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Does apparent stress vary with earthquake size?

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Abstract. Seismic energy is distributed across a wide frequency band so that limited bandwidth recording can lead to substantial underestimates of the radiated seismic energy or introduce an artificial upper bound of radiated energy. We estimate an adjustment factor to account for the probable missing energy and apply it to three previously studied data sets with limited recording bandwidth. We find that this adjustment, together with accounting for possibly missing events, eliminates much of the moment dependence of radiated energy found previously. We obtain a nearly constant ratio of radiated energy to seismic moment, 3×10^{-5}, or 1 MPa of apparent stress drop, over 17 orders of seismic moment. This suggests that deviation from similarity of the energy radiation for seismic events essentially the entire observable range of earthquake size may not yet be resolved.

Introduction

The energy radiated from the seismic source is a fundamental source parameter and can be estimated from the energy flux by integrating the squared velocity seismogram in the time or frequency domains [e.g., *Kanamori et al.,* 1993; *Choy and Boatwright,* 1995]. *McGarr* [1999] summarized the results for earthquakes of various sizes and found that the maximum of apparent stress, $\Delta\sigma_a$, defined as the rigidity multiplied by the ratio between radiated energy and seismic moment, is nearly constant over 19 orders of seismic moment. The major data sets in his study are the micro earthquake studies of Underground Research Laboratory, Canada [*Gibowicz et al.*, 1991] and KTB borehole, Germany [*Jost et al.*, 1998], the micro earthquake studies of *Abercrombie* [1995] and the relatively large earthquake studies of *Kanamori et al.* [1993] in California.

Despite the nearly uniform apparent stress over such a wide range of earthquake size, individual studies show strong size dependence. *Kanamori and Heaton* [2000] interpret the combined trend seen in the results of *Abercrombie* [1995] and *Kanamori et al.* [1993] as a difference in frictional behavior during rupture between small and large earthquakes. *Abercrombie* [1995] and *McGarr* [1999] both suggested trends might arise from underestimation of energy due to limited recording bandwidth.

Boore [1986], *Di Bona and Rovelli* [1988], and *Singh and Ordaz* [1994], have previously discussed the influence of finite bandwidth on the estimation of source parameters. They demonstrated that radiated energy can be severely underestimated when high frequencies are not recorded. In this paper, we show that seismic energy is widely distributed in more than a decade of frequency assuming an omega square model, and account for this bias to obtain an adjustment value of the radiated seismic energy.

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Effect of Finite Bandwidth on Energy Estimation

We first examine the range of frequencies that make a significant contribution to the estimate of the energy flux. We follow *Boore* [1986], *Di Bona and Rovelli* [1988] and assume a simple omega square model [*Aki*, 1967]. We approximate the velocity spectrum as:

$$\dot{u}(f) \approx M_o f/(1+(f/f_0))^2 \,, \tag{1}$$

where M_o is seismic moment and f_0 is the corner frequency [*Brune*, 1970]. Squaring and integrating, the estimate of radiated energy E is proportional to:

$$E \approx (1/2) M_o^2 f_0^3 [F(f,f_o)]_0^\infty = (1/4)\pi M_o^2 f_0^3 \tag{2}$$

$$F(f,f_o) = (-f/f_0)/(1+(f/f_0))^2 + \arctan(f/f_0) \tag{3}$$

In practice, the highest frequency is fixed by some cutoff value determined by instrumental characteristics and/or attenuation. When the upper limit is f_M, equation (2) is changed to

$$(1/2) M_o^2 f_0^3 [F(f,f_o)]_0^{f_M} = (1/2) M_o^2 f_0^3 F(f_M, f_o). \tag{4}$$

The ratio R between the estimated energy and the true energy is a function of f_M and f_0,

$$R(f_M, f_o) = (2/\pi) F(f_M, f_o). \tag{5}$$

This is the same equation derived by *Di Bona and Rovelli* [1988]. Figure 1 shows the shape of this function together with original form of omega-square velocity spectrum. Equation (5) indicates that over 80% of the radiated seismic energy will be carried by waves of higher frequency than the corner frequency. Fig. 1 also shows that integration up to approximately ten times the corner frequency is necessary to approach 90% of the seismic energy. This condition is not often met for seismic observations and suggests that estimates of the seismic energy that don't account for this effect may be biased.

Some studies use a different style of omega-square model [*Boatwright*, 1978] as

$$\dot{u}(f) \approx M_o f/\sqrt{1+(f/f_0)^4} \,. \tag{6}$$

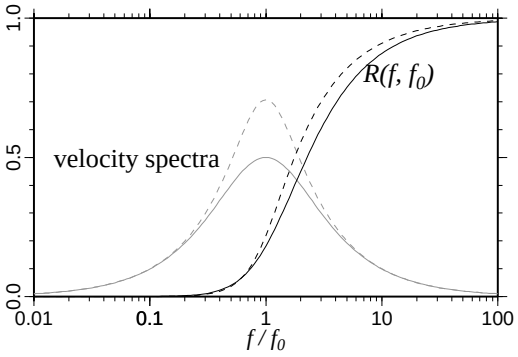

Figure 1. The shape of omega-square velocity spectrum and function R in eqs. (1) and (5) (solid lines) and eqs. (6) and (7) (dotted lines).

In this case, the ratio R is given as

$$R(f_M, f_o) = \frac{1}{2\pi}\{\log \frac{1-\sqrt{2}f_M/f_0+(f_M/f_0)^2}{1+\sqrt{2}f_M/f_0+(f_M/f_0)^2} + 2(\arctan(1+\sqrt{2}f_M/f_0) - \arctan(1-\sqrt{2}f_M/f_0))\}. \quad (7)$$

The corresponding curves for this model are also shown in Fig. 1 and while the energy is somewhat more concentrated near the corner frequency, the possibility of a strong bias due to limited recording bandwidth persists.

Re-interpretation of Previous Results

We now re-examine results of energy estimation from four studies summarized by *McGarr* [1999]. The following calculation rests on the assumption that the seismic moment is related to the corner frequency as *Brune* [1970, 1971] assumes, namely:

$$\Delta\sigma = (7/16)M_o(2\pi f_0/2.34\beta)^3 \quad (8)$$

where $\Delta\sigma$, is the stress drop, β is the shear wave velocity, which we assumed to be 3.5 km/s.

Kanamori et al. [1993] estimated the energy of 66 events from $1 \le M_L \le 7$ in California. For the 19 largest events, the ratio between energy and seismic moment shows a slight size dependence for the events smaller than M_w 4 (Figure 2a). As discussed by *Abercrombie* [1995], the frequency band for their result is limited to less than 7 Hz and such size dependence is reasonable. Assuming f_M of 7 Hz and stress drop of 1 MPa and 10 MPa, we can explain a slight decrease of E/M_o; however, the decrease seen in the data is larger and requires a stress drop of 100 MPa to be explained purely as an effect of limited recording bandwidth. This is unreasonably high for an average stress drop.

In the observations of microearthquakes, bandwidth constraints are much more serious. The instrument used in KTB has the Nyquist frequency of 500 Hz [*Jost et al.*, 1998]. A curve based on a value of f_M = 350 Hz, 75% of Nyquist frequency, fits the data fairly well (Fig. 2b). Moreover there is also the possibility of a limit imposed by a site-controlled f_{max} [*Hanks*, 1982]. In their data, the corner frequency only changes from 55 Hz to 108 Hz, while seismic moment changes by a factor of 1000. Although they emphasized that the correction of attenuation is complete, we suspect this almost constant corner frequency may be associated with the site-controlled f_{max}. If we assume f_M to be 100 Hz, the curves fit the data well at a reasonable earthquake stress drop of 1 or 10 MPa. Since a corner frequency of 100 Hz corresponds to stress drop and

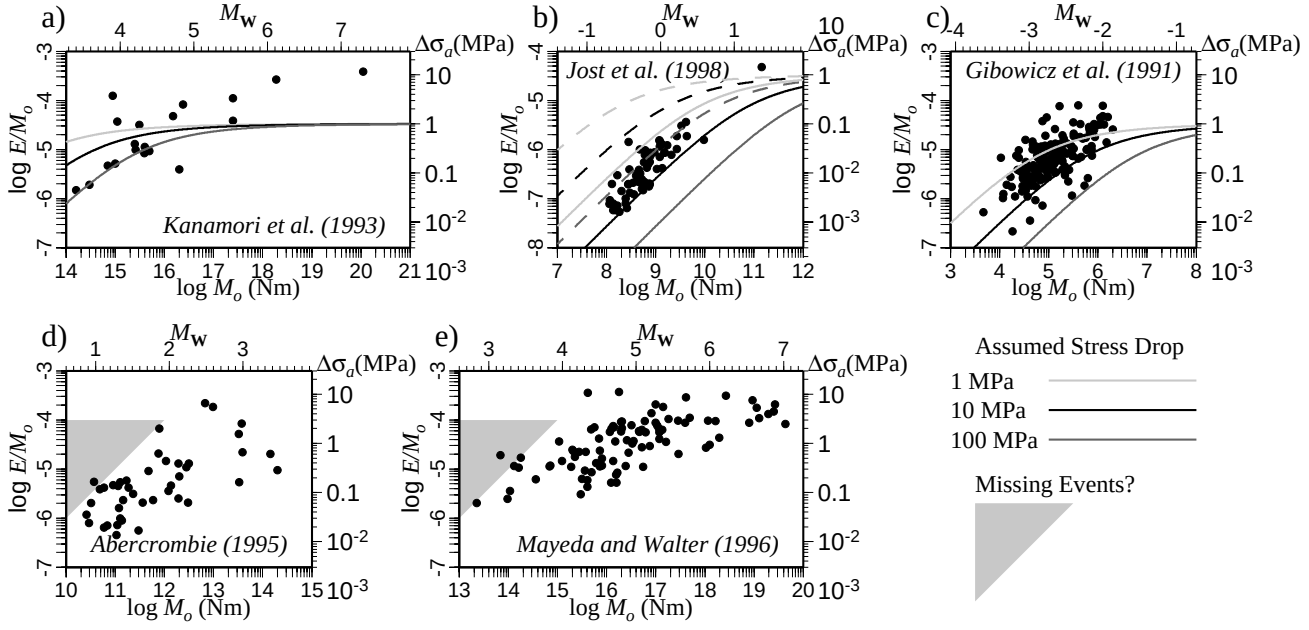

Figure 2. Energy-seismic moment ratio and apparent stress calculated with a rigidity of 30 GPa. a) Data from *Kanamori et al.* [1993]. Black and two gray curves represent possible underestimation curves assuming a geometrical similarity with constant stress drops as shown. Assumed f_M is 7 Hz. b) Data from *Jost et al.* [1998]. Solid curves are calculated for f_M of 100 Hz. Dotted curves are calculated for f_M of 350 Hz. c) Data from *Gibowicz et al.* [1991]. Curves are calculated with f_M of 500 Hz. d) Data from *Abercrombie* [1995]. Gray triangle represents the area where possibly missing events would locate. e) Data from *Mayeda and Walter* [1996].

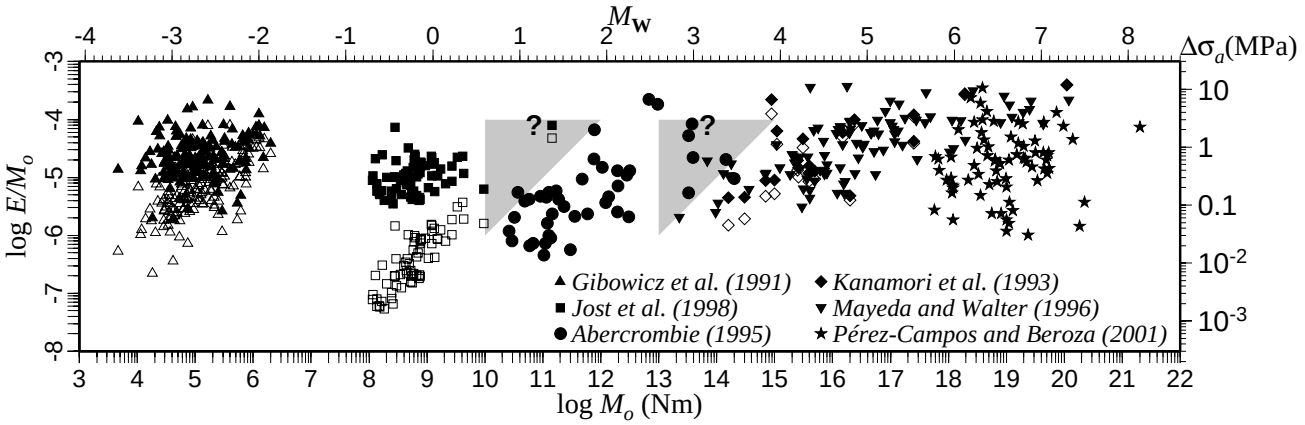

Figure 3. Energy-seismic moment ratio and apparent stress. Each symbol denotes a different data set as shown. Solid symbols are adjusted values or those that already accounted for the missing energy. Open symbols are the original values. Two gray triangles are the area where possibly missing events would locate.

seismic moment of 1 MPa and 5×10^9 N-m, respectively, this assumption means that most of the corner frequencies in their study may be higher than the upper limit of observation.

Gibowicz et al. [1991] determined the energy and the moment of 155 earthquakes $-3.6 \leq M_w \leq -1.9$. Because the bandwidth in their study ranges from 0.5 kHz to 5 kHz, it spans only one decade in frequency and there is a strong likelihood that energy is underestimated for almost all events. Here again, the simulated curves fit data well with a reasonable stress drop of 1 MPa and 10 MPa (Fig. 2c).

The events of *Abercrombie* [1995] are in the range $-1 \leq M_L \leq 5$. In the present study, we use 43 events with seismic moments larger than 10^{10} N-m. The data is obtained at 2.5-km depth borehole and omega-square high-frequency asymptote is clear up to 100 Hz. Since she analyzed only events with corner frequencies five times lower than the upper limit of analysis range ($\sim$ 150 Hz), little adjustment is necessary for these events. However, this event selection criterion may introduce an artificial trend in the seismic moment range from 10^{10} to 10^{12} N-m. In Fig. 7 of *Abercrombie* [1995] there are a number of events with corner frequencies higher than 30 Hz and the energies of these events would naturally be higher than those in the same moment range with lower corner frequency that would not be excluded. Since energy scales with the square of the seismic moment when the corner frequency is the same (eq. (2)), neglecting these events introduces an artificial upper limit and these excluded events would be located in or above the gray triangular field shown in Fig. 2d. If these events were included in the same figure, the scaling between apparent stress and seismic moment would be less significant.

A similar criterion is used in the study of *Mayeda and Walter* [1996]. They estimated energies of western United States earthquakes with magnitudes ranging from $3.3 \leq M_w \leq 7.3$. They analyzed only events with 70 % of the energy in the observed frequency range of up to 10 Hz. This corresponds to excluding events with corner frequencies larger than about 3 Hz. As shown by *Abercrombie* [1995], there are many earthquakes of $M_w 3-4$ having higher corner frequencies than 3 Hz and it is likely that if the energy of these events were estimated it would lie in or above the triangular field in Fig. 2e. Since we can see the scaling between apparent stress and seismic moment even above $M_w 5$ some systematic variation remains after including these events.

Constant apparent stress over 17 orders of seismic moment

In the previous section, we showed how limited recording bandwidth can introduce an apparent scaling of the apparent stress with seismic moment. Next, we summarize all the data together assuming the underestimated energy can be adjusted using R in equation (5). Assumed values of f_M are 500, 100, and 7 Hz for *Gibowicz et al.* [1991], *Jost et al.* [1998], and *Kanamori et al.* [1993], respectively. The values of stress drop are 3 MPa for *Kanamori et al.* [1993] and 1 MPa for other two studies.

Figure 3 shows the adjusted data (*Gibowicz et al.* [1991], *Jost et al.* [1998], and *Kanamori et al.* [1993]) and original data with missing events region (*Abercrombie* [1995] and *Mayeda and Walter* [1996]) together with the study of *Pérez-Campos and Beroza* [2001], in which they estimated radiated energy using far-field body wave and added missing energy by extrapolation of high-frequency asymptote. From their original data, we excluded the earthquakes that occurred near subduction zones and ridges where the tectonic environment is markedly different from the continental setting of the other studies.

As pointed out by *McGarr* [1999], the ratio between energy and seismic moment has an uniform upper bound. In the present study the variance about a constant apparent stress value decreases and most of the data are in the range between 0.1 and 10 MPa in apparent stress over the entire range of earthquake sizes. Some data of *Kanamori et al.* [1993] and *Mayeda and Walter* [1996] have large E/M_o ratio for $M_w > 6$. However, the estimation of energy for large events is problematic. It may depend on event type [*Choy and Boatwright*, 1995; *Shi et al.*, 2000; *Pérez-Campos and Beroza*, 2001], and even more strongly on the estimation method [*Singh and Ordaz*, 1994; *Pérez-Campos and Beroza*, 2001]. In particular, estimates based on regional versus teleseismic data often disagree with each other by an order of magnitude for the same earthquake. *Boatwright et al.* [2000] have suggested that estimates of radiated energy based on regional data may be biased too high by a factor of approximately five.

It should be noted that geometrical similarity based on constant stress drop has been assumed in our calculations for the adjustment factor that accounts for the unobserved part of the spectrum, and that assumption may break down at small or large magnitudes. There is, however, no such assumption on the measured energy radiation. So our conclusions on the similarity of energy radiation should not be strongly dependent on the assumption of geometric similarity of the source.

Conclusions

Finite bandwidth affects radiated energy estimates as shown in Fig. 2. Although we cannot prove it for each data set since there is no observation beyond the upper frequency limit of each study, we suspect that many previous energy estimates are underestimates. Once we account for the probable effects of band limitation, we obtain an almost constant ratio of radiated energy to seismic moment over 17 orders of seismic moment. The ratio is approximately 3×10^{-5}, or ~ 1 MPa of apparent stress drop. This suggests that the similarity of seismic events as expressed by the radiated seismic energy may hold over the entire observable range of earthquake size. Perhaps the strongest evidence for a break in scaling of energy is from borehole recordings for events in the range of M 1 - 2 where both *Abercrombie* [1995] and *Prejean and Ellsworth* [2001] find a decrease in apparent stress with decreasing moment. A universal break in scaling in this size range could occur if the *Jost et al.* [1998] energy values, which are highly uncertain, are lower than we estimate, and if the *Gibowicz et al.* [1991] results pertain to fracture of fresh rock, rather than frictional failure. Further study of borehole seismograms should allow a resolution of this issue.

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