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IS THE SEQUENCE OF EARTHQUAKES IN SOUTHERN CALIFORNIA, WITH AFTERSHOCKS REMOVED, POISSONIAN?

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Abstract

Yes.

Early attempts to determine whether or not earthquake events were Poisson-distributed led to the conclusion that main sequence events were significantly non-Poissonian, whether worldwide catalogs (Aki, 1956), or local catalogs (Knopoff, 1964) were considered. Such a conclusion is of importance in earthquake prediction; a demonstration of nonrandomness in earthquake catalogs gives rise to the hope that a basis for prediction of occurrence times can be found by decoding the sequential message in the list of events. Conversely, if earthquakes are random events in time, no hope exists for finding order in earthquake catalogs and, hence, attempts to predict times of occurrence based on studies of catalog sequences are very likely doomed. Nevertheless, a result which shows that earthquake sequences are Poissonian can be of use in estimating seismic risk (Molchan, *et al.*, 1970) by virtue of the simple statistical models which result; we assume that more complicated statistical models will make the analytical procedures more complicated, although such analyses have not been carried out in detail.

Aftershocks are earthquake events which are causally connected with a parent event which is usually large. Typical earthquake catalogs must be non-Poissonian if they are mixtures of two populations, aftershock clusters which are not Poissonian, and main-sequence events which may or may not be. Interest in randomness or nonrandomness of earthquake sequences centers on main-sequence events. Immediately after a large earthquake, numerous aftershocks occur on a short time scale. Later in aftershock sequences, the time interval between earthquakes becomes longer. The clustering property implies short-term predictability.

The success of any attempt to study the randomness of main-sequence events depends on the skill with which aftershocks are identified and removed from catalog listings. If this is not done skillfully, the residual is certain to be non-Poissonian. In one of the first attempts to delete aftershocks from an earthquake catalog (Knopoff, 1964), it was found that successively more realistic definitions of aftershocks led to main-sequence catalogs which were successively more Poissonian. Nevertheless, after the best attempt to remove aftershocks was made, a significantly non-Poissonian residual catalog remained. The result of this early investigation left the basic question unresolved: Was the non-Poissonian result the consequence of a still incomplete identification of aftershocks, or is there a genuine non-Poissonian character to a catalog of main-sequence earthquakes (i.e., was the removal of aftershocks the best that could be done)? We describe an empirical method of culling aftershocks from an earthquake catalog, which is an alternative to the earlier (Knopoff, 1964) procedure. The main-sequence list that remains is Poissonian. We believe that this analysis demonstrates the inadequacy of the earlier technique to cull aftershocks.

In the earlier technique to separate aftershocks (Knopoff, 1964), the occurrence of intervals of time in which an excessive number of earthquakes occurred was taken to be an identifying marker that an aftershock sequence was present in the interval. Adjoining intervals were deleted as well. The disadvantage of this technique is that some main shocks are also deleted from the catalog, while some short aftershock sequences remain. This procedure, as noted, led to non-Poissonian results for the main shock residual.

A more reliable procedure for identifying aftershocks is the manual compilation of a list of earthquakes occurring near a large event in both space and time. Such lists are commonly part of the documentation associated with the description of selected large earthquakes, but the compilation of such lists for smaller earthquakes, such as main shocks of magnitude 4 or 5, is not normally made in view of the amount of labor involved in this type of work.

We attempted a rough estimation of the second procedure in the following way. The list of Southern California earthquakes was scanned visually to identify events close in space and time to other events, in as thorough a manner as possible. A plot of the durations T of aftershock sequences was made as a function of the magnitude M of the largest shock in the sequence. A least upper bound (*lub*) or envelope to the $T(M)$ data of the form

$$\log T = a_1 M + b_1$$

was found to a reasonable approximation so that all aftershock sequences had $T(M)$ values below the value given by the expression. The extent of the aftershock zone was estimated from the magnitude-length $M(L)$ data derived from the small number of fault trace observations of Southern California earthquakes. A *lub* relationship of the form

$$\log L = a_2 M + b_2$$

was selected in a similar way to the $T(M)$ bounds. These *lub's* were enlarged slightly to account for stray series that might have been missed or might occur in the future.

These *lub's* were programmed for computation in such a way that a window $[T(M), L(M)]$, of the form above, was applied to the entire catalog. For each event in the catalog, a scan within distance $L(M)$ and time $T(M)$ was initiated. In this computational procedure, the first shock is not necessarily the largest shock in the sequence; thus a small foreshock is considered to be the first event of an aftershock sequence. The largest shock in the series often enlarges the window beyond the value used for the first shock in the series. The residual catalog was visually scanned for inadequacies in the identification and the windows were adjusted appropriately. An application of this windowing procedure has been given elsewhere (Knopoff and Gardner, 1972).

The windowing technique was applied to the catalog of earthquakes in Southern California described by Allen *et al.* (1965). We report the results of winnowing of aftershocks applied to two subsets of the Southern California catalog, namely, for the "Southern California Local Area" (SCLA) and the "Southern California Local Area with Baja California Extension" (SCBE) catalogs. From each of these two catalogs, further subcatalogs were derived:

SCLA, 1932 through 1971, all events with $M \geqq 3.8$
SCLA, 1952 through 1971, all events with $M \geqq 2.8$
SCBE, 1932 through 1971, all events with $M \geqq 3.8$
SCBE, 1952 through 1971, all events with $M \geqq 2.8$

The threshold magnitudes 3.8 and 2.8 were derived from a study of reliability of reporting

TABLE 1

WINDOW ALGORITHM FOR AFTERSHOCKS

M	L (km)	T (days)
2.5	19.5	6.
3.0	22.5	11.5
3.5	26.	22.
4.0	30.	42.
4.5	35.	83.
5.0	40.	155.
5.5	47.	290.
6.0	54.	510.
6.5	61.	790.
7.0	70.	915.
7.5	81.	960.
8.0	94.	985.

of events by Knopoff and Gardner (1969). A listing of selected values for the windows is given in Table 1; the computational routine uses an interpolation among the values listed. As an example, any earthquake within 510 days after a magnitude $M = 6.0$ earthquake, and with epicenter within 54 km of the epicenter of the $M = 6$ shock, was identified as an aftershock. For $M > 6.4$, the slope of the $T(M)$ window is less than for $M < 6.4$ to conform with improved estimates of the shape of the envelope. We do not have any strong affection for these particular windows. The reader is free to try other models.

The result of applying the windowing routine to the four catalogs is given in Table 2. The fraction of earthquakes identified as aftershocks is surprisingly high. Approximately

TABLE 2

PROBABILITY OF FIT OF SOUTHERN CALIFORNIA EARTHQUAKES

Catalog	Dates (inclusive)	Lower Magnitude Threshold	Total No. of Shocks	No. of First Shocks	χ^2	n-Degrees of Freedom	$\chi^2_{95 \text{ per cent}}$	Interval (days)
SCLA	1932–71	3.8	1751	503	0.62	2	5.99	10
SCLA	1952–71	2.8	4868	1654	3.70	6	12.59	10
SCBE	1932–71	3.8	2331	647	0.50	2	5.99	10
SCBE	1952–71	2.8	5506	1841	5.76	6	12.59	10

$P(\chi^2 < \chi^2_{95 \text{ per cent}})_n = 0.95$

two-thirds of the events of each of the four raw subcatalogs were identified as aftershocks. A visual inspection or the catalog showed no significant number of events that were not identified by the earlier aftershock algorithm. The shocks remaining after aftershocks are removed are identified as "first shocks" in Table 2.

The catalog with aftershocks removed can now be analyzed for randomness. We can analyze the sequence of largest shocks in each aftershock series as well as the sequence of first shocks in each aftershock series. We choose the latter sequence to report here. This probably represents the events unbiased by the aftershock sequences, although on the time scale of this analysis, the two sequences are remarkably similar to each other; often, but not always, the first shock is also the largest.

We divide the interval 1932 through 1971 or 1952 through 1971 into decades, or 10-day intervals (Knopoff, 1964), and count the number of decades with N first shocks. The results could be used as input data for drawing histograms. The observations are compared with the values predicted by Poisson random processes by χ^2 calculations. In each case, the hypothesis that the catalog of first shocks was generated by a Poisson random process is accepted at a probability level of better than 95 per cent.

We conclude that the removal of aftershocks by the windowing procedure leads to residual catalogs which are Poissonian in character for local catalogs. This is at variance with predictions based on stochastic models in a sequence of earthquake occurrence (Knopoff, 1971). The reasons for this will be discussed separately.

We will also discuss elsewhere the frequency-magnitude relations obtained for the various catalogs and their fragments. Nevertheless we note that, as usual, the catalogs are dominated numerically by the smallest shocks in the listing. Thus, the residual catalogs, after as many aftershocks are removed as we can identify, are influenced mainly

TABLE 3

PROBABILITY OF FIT OF SOUTHERN CALIFORNIA EARTHQUAKES

Catalog	Dates (inclusive)	Lower Magnitude Threshold	χ^2	n-Degrees of Freedom	χ^2_{95} per cent	Interval (days)
SCLA	1932–1971	3.8	0.62	2	5.99	10
		4.3	4.26	2	5.99	50
		4.8	1.13	1	3.84	50
		5.3	2.59	1	3.84	300
SCBE	1932–1971	3.8	0.50	2	5.99	10
		4.3	1.25	2	5.99	50
		4.8	2.32	1	3.84	50
		5.3	0.49	1	3.84	300

by what is apparently a strong Poissonian character to the smallest shocks. The larger shocks have only a small influence on the statistical character of the ensemble. To study the time relationship among larger events, we have analyzed our longest catalogs, 1932 to 1971 inclusive, retaining only the largest events. A succession of such analyses for catalogs with $M \geqq 3.8$, $M \geqq 4.3$, $M \geqq 4.8$ and $M \geqq 5.3$, with aftershocks removed, is presented in Table 3. We see no significant change in the character of these subcatalogs from the largest catalog with the smallest magnitude shocks. We see no evidence for any less disorder with increasing magnitude. There are insufficient events with $M \geqq 5.8$ to draw any conclusions.

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REFERENCES

Aki, K. (1956). Some problems in statistical seismology, *Zisin* **8**, 205–228.

Allen, C. R., P. St. Amand, C. F. Richter, and J. M. Nordquist (1965). Relationship between seismicity and geologic structure in the Southern California region, *Bull. Seism. Soc. Am.* **55**, 753–797.

Knopoff, L. (1964). Statistics of earthquakes in Southern California, *Bull. Seism. Soc. Am.* **54**, 1871–1873.

Knopoff, L. (1971). A stochastic model for the occurrence of main sequence earthquakes, *Rev. Geophys.* **9**, 175–188.

Knopoff, L. and J. K. Gardner (1969). Homogeneous catalogs of earthquakes, *Proc. Nat. Acad. Sci.* **63**, 1051–1054.

Knopoff, L. and J. K. Gardner (1972). Higher seismic activity during local night on the raw worldwide earthquake catalog, *Geophys. J.* **28**, 311–313.

Molchan, G. M., V. I. Keilis-Borok, and G. V. Vilkovich (1970). Seismicity and principal seismic effects, *Geophys. J.* **21**, 235–411.

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