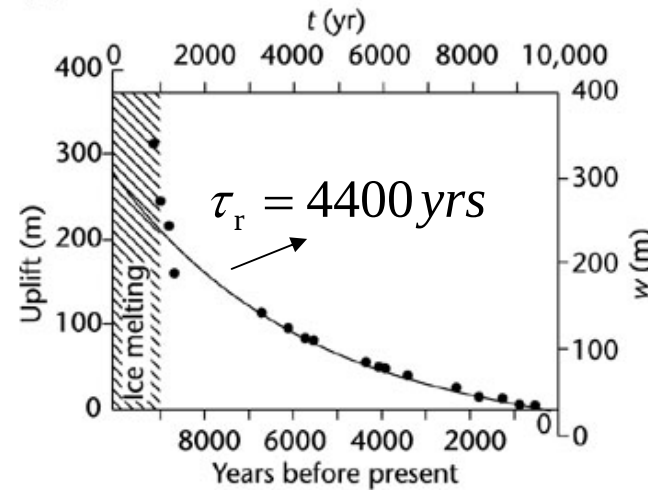
The relaxation time is the time (in years) required for the deviation from isostasy to be reduced to $1/e$ of its initial value, where e is the base of the natural logarithm ($e \sim 2.718$).

(a)



For a relaxation time of 4400 yrs derived from eq. 2.67, assuming $\lambda = 3000$ km, the viscosity of mantle (μ) is 1.1×10^{21} pas.

(b)



Last Glacial Maximum:
22 kyr - 19 kyr.
(Yokoyama et al.,
2000, Nature 406)

Fig. 2.23 Postglacial rebound of Scandinavia. (a) Present day rates of uplift occurring in Scandinavia (after Flint 1971; (b) Postglacial uplift of the mouth of the Angerman River, Sweden over the last 10000 years compared with the exponential relation in equation (2.67) with a constant viscosity of 10^{21} Pas.

Region	Depth (km)	Dynamic or absolute viscosity	
		(Pas)	(Poise)
Lithosphere	0-100	4×10^{19} 10^{21}	Elastic 4×10^{20} 10^{22}
Asthenosphere	100-175		
Mesosphere	175-2848		

Table. 2.5 Viscosity of the mantle from glacial rebound studies (Cathles 1975).

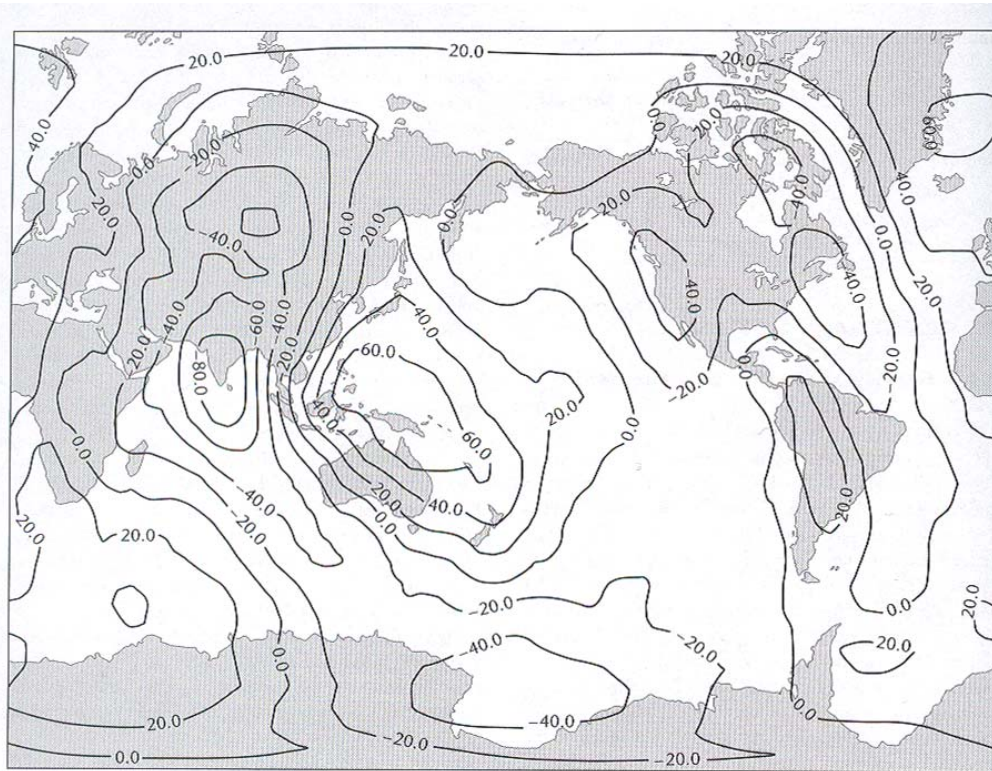
2.3 Gravity and isostasy

Gravitational acceleration of a point outside a spherical mass:

$$g_m = \frac{GM}{\gamma^2}$$

where G is the universal gravitational constant (equal to $6.673 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$), M is the mass of the Earth ($73.483 \times 10^{21} \text{ kg}$) and γ is the radial distance from the center of mass to the point of interest.

2.3.1 Gravity anomalies and the geoid



Geoid: The reference equipotential surface on earth. The geoid is close to being a spherical surface, with a certain eccentricity (flattening). The difference between polar and equatorial radii of the reference geoid is 21 km.

Geoid anomalies (in height): Small departure from reference geoid. The largest geoid anomaly is about 100 m.

Bouguer gravity formula: the surface gravity anomaly at a point

$$g_y = 2\pi G \int_0^h \rho(y) dy$$

where g_y is the surface gravity anomaly at a point, G is the universal gravitational constant ($6.673 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$) and the integral gives the mass excess or deficiency below that point, h is the height of the body of anomalous density and y is the vertical coordinate.

For the case of topography, h is the height of the topography and $\rho(y)$ is the crustal density ρ_c . Bouguer gravity formula becomes:

$$\Delta g = 2\pi\rho_c Gh$$

where Δg is the gravity anomaly due to topography.

A number of corrections are applied to surface gravity measurements in order to obtain the surface gravity anomaly.

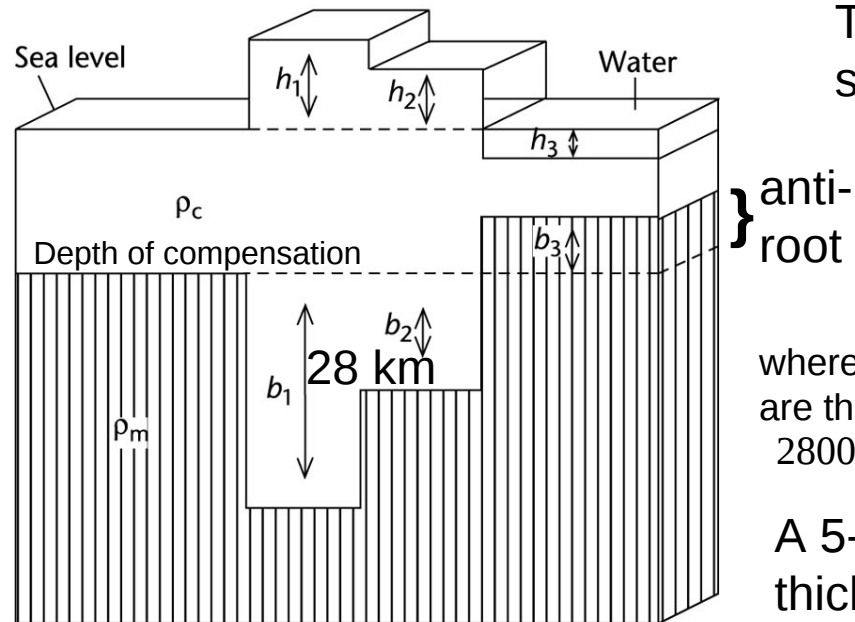
- The reference gravity field is firstly subtracted, which contains a latitude correction;
- the variation of gravity with height is then subtracted (elevation or free-air correction). This correction is 3.07 mms^{-2} for an elevation of 1km. The resulting anomaly after this second step is called a free-air gravity anomaly;
- the gravitational attraction of local (short-wavelength) topography is removed with the Bouguer gravity formula to give the Bouguer gravity anomaly. If the topography is particularly steep, a terrain correction is also applied.

2.3.2 Models of compensation

- **Airy compensation** (or Airy isostasy): Compensation for topography by **changes in the thickness** of the underlying crust. This hypothesis assumes that the lithosphere is very weak and it responds to load **locally**.
- **Pratt compensation**: Compensation for topography by changes in density in a lateral direction (because of a change in temperature). (e.g., the oceanic crust where density increases away from the oceanic ridges; plume-related swells (the Hawaii swell)). This hypothesis assumes that the lithosphere is very weak and it responds to load **locally**.
- **Flexural isostasy**: Loads are compensated **regionally** by a lithosphere approximating an elastic sheet overlying a fluid substratum.

Airy compensation

(a)



Thickness of the root (b) beneath the surface of hydrostatic equilibrium

$$b = \frac{\rho_c h}{\rho_m - \rho_c} \quad (2.72)$$

where h is the height of the mountain belt, and ρ_m and ρ_c are the mantle and crustal densities respectively. If ρ_c is 2800 kg m^{-3} and ρ_m is 3300 kg m^{-3} ,

A 5-km high mountain would require a root thickness of $b = 28 \text{ km}$

Fig. 2.25(a) Airy model of isostatic compensation.

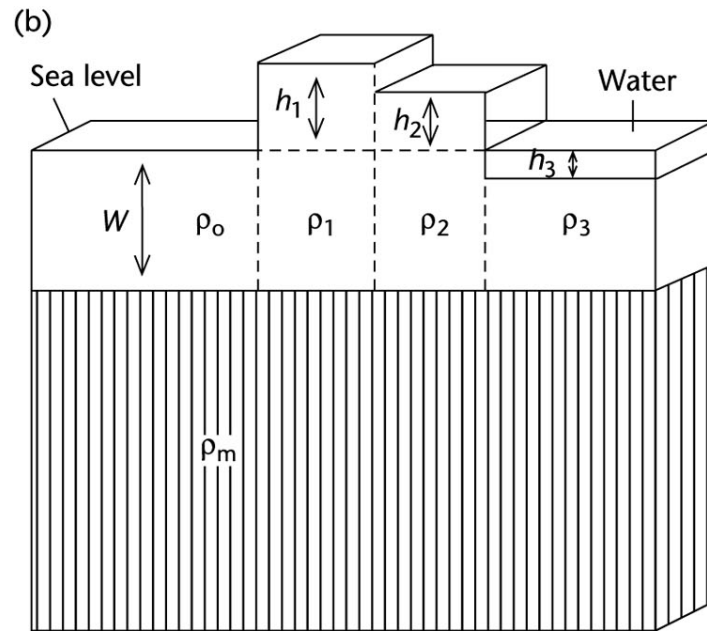
Thickness of an “anti-root” beneath a marine basin

$$b = \left(\frac{\rho_c - \rho_w}{\rho_m - \rho_c} \right) h \quad (2.73)$$

If the topography is negative (h is negative), representing a marine basin, the balancing of lithospheric columns gives where ρ_w is the density of ocean water.

Using the same density terms, the “anti-root” beneath a marine basin of depth 1 km is 3.6 km.

Pratt compensation



The density of lithospheric column elevated at h is given by:

$$\rho_p = \rho_0 \left(\frac{W}{W + h} \right) \quad (2.74)$$

where ρ_0 is the reference density for a lithospheric column with its surface at sea level.

For negative topography (h is negative), the variable density of lithospheric columns below sea level is given by:

$$\rho_p = \frac{\rho_0 W + \rho_w h}{W + h} \quad (2.75)$$

Fig. 2.25b Pratt model of isostatic compensation.

Example: The Hawaii swell reaches water depths of 4 km at its crest. The swell is situated in an abyssal plain at -6 km. The density change required to elevate the Hawaii swell 2 km above the adjacent abyssal plain is 3281 kgm^{-3} if $\rho_0 = 3300 \text{ kgm}^{-3}$, $W = 150 \text{ km}$. The density reduction is 0.6%. Since $\alpha_v = 3 \times 10^{-5} \text{ K}^{-1}$ (eq. 2.64), this is equivalent to an average heating down to 120 km by c. 190°C .

2.3.3 Flexural isostasy

General flexural equation:

$$D \frac{d^4 W}{dx^4} + (\rho_m - \rho_s) g w = \rho_s g h$$

The maximum deflection (W_0)

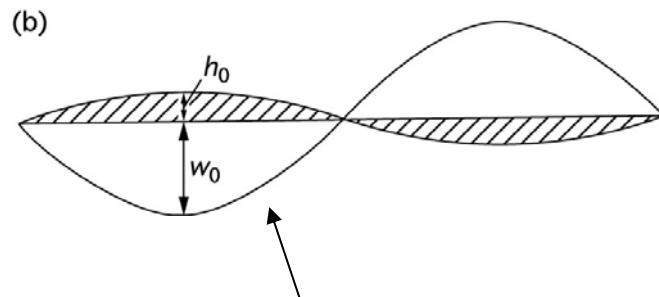
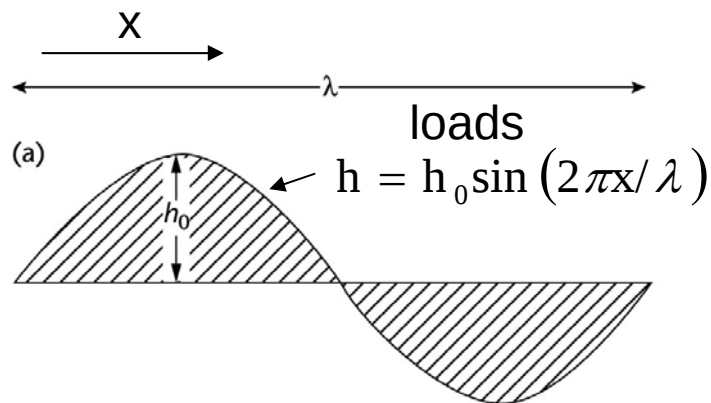
$$w_0 = \frac{h_0}{\frac{\rho_m}{\rho_s} - 1 + \frac{D}{\rho_s g} \left(\frac{2\pi}{\lambda}\right)^4} \quad (2.81)$$

while the wavelength of the load (λ) is short, w_0 is very small. The lithosphere therefore appears to behave very rigidly to loads of this scale. If the wavelength is long then:

$$w = w_{0\infty} = \frac{\rho_s h_0}{(\rho_m - \rho_s)} \quad (2.82) \quad \text{This is Airy isostasy!}$$

Meaning that at long wavelengths, the lithosphere appears to have no rigidity.

Fig. 2.26 Deflection of the lithosphere under a periodic (sinusoidal) load. In (a) the wavelength of the load is short and there is no deflection of the lithosphere. In (b) the wavelength of the load is long, leading to isostatic compensation of the load by a deflection of the lithosphere. w_0 is the maximum deflection, h_0 is the maximum elevation of the load (after Turcotte and Schubert 1982).



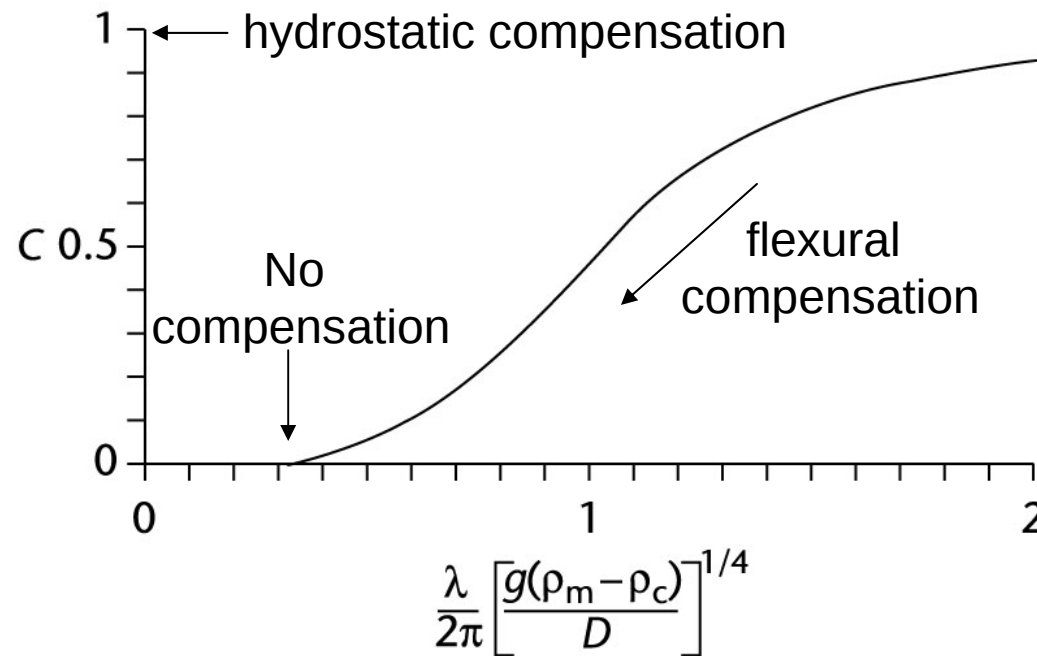
deflection $w = w_0 \sin(2\pi x / \lambda)$

How do we know the degree of compensation for a particular load?

The degree of compensation C of the load is the ratio of the actual deflection compared with the maximum (Airy) or hydrostatic deflection.

$$C = \frac{W_0}{W_{0\infty}}$$

$$C = \frac{(\rho_m - \rho_s)}{\rho_m - \rho_s + \frac{D}{g} \left(\frac{2\pi}{\lambda} \right)^4}$$



$(2\pi/\lambda)$ is sometimes termed the **wave number** (e.g., Watts 1988).

Generally, topography with wavelength < 100 km (i.e. the width of the mountain belt is < 50 km) is not compensated; whereas for $\lambda > 1000$ km, the topography is fully compensated.

Fig. 2.27 Dependence of the degree of compensation C , on the nondimensional wavelength of periodic (sinusoidal) topography (after Turcotte and Schubert 1982). D is the flexural rigidity, λ is the wavelength of the load, ρ_m and ρ_c are the mantle and crustal densities respectively and g is the gravitational acceleration.

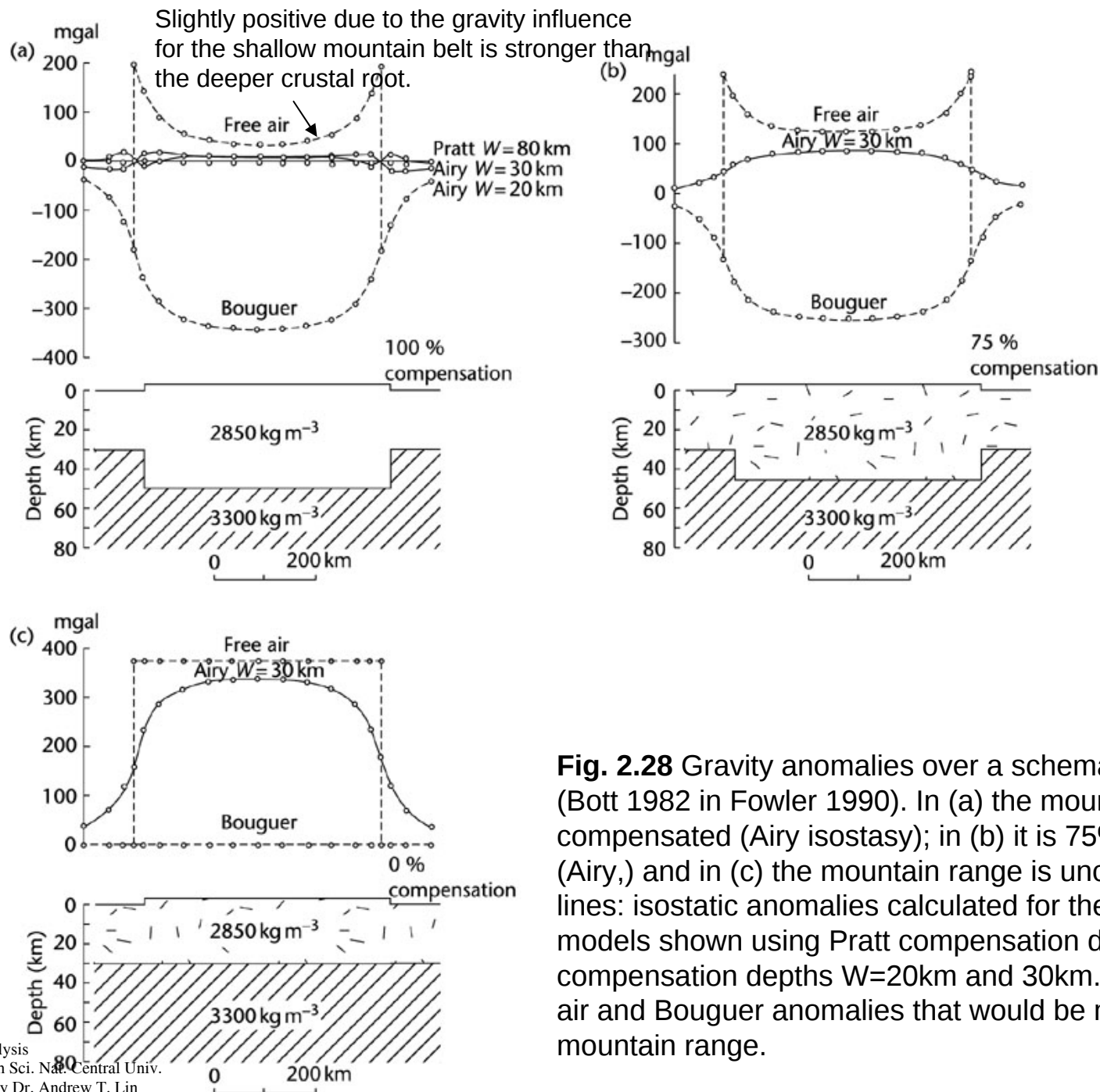
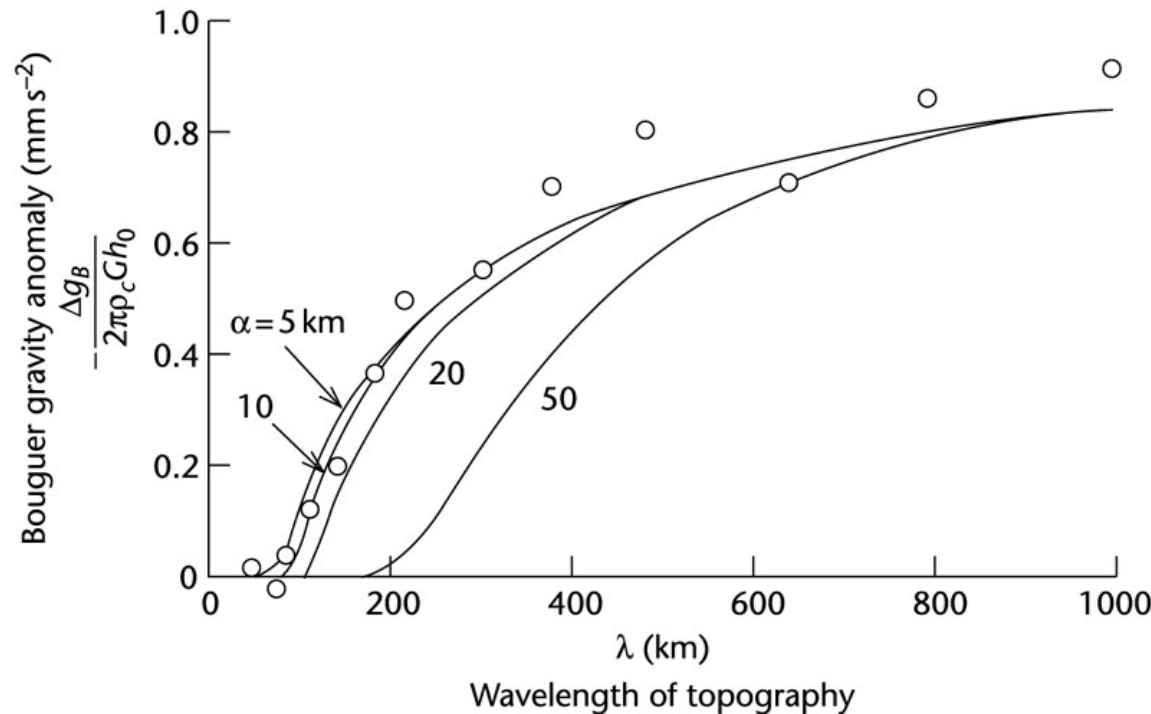


Fig. 2.28 Gravity anomalies over a schematic mountain range (Bott 1982 in Fowler 1990). In (a) the mountain range is 100% compensated (Airy isostasy); in (b) it is 75% compensated (Airy,) and in (c) the mountain range is uncompensated. Solid lines: isostatic anomalies calculated for the different density models shown using Pratt compensation depth $W=80\text{km}$, Airy compensation depths $W=20\text{km}$ and 30km . Dashed lines, free-air and Bouguer anomalies that would be measured over the mountain range.

Degree of compensation: an admittance analysis

The very common negative Bouguer anomalies associated with mountain belts suggests that they are compensated to some degree, depending on the flexural rigidity of the lithosphere. The degree of compensation can be estimated from the correlation of Bouguer gravity anomalies with topography as a function of its wavelength. This technique is called admittance.

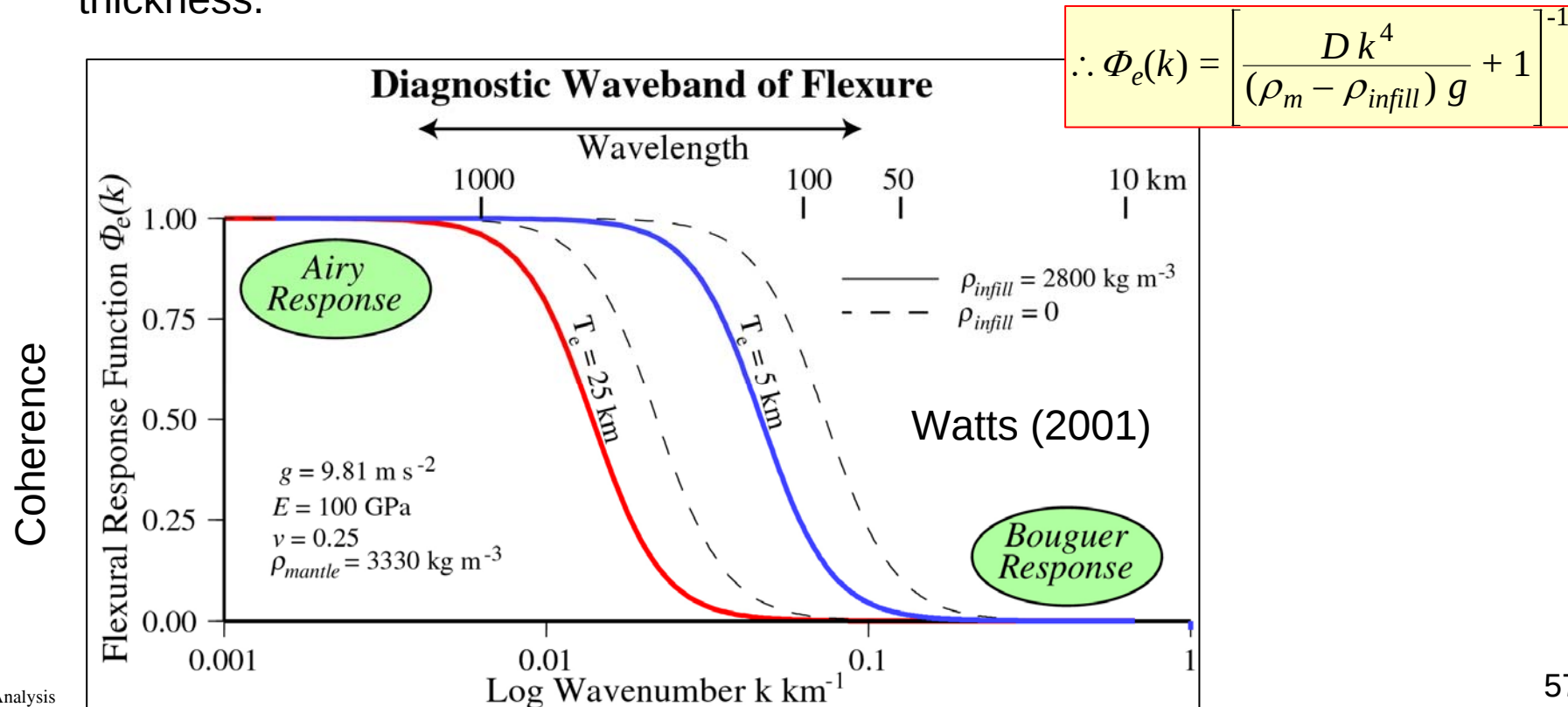


Generally, topography with wavelength < 100 km is not compensated; whereas for $\lambda > 1000$ km, the topography is fully compensated.

Fig. 2.29 Correlation of Bouguer gravity anomalies with topography (**admittance**) for the United States (Dorman and Lewis 1972), compared with the gravity formula for loading by periodic topography for different values of the **flexural parameter** $\alpha = [4D(\rho_m - \rho_c)g]^{1/4}$ in km. the best agreement is with $\alpha \cong 20$ km, or flexural rigidity $D \cong 10^{21}$ Nm, which with $E = 60$ GPa and $\nu = 0.25$, gives an equivalent elastic thickness T_e of 6 km.

Degree of compensation: a coherence analysis

The coherence function is the square of the correlation coefficient between the topographic and gravity signals. The coherence indicates the degree of compensation for any topographic loads or density contrasts. A coherence of 1 indicates complete compensation on a plate with no flexural strength, and a coherence of 0 indicates that the load is completely supported by the strength of the plate. On a plot of coherence versus wavenumber ($k=2\pi/\lambda$), the roll-over from a coherence of 1 to 0 gives a measure of the flexural rigidity or equivalent elastic thickness.



2.4 Rock rheology

2.4.1 Fundamentals

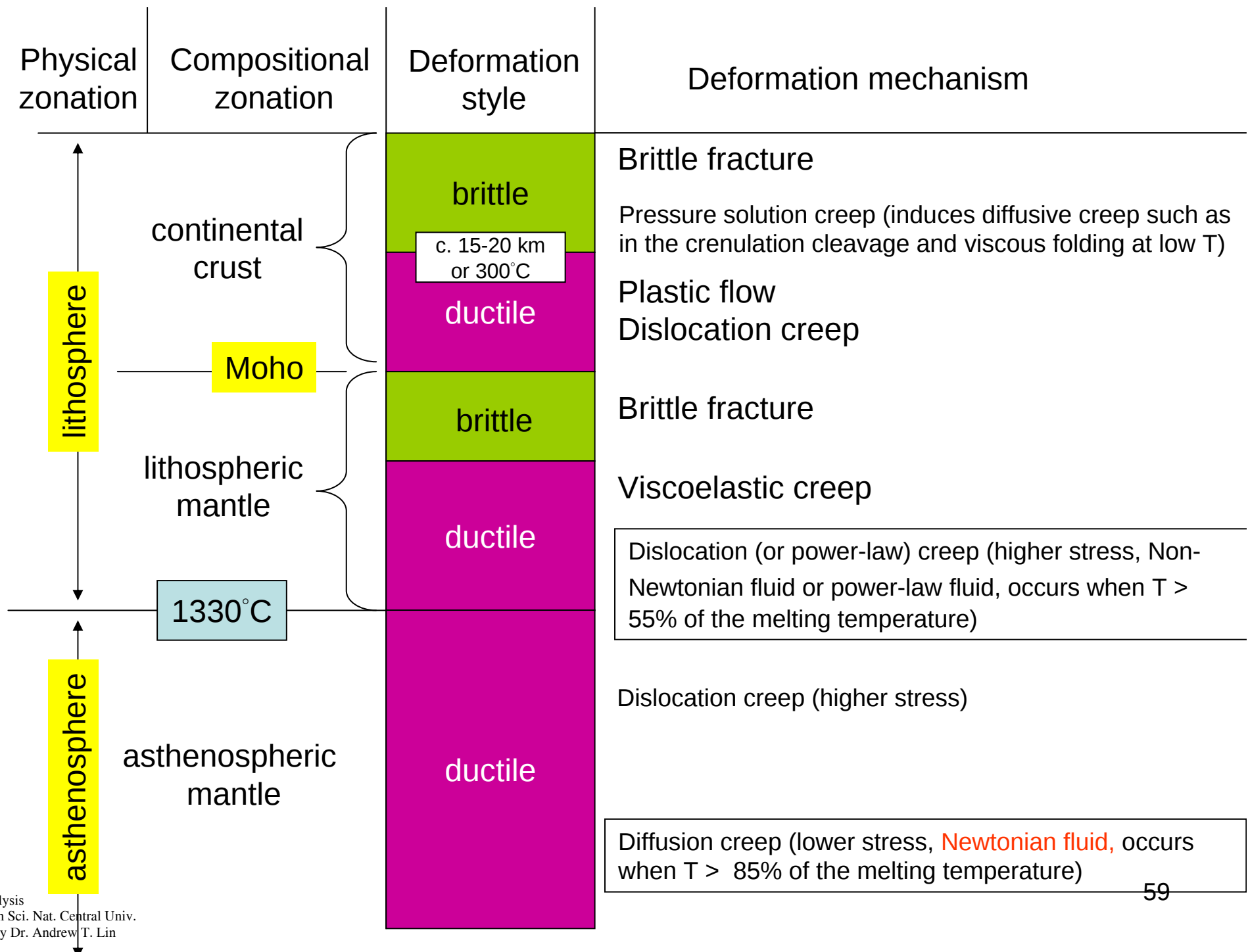
Three parameters are important in determining the brittle to ductile transition: **pressure**, **temperature**, and **strain rate**.

決定岩石圈強度的因素

Temperature: 礦物所在的溫度若約大於其solidus(固相線、初熔)溫度的一半 (Ranalli, 1997)，則此礦物(或岩石)若受力會呈韌性變形。以下成碎性變形。
Quartz, $T > 250-300^{\circ}\text{C}$: ductile deformation; Olivine, $T > 600-700$: ductile deformation.

Composition: multi-layer composition (e.g., quartz/olivine, continental lithosphere, multi-layer composition may have weak lower crust that leads to the decouple of upper crust and upper mantle); single layer composition (olivine, oceanic lithosphere);

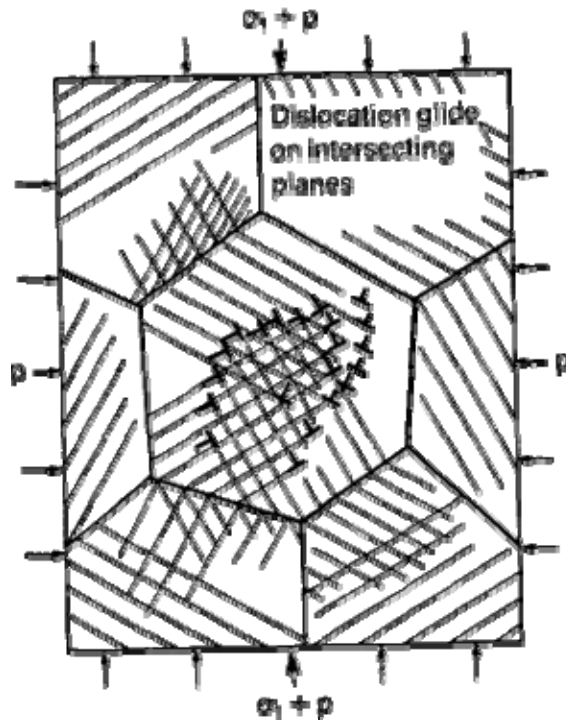
Strain rate: high strain rate, high strength (e.g. 地函在 $1\sim 10^4$ 秒呈彈性體；在 $10^{11}\sim 10^{17}$ 秒 (3000年~30億年)呈viscous fluid (Turcotte & Schubert, 2002, p.330)



Ductile Deformation

Three forms: **plastic flow**; **dislocation (power-law) creep**; and **diffusion creep**. Each of these mechanisms has an exponential constitutive relation in which strain increases exponentially with applied stress.

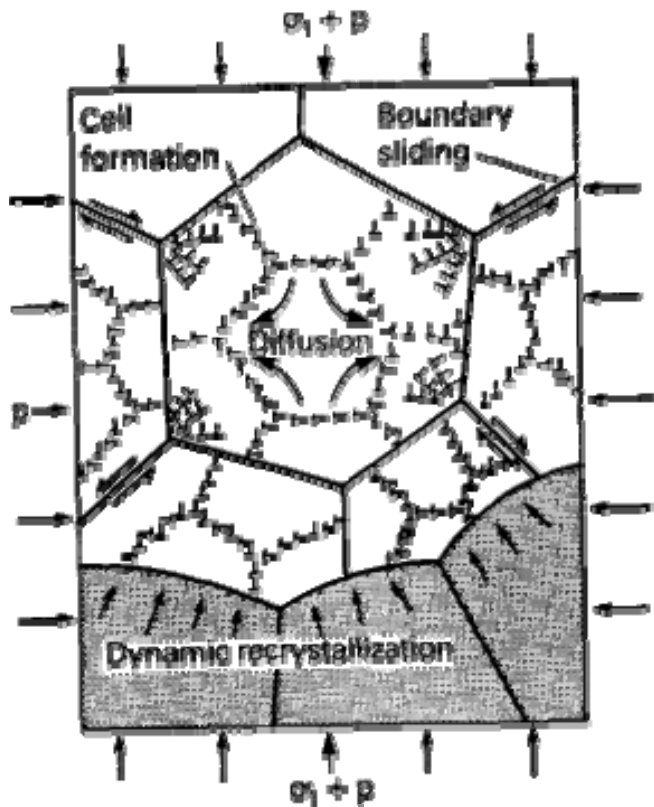
Plastic flow



Plastic flow produces limited, permanent strains at high stresses when the yield strength of mineral grains is exceeded and they deform by gliding along internal dislocation planes or grain boundaries. Such crystal-structure dislocations, as in this microscopic view, can heal quickly and may actually strengthen the rock (work hardening).

From: <http://www.seismo.unr.edu/ftp/pub/louie/class/plate/deformation.html>

Dislocation (power-law) creep

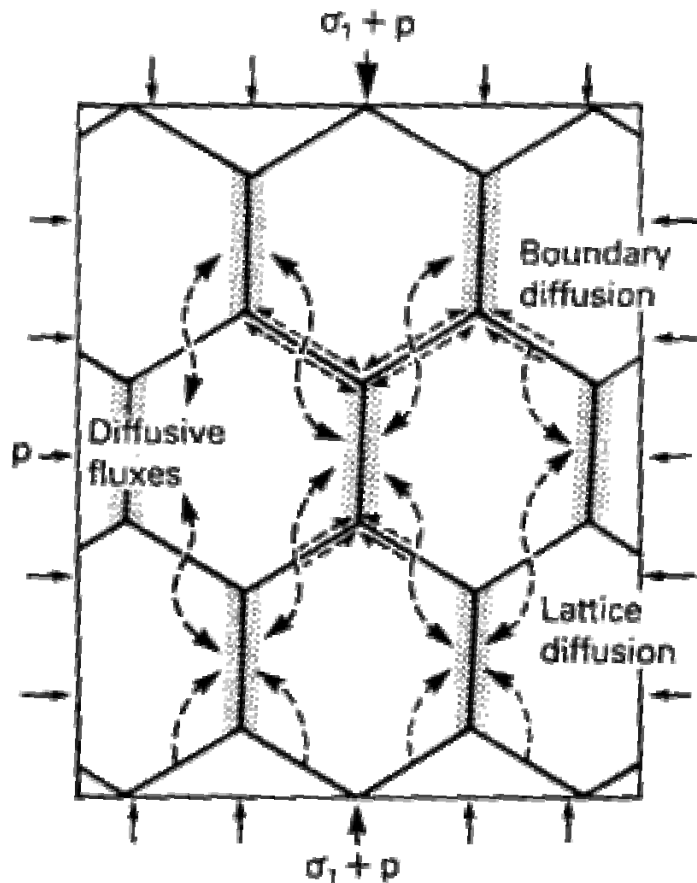


Power-Law Creep takes place only at high temperature, at least 55% of the melting temperature. Most minerals melt at temperatures between 400 and 1800 C, so power-law creep takes place in the lower crust or deeper. For this mechanism the constitutive relation gives the *strain rate* with time as proportional to an exponential power of the applied stress, with the exponent being 3 or greater. A newtonian viscous fluid has strain rate proportional to the stress with an exponent of one.

Internal gliding dislocation, as with plastic flow, is the main mechanism of deformation for power-law creep. However, the high temperature allows diffusion of atoms and recrystallization, so creep can continue to strains in excess of 100% at relatively low stresses.

Power-law creep is likely the dominant mode of deformation in the lithospheric mantle.

Diffusion Creep



Diffusion Creep occurs when temperatures exceed 85% of the melting temperature. Such temperatures lead to rapid diffusion and atom migration along stress gradients, promoting continual recrystallization.

Diffusion creep has a newtonian constitutive relation with strain rate proportional to the first power of stress, so such materials flow like fluids. This mechanism should be operating in regions just below the solidus temperature, like the LVZ or *Asthenosphere* that underlies the oceanic lithosphere.

From: <http://www.seismo.unr.edu/ftp/pub/louie/class/plate/deformation.html>

Power-law fluid: A fluid whose strain rate or velocity gradient is proportional to the power n of the stress.

$$\dot{\varepsilon} = A \sigma^n \exp\left(\frac{-E_a}{RT}\right) \quad (2.94)$$

$\dot{\varepsilon}$: strain rate

A: material coefficient

E_a : activation energy

n : constants dependent on minerals
(for Newtonian fluid, $n=1$)

R : universal gas constant

T : absolute temperature

For olivine, $A = 4.2 \times 10^5 \text{ MPa}^{-3} \text{ s}^{-1}$, E_a is the activation energy = 523 kJ mol^{-1} , and $n=3$.

$$\mu_{\text{eff}} = \tau / \frac{du}{dy} \quad (2.85)$$

where μ_{eff} : effective viscosity; τ : shear stress; u : velocity

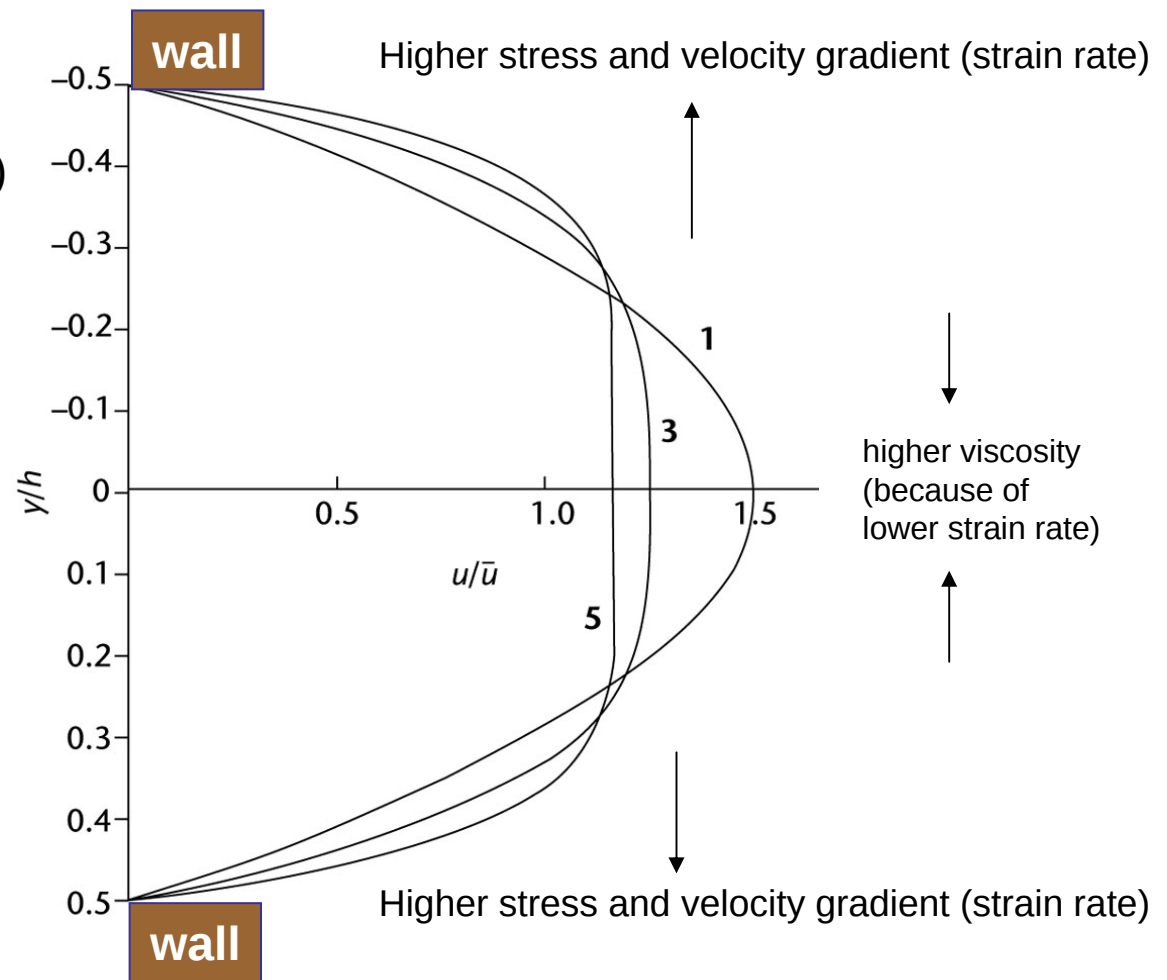


Fig. 2.30 Velocity profiles in a channel of thickness h for power-law fluid rheologies with $n=1$ (Newtonian), $n=3$, and $n=5$. distance from the channel wall is expressed by y/h . The velocity u is scaled by the average velocity $\bar{u}$.

2.4.2 Rheology of the mantle

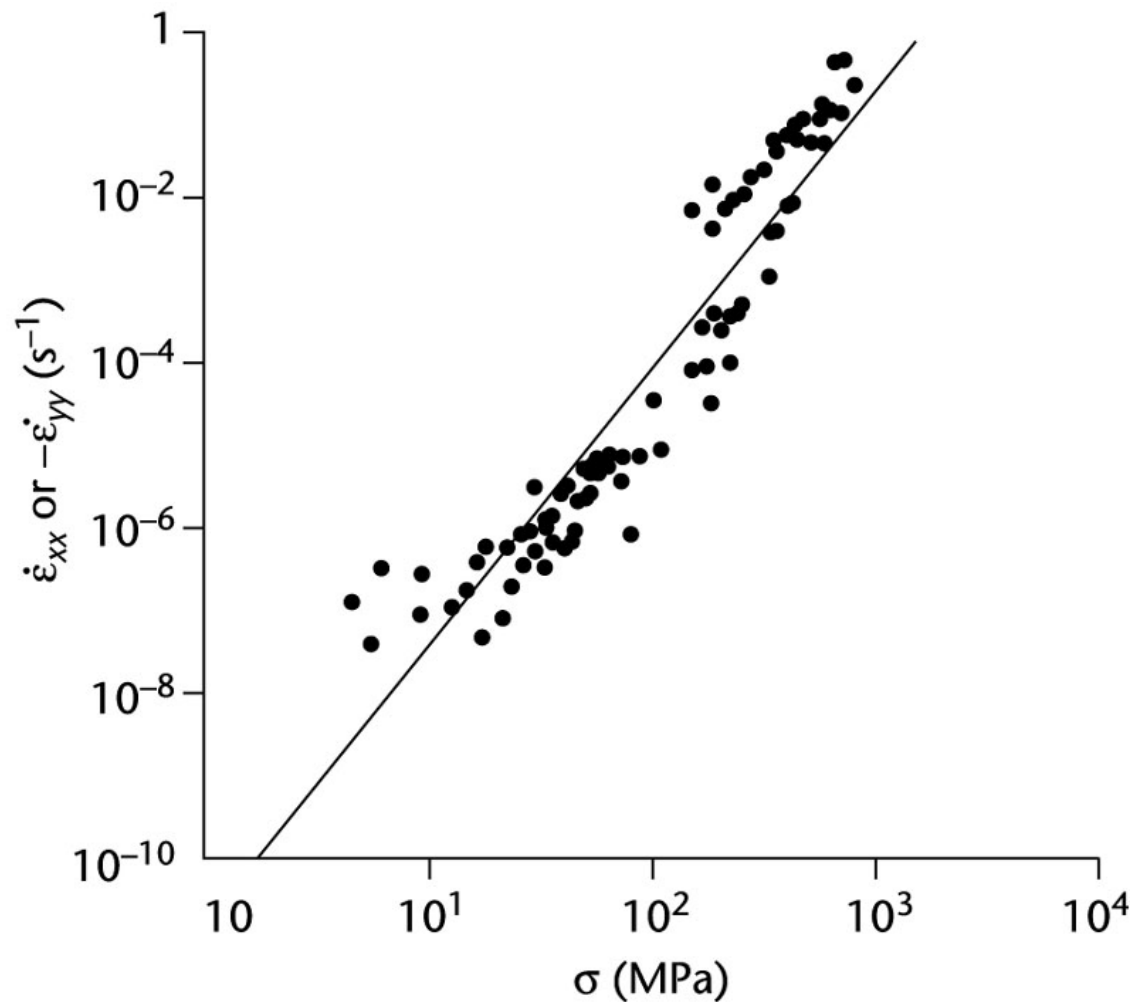


Fig. 2.31 Observed dependence of strain rate on stress for olivine at a temperature of 1400°C (after Ashby and Verall 1977). Dry olivine obeys an approximate **cubic power law** rheology in these laboratory experiments.

2.4.3 Rheology of the crust

There is a roughly constant ratio between the *frictional force* on a potential failure plane **F** and the *normal stress* **N**. This ratio is equal to the coefficient of friction *f*, or the tangent of the angle of sliding friction ϕ .

$$\frac{F}{N} = f = \tan \phi \quad (2.86)$$

Friction coefficient depends on rock types at low pressures (< 5 MPa); at moderate pressure (5-100 MPa) the correlation was poor, and at high pressures (200-2000 MPa) there is no rock type dependence.

Byerlee's law states:

$$F = aN + b \quad (2.87)$$

where the coefficients *a* and *b* are 0.6 and 0.5 kbar at normal pressures of > 2 kbar (200 MPa).

Byerlee's law has the same form as the *Navier-Coulomb failure criterion*:

$$\tau_c = C + \sigma \tan \phi \quad (2.88) \quad C \text{ of (2.88) } = b \text{ of (2.87)}$$

where the shear stress acts in the direction of and balances the frictional force on a fracture plane, τ_c is the critical shear stress for failure, σ is the normal stress, ϕ is the angle of internal friction (in the range 0.7-1.7 for compact, coherent rocks), and *C* is the strength that exists even at zero normal stress (pressure) called cohesion.

The weight of a column of rock of density 2800 kgm⁻³, height 1 km and $g = 10 \text{ ms}^{-2}$, is 28 MPa. So 200 MPa of pressure is at depths of 7-8 km in the crust.

For normal pressure > 200 MPa (c. > 7-8 km)
friction coefficient is independent of rock types and
is in the range of c. 0.6-0.85.

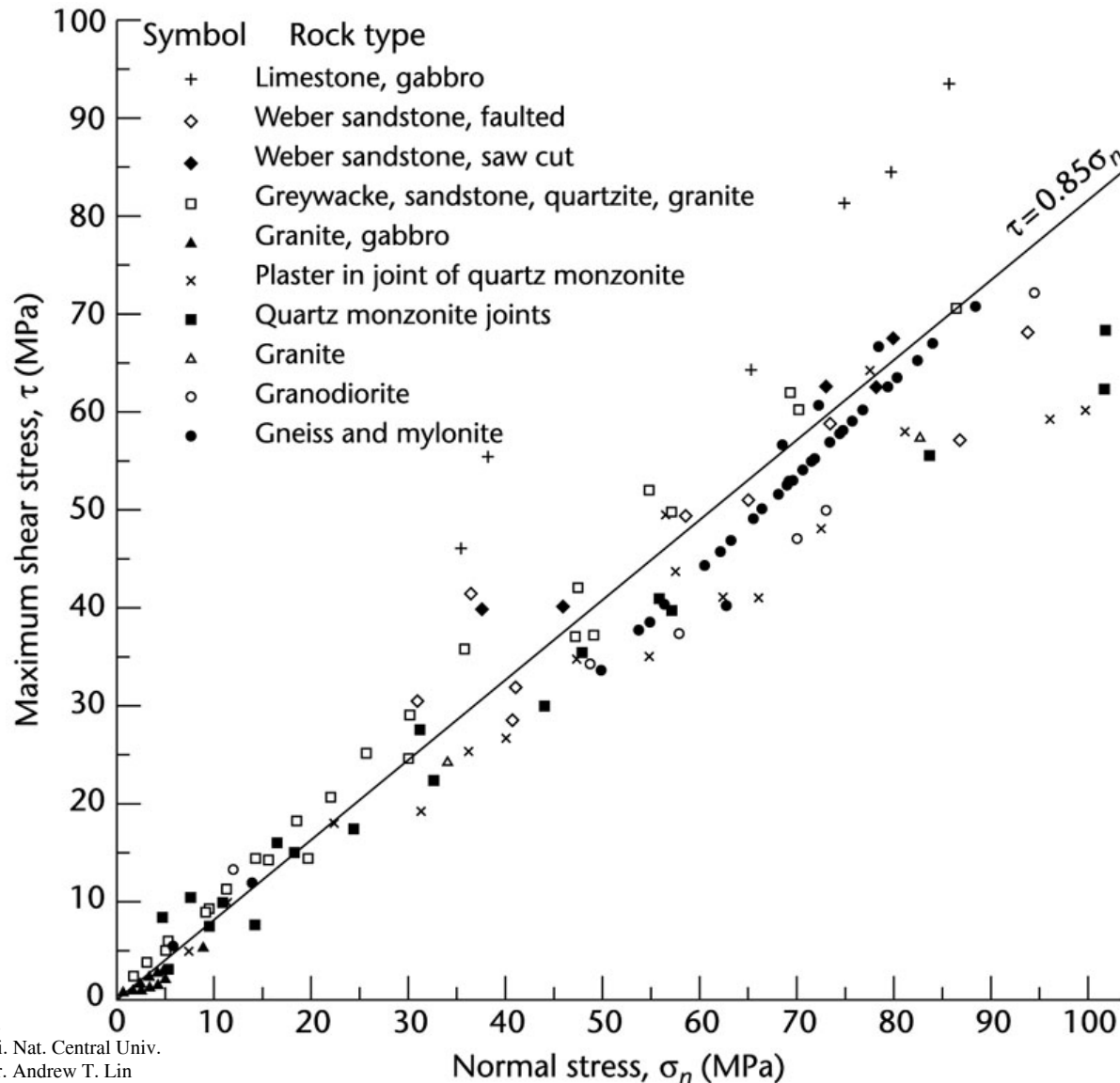


Fig. 2.32 Byerlee's law for the maximum shear stress τ to initiate sliding as a function of the normal stress σ_n , for a variety of rock types. The linear fit defines a maximum coefficient of static friction of 0.85 (Byerlee 1977).

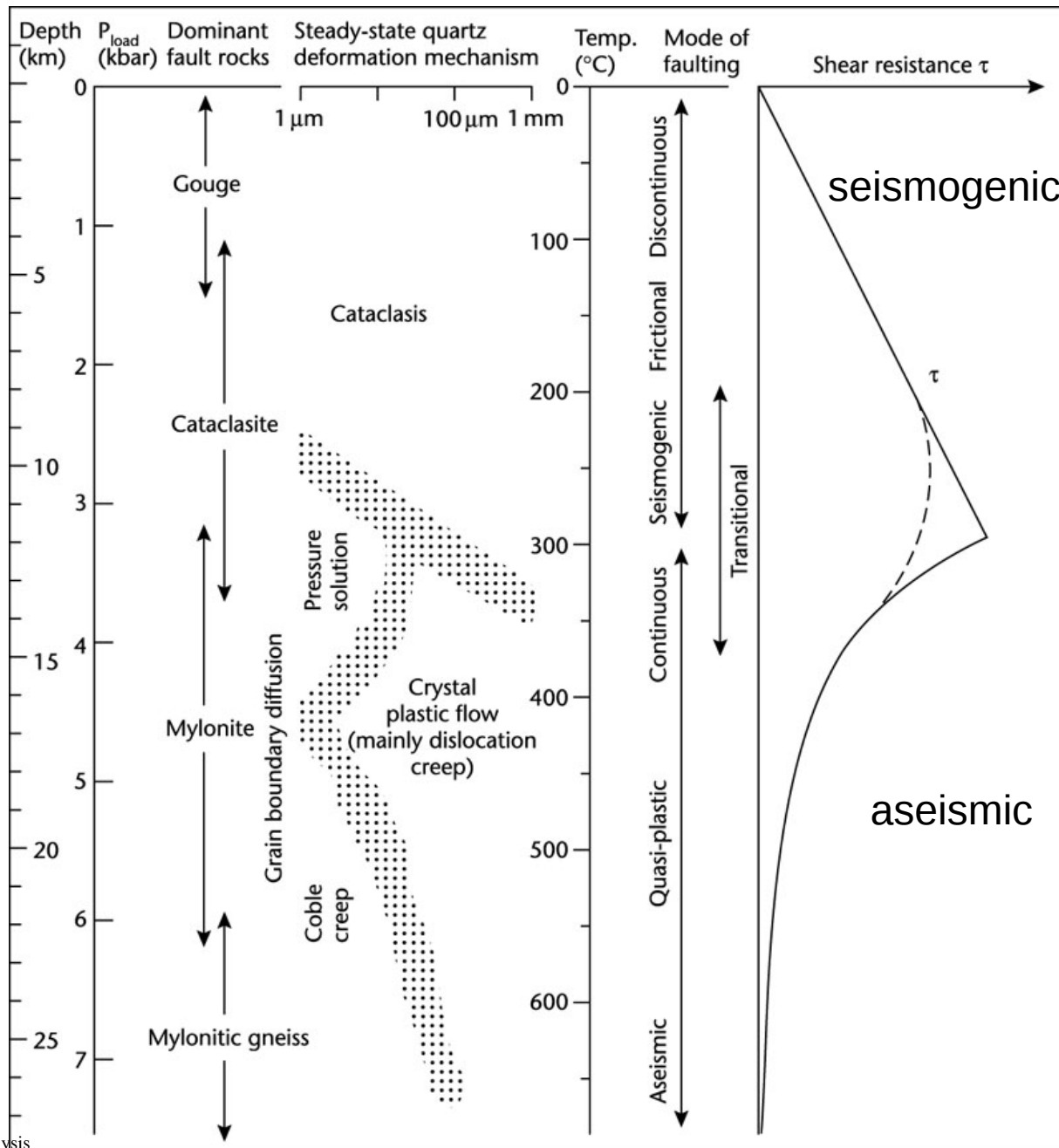


Fig. 2.33 Sibson's (1983) conceptual model for a major fault zone in the continental crust showing the dominant deformation mechanisms.

300 °C

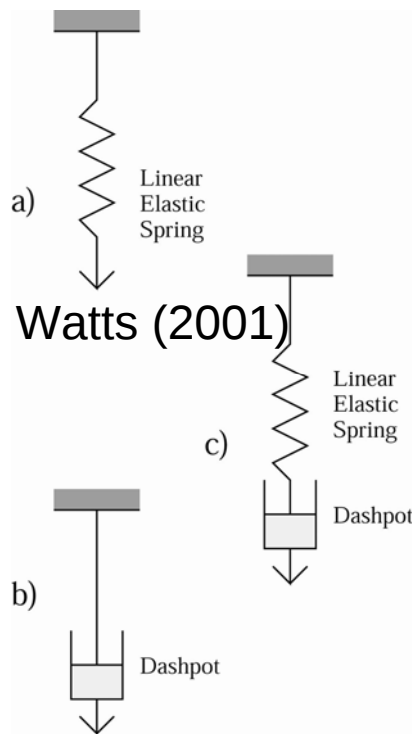
450 °C

Fully ductile
continuous
deformation

2.4.4 Viscoelasticity

A material that behaves as an elastic solid on short time scales but viscously on long time scales is known as a viscoelastic or Maxwell material.

(a) Elastic model $\sigma = E \varepsilon_e$



(C) Maxwell viscoelastic model (a+b)

Total strain: $\varepsilon = \varepsilon_e + \varepsilon_v$

If the system is initially unstrained – that is, $\varepsilon=0$ at $t=0$ – then

$$\varepsilon_e = \frac{\sigma}{E} \quad \text{and} \quad \dot{\varepsilon}_v = \frac{d\varepsilon_v}{dt} = \frac{\varepsilon_v(t) - \varepsilon_v(0)}{t - t_0} = \frac{\varepsilon_v}{t} \longrightarrow \varepsilon_v = \frac{t\sigma}{\eta}$$

$$\therefore \varepsilon = \frac{\sigma}{E} + \frac{\sigma \cdot t}{\eta}$$

The viscoelastic strain is made up of both an elastic and a viscous part.

Viscoelastic relaxation time (τ):

The time when the stress relaxes to 1/e of its original.

$$\tau = \frac{2\eta}{E}$$

For asthenosphere, τ is of the order of 30 to 40 years.
For lithosphere ~ 0.1 to several Myr or 1 to 100 Myr (Watts, 2001, p.244)

(b) Viscous model

$$\sigma = \eta \dot{\varepsilon}_v$$

Parameters:

E = Young's modulus

η = viscosity

ε_e = elastic strain

$\dot{\varepsilon}_v$ = viscous strain rate

σ = stress

τ = Maxwell relaxation time

2.4.5 Elastic-perfectly plastic rheology

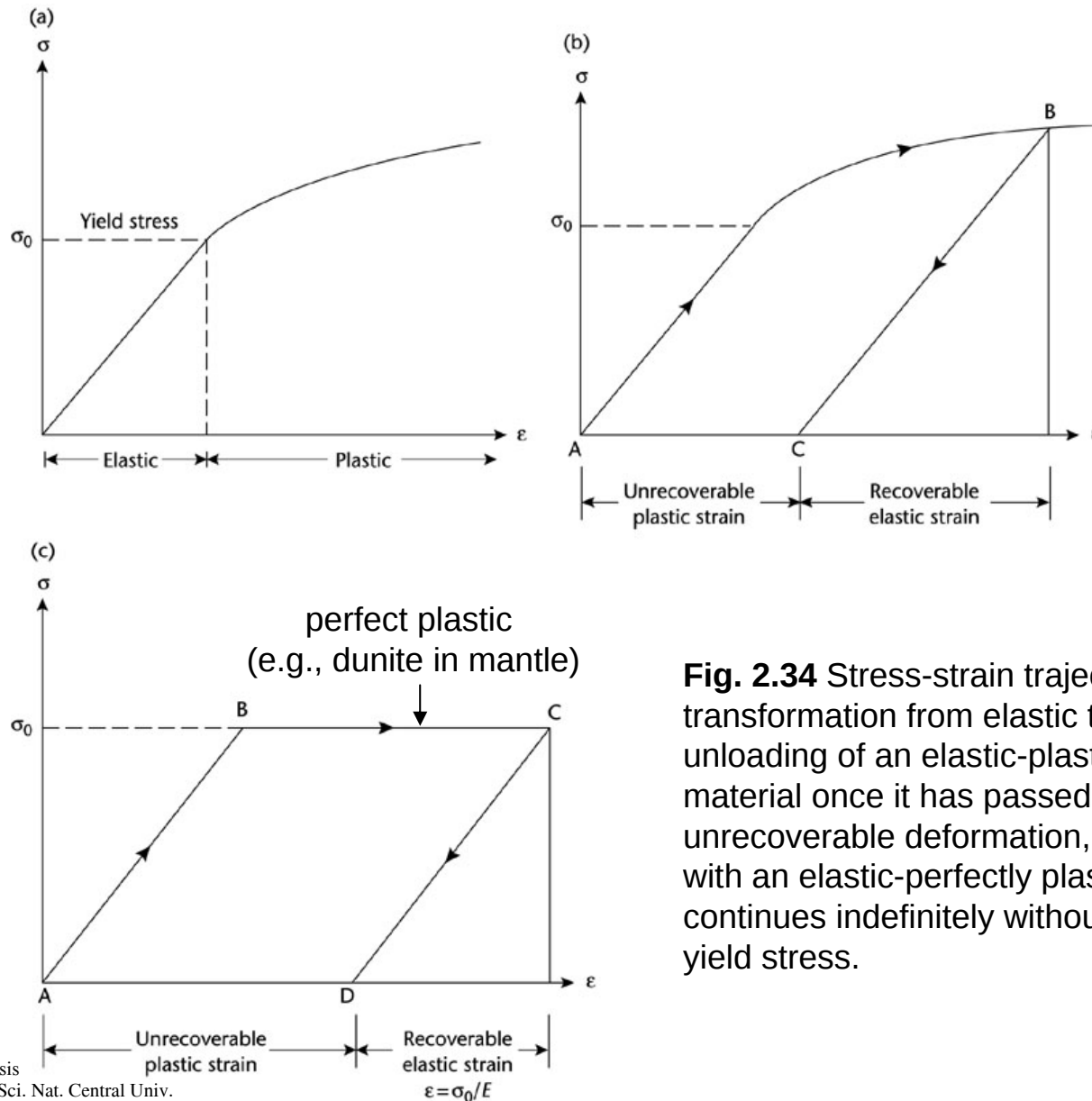


Fig. 2.34 Stress-strain trajectories for (a) a solid showing a transformation from elastic to plastic behavior; (b) loading and unloading of an elastic-plastic material. Unloading of the material once it has passed into the plastic field results in an unrecoverable deformation, or plastic strain; (c) a material with an elastic-perfectly plastic rheology. Plastic strain continues indefinitely without any addition of stress above the yield stress.

2.4.6 Strength profiles of the lithosphere

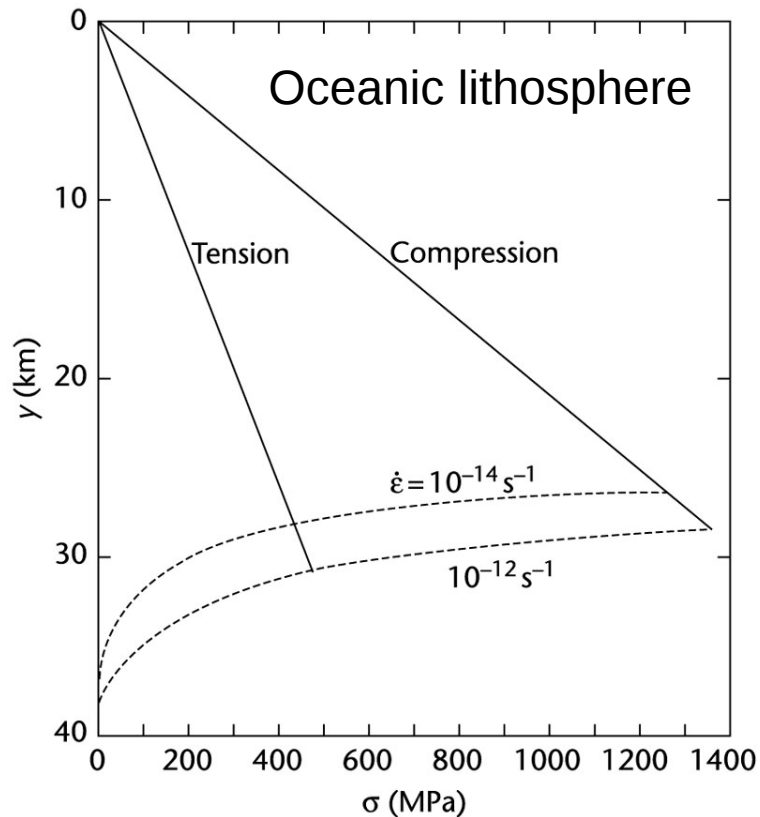


Fig. 2.35 Strength profile for the oceanic lithosphere calculated using equation (2.92) for tension and equation (2.93) for compression. The dashed lines are the stresses associated with creep in the lithosphere at the strain rates indicated.

Brittle deformation:

Tension:
$$\Delta \sigma_{xx} = \frac{-2f_s(\rho g y - p_w)}{(1 + f_s^2) + f_s} \quad (2.92)$$

Compression:
$$\Delta \sigma_{xx} = \frac{2f_s(\rho g y - p_w)}{(1 + f_s^2) - f_s} \quad (2.93)$$

where $\Delta \sigma_{xx}$ is the deviatoric stress, f_s is that static coefficient friction, $\rho g y$ is the lithostatic stress and p_w is the pore pressure.

Ductile deformation

$$\dot{\epsilon} = A \sigma^n \exp\left(\frac{-E_a}{RT}\right) \quad (2.94)$$

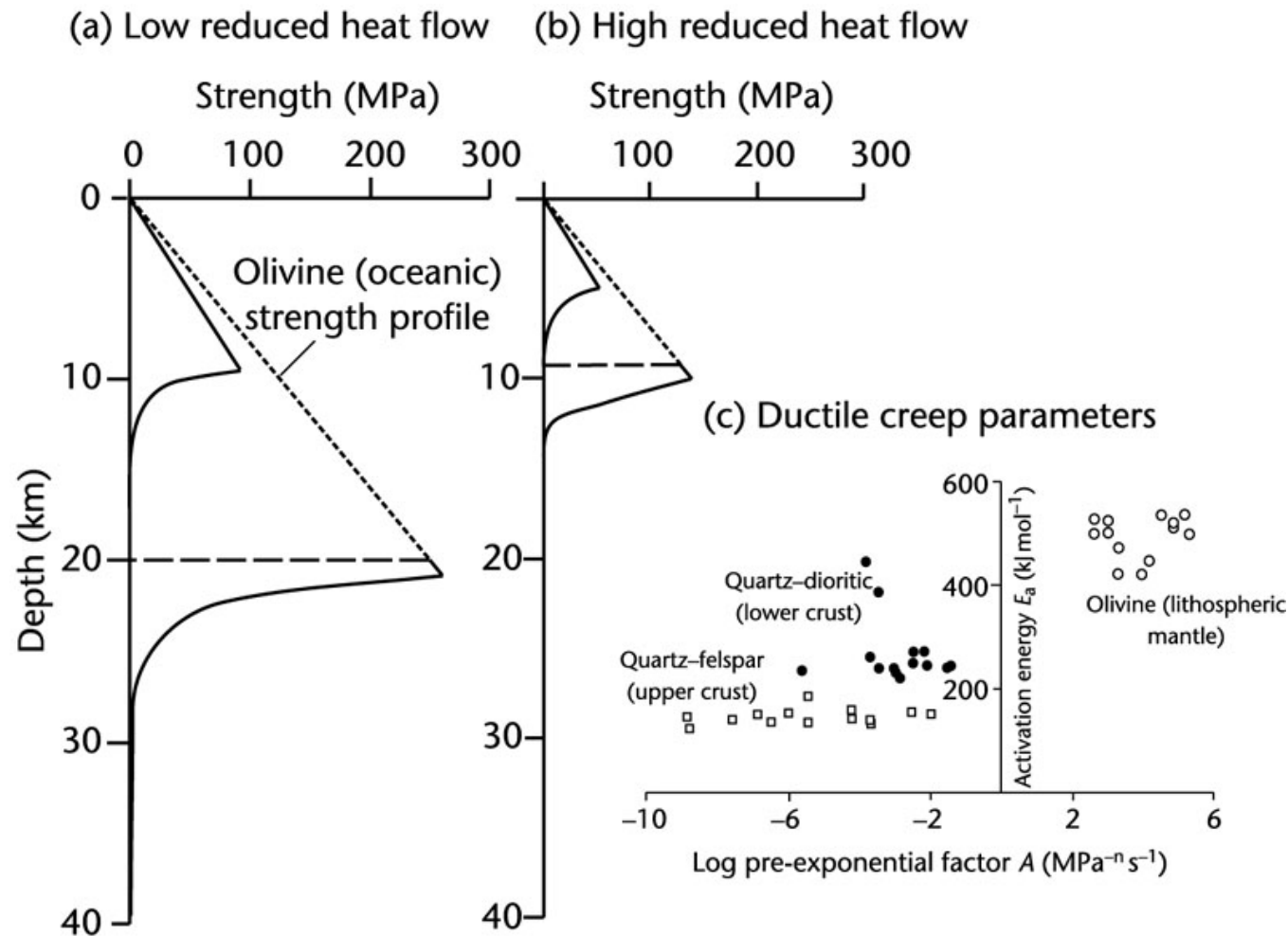


Fig. 2.36 Strength profiles of the **continental lithosphere** for different geothermal gradients, using (a) a low reduced heat flow of 25 mW m^{-2} for a cold continental shield area, and (b) a higher reduced heat flow of 59 mW m^{-2} for an area undergoing extension. $\kappa = 2.5 \text{ W m}^{-1} \text{ K}^{-1}$, $A_0 = 2.1 \text{ m W m}^{-3}$, $T_0 = 15^\circ \text{C}$, Universal Gas Constant $= 8.31451 \text{ J mol}^{-1} \text{ K}^{-1}$, strain rate is 10^{-12} s^{-1} , $n=3$. For the olivine rheology, $A = 10^4 \text{ MPa}^{-3} \text{ s}^{-1}$, $E_a = 500 \text{ kJ mol}^{-1}$, and for brittle failure under tension $f_s = 0.6$ and $\rho = 3300 \text{ kg m}^{-3}$. For the lower crustal rheology, $A = 100 \text{ Pa}^{-3} \text{ s}^{-1}$, $E_a = 300 \text{ kJ mol}^{-1}$, and for brittle failure under tension $f_s = 0.6$ and $\rho = 2750 \text{ kg m}^{-3}$. (c) shows plot of activation energy E_a versus the pre-exponential factor A of a range of rock types typical of the lithospheric mantle, lower crust, and upper crust (derived from Table 1, Fernandez and Ranalli 1997).