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Geological noise in magnetotelluric data: a classification of distortion types

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ABSTRACT

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Decomposition of the magnetotelluric impedance tensor into parameters relevant to a general Earth model that allows for galvanic distortion and regional induction has become a powerful data evaluation tool. Two similar techniques that incorporate superimposition of local three-dimensional and regional two-dimensional structures are considered. Both techniques have two serious limitations: (1) the conductivity structure might be less complex than assumed in the general model and therefore irrelevant model parameters are derived; (2) the regional conductivity structure may be more complicated than indicated by a two-dimensional model. The first problem is addressed in this paper by considering seven classes of general model of increasing complexity. Procedures are suggested that can be used to assign a particular datum to only one of the model classes. Therefore dimensionality parameters are suggested which include conventional and regional skew as well as local and regional structural dimensionality indicators. To address the second problem, an extension of the decomposition technique is presented that allows for a departure from the purely two-dimensional case for regional structures. An example, together with field data, is provided from the German deep drilling site. It explains how the decomposition technique recovers the two impedance phases belonging to a large regional anomaly although the impedance tensors are influenced by strong local distortion. This example also illustrates how the length scale of inductive structures can be estimated from the frequency dependence of the structural dimensionality parameters.

1. Introduction

The most important improvement in our understanding of experimental magnetotelluric data arises from techniques that evaluate all four complex elements of the magnetotelluric impedance tensor. These methods provide quantitative solutions for cases in which the measured impedance tensor does not conform to the ideal two-dimensional tensor. They may be split into two groups: (1) decomposition schemes which assume a priori general conductivity models and extract the parameters of a particular model from the elements of the tensor (Larsen, 1977; Zhang et al., 1987; Bahr, 1988; Groom and Bailey, 1989); (2) mathematical treatments of the impedance tensor as a rank 2 matrix (Eggers, 1982; Spitz, 1985; Cevallos, 1986; LaTorraca et al., 1986).

The latter group has recently been reviewed by Groom and Bailey (1990). The concepts offered by these techniques have seldom been applied to experimental data, probably because they do not take into account static shifts which seriously affect the measured impedance in many field situations. In contrast, in the decomposition schemes of the first group, a part of the general model is used to describe local conductivity structures that are responsible for static shifts. It has become

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evident that 'telluric distortion' is a combination of both static shifts and angular deviations of the induced telluric fields.

In this paper two recent decomposition techniques based on the principal superimposition model of a regional two-dimensional (2D) structure and a local three-dimensional (3D) structure (Bahr, 1988; Groom and Bailey, 1989) are considered in a synoptic manner. The 'telluric vector technique' of Bahr (1988) is expanded to take into account moderate departures of the regional structure from the exact 2D case. The resulting formulae provide analytical solutions for the model parameters, e.g. impedance phases, strike and skew, even for strongly distorted tensor impedances. Field data are treated from a magnetotelluric target area in the vicinity of the German deep drilling site where extreme distortions occur. For practical use, the telluric distortion is subdivided into seven 'classes' of increasing model complexity, and appropriate model parameters are derived for each class.

2. Basic concepts for the case of a departure from two-dimensionality

In magnetotellurics one assumes that in the frequency domain the horizontal electric field E and the horizontal magnetic field B are linked through a complex impedance tensor $\mathbf{Z}$:

$$E = \mathbf{Z}B, \mathbf{Z} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \tag{1}$$

Here and in the following sections, Cartesian coordinates (x, y) refer to observations, and coordinates (x', y') refer to a regional 2D structure, with x' normal to strike.

In the conventional 2D evaluation, only the off-diagonal elements Z_{xy} and Z_{yx} are interpreted by the use of 1D or 2D models. The main diagonal elements Z_{xx} and Z_{yy} are used only to compute the skew parameter, which is not interpreted by the model but rather describes the deviation of the conductivity distribution from the exact 2D case. In this section, two methods are reviewed that take into account all the elements of the impedance tensor.

(1) 'Telluric vectors' (Schmucker and Wiens, 1980; Bahr, 1988). Let $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ be unit vectors in the north and east direction, respectively. The two columns of the impedance tensor are presented in terms of two complex 'telluric vectors',

$$\mathbf{e}_x = Z_{xx}\hat{\mathbf{x}} + Z_{yx}\hat{\mathbf{y}} \text{ and } \mathbf{e}_y = Z_{xy}\hat{\mathbf{x}} + Z_{yy}\hat{\mathbf{y}} \tag{2}$$

$\mathbf{e}_x$ is the electric field induced by a magnetic field B_x which is linearly polarized in the north–south direction. Only in the special case where $Z_{xx} = 0$ is this electric field linearly polarized in the west–east direction. Similarly, $\mathbf{e}_y$ is induced by B_y. Graphically, each complex telluric vector $\mathbf{e}_x$ or $\mathbf{e}_y$ may be shown by two vectors: the real vector representing the electric field in phase with the inducing magnetic field, and the out-of-phase vector.

Although, the telluric vectors only provide a method of presentation of the information contained in the impedance tensor, the second method tries to adapt a principal conductivity model to this tensor. For example, Cagniard (1953) and Swift (1967) used models of a 1D and 2D conductivity distribution, respectively, and Swift (1967) was the first to propose the use of a parameter, the skew, as a measure of deviations from his principal model.

(2) The model of a local 3D anomaly over a layered Earth (Larsen, 1977). The local model can be thought of as a top layer structure of locally varying conductance. The model yields the general impedance tensor

$$\mathbf{Z} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 0 & Z_n \\ -Z_n & 0 \end{pmatrix} \tag{3}$$

Z_n is the normal impedance of the layered Earth. The distortion matrix elements a_{11}, a_{12}, a_{21} and a_{22} are real and independent of frequency at low frequencies, for which the top layer structure is small compared with the penetration depth. Therefore, all elements of the measured tensor must have the same phase if they are to be explained by eqn. (3).

Larsen (1977) evaluated long-period data with penetration depths as great as 100 km and more, and these data fulfilled the above condition. His general model, however, has also been applied to shorter periods. I will therefore present a dimensionality parameter that describes how much particular data depart from the model (3).

Consider the modified impedances

$$S_1 = Z_{xx} + Z_{yy}, \quad S_2 = Z_{xy} + Z_{yx},$$
$$D_1 = Z_{xx} - Z_{yy}, \quad D_2 = Z_{xy} - Z_{yx} \tag{4}$$

S_1 and D_2 are rotationally invariant. Swift's (1967) skew is

$$\kappa = |S_1|/|D_2| \tag{5}$$

The phase differences between two complex numbers $C1$ and $C2$ and the corresponding amplitude products are now abbreviated by the commutators

$$[C1, C2] = \mathrm{Im}(C2C1^*)$$
$$= \mathrm{Re}\, C1 \,\mathrm{Im}\, C2 - \mathrm{Re}\, C2 \,\mathrm{Im}\, C1$$

and

$$\{C1, C2\} = \mathrm{Re}(C2C1^*)$$
$$= \mathrm{Re}\, C1 \,\mathrm{Re}\, C2 + \mathrm{Im}\, C2 \,\mathrm{Im}\, C1 \tag{6}$$

(* indicates the complex conjugate). A rotationally invariant measure of phase differences in the impedance tensor is

$$\mu = (|[D_1, S_2]| + |[S_1, D_2]|)^{1/2}/|D_2| \tag{7}$$

It should be noted that this measure becomes unstable if the skew κ defined by (5) is very small. Larsen (1977) pointed out that only the phase is obtained, and instead of the magnitude of the normal impedance Z_n, the magnitude of a 'shifted' impedance $D \cdot Z_n$ is revealed. This is obvious from the decomposition of eqn. (3), which consists of six independent parameters, the four distortion matrix elements and the complex normal impedance. On the other hand, if $\mu = 0$, a measured impedance tensor will provide only five parameters: the common phase of all the tensor elements and multiples of the four distortion matrix elements:

$$\mathbf{Z} = \begin{pmatrix} a_{11}/D & a_{12}/D \\ a_{21}/D & a_{22}/D \end{pmatrix} \cdot \begin{pmatrix} 0 & DZ_n \\ -DZ_n & 0 \end{pmatrix} \tag{8}$$

The parameter D, which cannot be resolved from the measured tensor impedance, is referred to as the 'static shift' factor.

3. The principal superimposition model: a local 3D anomaly over a regional 2D structure

In an earlier paper (Bahr, 1988), Swift's (1967) and Larsen's (1977) model were combined to obtain a principal model that allows for both skew and phase differences in the tensor. This model is only briefly summarized here. It consists of a thin top layer of varying conductance over a regional 2D structure. It is referred to as the 'superimposition model' in the following discussion. In the (x', y') coordinate system of the regional 2D structure the impedance tensor is

$$\mathbf{Z} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 0 & Z_{nx'y'} \\ -Z_{ny'x'} & 0 \end{pmatrix}$$
$$= \begin{pmatrix} -a_{12}Z_{ny'x'} & a_{11}Z_{nx'y'} \\ -a_{22}Z_{ny'x'} & a_{21}Z_{nx'y'} \end{pmatrix}$$

or $\mathbf{Z} = \mathbf{A}\mathbf{Z}_2$, (9)

The regional impedances $Z_{nx'y'}$ and $Z_{ny'x'}$ provide the two regional phases. Within each column only one phase occurs. In the 'telluric vectors' concept (eqn. (2)) this means that the in-phase and out-of-phase vectors of $\mathbf{e}_x$ are parallel, and the in-phase and out-of-phase vectors of $\mathbf{e}_y$ are parallel as well.

In an arbitrary (x, y) coordinate system the impedance tensor

$$\mathbf{Z} = \mathbf{T}_\alpha^T \mathbf{A} \mathbf{Z}_2 \mathbf{T}_\alpha \tag{10}$$

is found. Its elements are linear combinations of $Z_{nx'y'}$ and $Z_{ny'x'}$. $\mathbf{T}_\alpha$, $\mathbf{T}_\alpha^T$ are rotation tensors, where the superscript T indicates a transpose and the label α refers to a regional strike angle α. This can be found by using the condition that the two elements of the impedance tensor of eqn. (9) which belong to the same telluric vector $\mathbf{e}_x$ have the same phase, and the result is

$$\tan(2\alpha) = ([S_1, S_2] - [D_1, D_2])$$
$$/([S_1, D_1] + [S_2, D_2]) \tag{11}$$

using the abbreviations of eqn. (6) (Bahr, 1988). A rotationally invariant dimensionality parameter that explains the extent to which a particular data set can be interpreted with the superimposition model is the regional skew

$$\eta = (|[D_1, S_2] - [S_1, D_2]|)^{1/2}/|D_2| \tag{12}$$

(Bahr, 1988). Formally, this parameter is found from the condition that in the coordinate system of the regional 2D structure, $Z_{x'x'}$ has to have the same phase as $Z_{y'x'}$ and also $Z_{y'y'}$ has to have the same phase as $Z_{x'y'}$, as any departure from these two conditions results in a regional skew larger than zero. It should be noted that eqn. (11) yields an unstable result for the regional strike if $\mu \approx 0$. In that case, the application of the superimposition model would result in an overestimation, because $\mu \approx 0$ indicates that the less complex model of eqn. (3) for a regional 1D Earth would be sufficient and therefore a regional strike would make no sense. μ can therefore be referred to as an indicator of regional one-dimensionality.

The decomposition of eqn. (9) requires 10 parameters (Bahr, 1988): two complex regional impedances, four distortion matrix elements, the regional strike α and the regional skew η. As the impedance tensor provides only four complex elements, two unknown static shift factors occur in the decomposition as shown below.

In the coordinate frame of the regional conductivity structure, the angular deviations β_1 and β_2 of the telluric vectors $\mathbf{e}_x$ and $\mathbf{e}_y$ are given by

$$\tan(\beta_1) = -Z_{x'x'}/Z_{y'x'} = -a_{12}/a_{22}$$

and

$$\tan(\beta_2) = Z_{y'y'}/Z_{x'y'} = a_{21}/a_{11}. \qquad (13)$$

and are determined from ratios of the distortion matrix elements. They are referred to hereafter as 'skew angles'. The magnitudes of the distortion matrix elements are, however, inseparably linked to the magnitudes of the two regional impedances:

$$Z_E = \left(a_{12}^2 + a_{22}^2\right)^{1/2} Z_{ny'x'} = D' Z_{ny'x'}$$

$$Z_B = \left(a_{11}^2 + a_{21}^2\right)^{1/2} Z_{nx'y'} = D'' Z_{nx'y'} \qquad (14)$$

D' and D'' are the two unknown static shift parameters corresponding to Larsen's (1977) static shift D for the regional 1D case.

Recently, Groom and Bailey (1989) presented a decomposition that also incorporates the superimposition model, as does the decomposition of eqn. (9). In their approach, the distortion matrix $\mathbf{A}$ itself is also factorized. The general impedance tensor according to Groom and Bailey (1989) is

$$\mathbf{Z} = \mathbf{T}_\alpha^T \mathbf{A} \mathbf{Z}_2 \mathbf{T}_\alpha$$

$$= g' \mathbf{T}_\alpha^T \begin{pmatrix} -(1-s)(e-t)Z_{ny'x'} & (1+s)(1-te)Z_{nx'y'} \\ -(1-s)(1+te)Z_{ny'x'} & (1+s)(e+t)Z_{nx'y'} \end{pmatrix} \mathbf{T}_\alpha \qquad (15)$$

where $g' = g/\left[(1+e^2)(1+t^2)(1+s^2)\right]^{1/2}$

The relationship between the distortion parameters g, s, t, e and the elements a_{11}, a_{12}, a_{21} and a_{22} of the distortion matrix is briefly summarized as follows.

(1) The parameters t and e ('twist' and 'shear') define the two angles

$$\beta_t = \text{arc}\tan(t) \text{ and } \beta_e = \text{arc}\tan(e), \qquad (16)$$

such that the sum and difference of these angles yield the skew angles given by eqn. (13), i.e.

$$\beta_e + \beta_t = \beta_2 = \text{arc}\tan(a_{21}/a_{11}),$$

$$\beta_e - \beta_t = \beta_1 = \text{arc}\tan(-a_{12}/a_{22}) \qquad (17)$$

(2) The static shift factors linked to the two regional impedances are

$$g\left[1 - \frac{2s}{(1+s^2)}\right]^{1/2} = D' = \left(a_{12}^4 + a_{22}^4\right)^{1/2}$$

$$g\left[1 + \frac{2s}{(1+s^2)}\right]^{1/2} = D'' = \left(a_{11}^4 + a_{21}^4\right)^{1/2} \qquad (18)$$

The relationship between the model parameters of the decomposition of eqn. (15) and the modified impedances may be explained by a simple nonlinear system of equations (Groom and Bailey, 1989):

$$S_1 = t\tilde{S}_2 + e\tilde{D}_2$$

$$D_2 = \tilde{S}_2 - et\tilde{D}_2$$

$$S_2 = \left(\tilde{D}_2 - et\tilde{S}_2\right)\cos(2\alpha) - \left(t\tilde{D}_2 + e\tilde{S}_2\right)\sin(2\alpha)$$

$$D_1 = \left(t\tilde{D}_2 + e\tilde{S}_2\right)\cos(2\alpha) - \left(\tilde{D}_2 - et\tilde{S}_2\right)\sin(2\alpha)$$

where $\tilde{S}_2 = Z_E + Z_B$

and $\tilde{D}_2 = Z_E - Z_B \qquad (19)$

The distortion parameters g and s cannot be calculated from an experimental data set for the same reasons for which D' and D'' are inaccessible. Because this system of equations does not incorporate a regional skew, only seven model parameters have to be computed: the complex impedances $\tilde{S}_2$, $\tilde{D}_2$, the regional strike α and the local distortion parameters e and t. As eight input parameters are available, the system of equations (19) is slightly overestimated. For its solution a least-squares fitting procedure can be used.

The sequence in which the model parameters are derived constitutes the principal difference between the decomposition according to Groom and Bailey (1989) and the method of Bahr (1988) based on 'telluric vectors': The latter method provides straightforward solution formulae for the regional strike α (eqn. (11)) and the regional skew η (eqn. (12)). After the impedance tensor has been rotated into the coordinate frame of the regional strike α the regional impedances and the distortion parameters (eqn. (13)) are found. In contrast, Groom and Bailey (1989) estimated these parameters simultaneously from an inversion of eqn. (19). The results are, however, identical if $\eta = 0$. For experimental data, it is always the case that $\eta > 0$, i.e. the regional conductivity distribution is not exactly two-dimensional. In that case, the least-squares solution of eqn. (19) sometimes yields more stable estimates of the regional strike than the telluric-vector technique (R.W. Groom, personal communication, 1989). The latter has the advantage that it can easily be implemented without numerical efforts for the solution of eqn. (19). A description of the linearisation procedure of eqn. (19) has been given by Ritter (1988). In the following section, it is attempted to construct a decomposition scheme that is as robust as the inversion solution of eqn. (19) but can be implemented as easily as eqn. (11).

Zhang et al. (1987) also applied a decomposition corresponding to eqn. (9). Their principal model is a special case of the superimposition model, because in their approach the local anomaly is considered to be 2D.

4. A straightforward robust decomposition method for the case of moderate departures from the principal superimposition model

If $\eta = 0$, then the phases of the two regional impedances which occur in the superimposition model can easily be obtained from

$$\phi_{x'y'} = \arg Z_{x'y'},\ \phi_{y'x'} = \arg Z_{y'x'} \tag{20}$$

in the coordinate frame given by eqn. (11). For non-zero η, these phases may be obtained from the phases of the telluric vectors as defined by

$$\begin{aligned}\tan(\phi_{e_x}) &= \left[(\mathrm{Im}\ Z_{xx})^2 + (\mathrm{Im}\ Z_{yx})^2\right]^{1/2} \\ &\qquad /\left[(\mathrm{Re}\ Z_{xx})^2 + (\mathrm{Re}\ Z_{yx})^2\right]^{1/2} \\ \tan(\phi_{e_y}) &= \left[(\mathrm{Im}\ Z_{xy})^2 + (\mathrm{Im}\ Z_{yy})^2\right]^{1/2} \\ &\qquad /\left[(\mathrm{Re}\ Z_{xy})^2 + (\mathrm{Re}\ Z_{yy})^2\right]^{1/2}\end{aligned} \tag{21}$$

(Bahr, 1988). For non-zero η, however, eqn. (11) may not yield a reliable regional strike because $\eta \neq 0$ means a departure from the principal superimposition model; in that case, no coordinate system exists in which the impedance tensor takes the simple form of eqn. (19). In the coordinate system found with eqn. (11), the phases in one column of the impedance tensor may then coincide perfectly whereas the phases of the other column may differ very much. This can also happen in the coordinate system found by a least-squares solution of eqn. (19) after Groom and Bailey (1989), in particular in the case of strong anisotropy. Then, only the telluric vector with the larger modulus determines the solution, whereas the phase of the other telluric vector is poorly resolved.

Therefore the superimposition model of eqn. (9) is now modified so that a regional skew η, if any, results in a 'phase deviation' in the main diagonal elements of $\mathbf{Z}$. The phases in one column of the tensor which would coincide if eqn. (9) describes the tensor exactly now differ by a phase deviation angle δ. To avoid any 'preference' for one of the telluric vectors, the ad hoc condition that the same phase deviation must occur in the two columns of the impedance tensor is added. The principal model impedance tensor in the coordinate system of the regional strike is then

$$\mathbf{Z} = \begin{pmatrix} -a_{12} Z_{ny'x'}\, \mathrm{e}^{i\delta} & a_{11} Z_{nx'y'} \\ -a_{22} Z_{ny'x'} & a_{21} Z_{nx'y'}\, \mathrm{e}^{-i\delta} \end{pmatrix} \tag{22}$$

In eqn. (22) the phase deviation angle δ establishes an additional model parameter that replaces the regional skew η as shown below. The regional strike coincides with a coordinate frame in which the impedance tensor takes the form of eqn. (22). This strike angle is found from two conditions for the two columns of the impedance tensor of eqn. (22), whereas the two variables α and δ are to be resolved. The following mathematical treatment is a generalization of the algebra presented in a previous paper (Bahr, 1988) in which it was a priori assumed that $\delta = 0$.

The condition that in the coordinate frame of the regional strike the phases of the two elements in the left column of the impedance tensor differ by δ is $\arg Z_{x'x'} - \arg Z_{y'x'} = \delta$ or

$$\frac{\mathrm{Re}(Z_{x'x'})\cos\delta + \mathrm{Im}(Z_{x'x'})\sin\delta}{\mathrm{Re}(Z_{y'x'})} = \frac{-\mathrm{Re}(Z_{x'x'})\sin\delta + \mathrm{Im}(Z_{x'x'})\cos\delta}{\mathrm{Im}(Z_{y'x'})} \quad (23)$$

This condition can be expressed by use of the modified impedances in eqn. (4). Transformation of these modified impedances into a new coordinate system which is rotated clockwise by an angle α yields

$$D_1' = D_1\cos(2\alpha) + S_2\sin(2\alpha),\ D_2' = D_2$$
$$S_2' = S_2\cos(2\alpha) - D_1\sin(2\alpha),\ S_1' = S_1 \quad (24)$$

and therefore

$$Z_{x'x'} = S_1' + D_1' = S_1 + D_1\cos(2\alpha) + S_2\sin(2\alpha)$$

and

$$Z_{y'x'} = -D_2' + S_2' = -D_2 + S_2\cos(2\alpha) - D_1\sin(2\alpha) \quad (25)$$

With eqn. (25), eqn. (23) becomes

$$-A\sin(2\alpha) + B\cos(2\alpha) + C + E\cos(2\alpha)\sin(2\alpha) = 0$$

where

$$A = A_1 + A_2 = ([S_1, D_1] + [S_2, D_2])\cos\delta + (\{S_1, D_1\} + \{S_2, D_2\})\sin\delta$$
$$B = B_1 + B_2 = ([S_1, S_2] - [D_1, D_2])\cos\delta + (\{S_1, S_2\} - \{D_1, D_2\})\sin\delta$$
$$C = C_1 + C_2 = ([D_1, S_2] - [S_1, D_2])\cos\delta + (\{D_1, S_2\} - \{S_1, D_2\})\sin\delta$$
$$E = E_2 = (\{S_1, S_1\} - \{D_2, D_2\})\sin\delta \quad (26)$$

where the 'commutators' [ ], { } are defined by eqn. (6). It should be noted that for $\delta = 0$ eqn. (26) becomes the condition used by Bahr (1988) to estimate the regional strike α from eqn. (11). The second condition is that in the coordinate frame of the regional strike the phases of the two elements in the right-hand column of the impedance tensor differ by $-\delta$. This yields a relation similar to eqn. (26):

$$-A^+\sin(2\alpha) + B^+\cos(2\alpha) + C^+ + E^+\cos(2\alpha)\sin(2\alpha) = 0$$

where

$$A^+ = A_1 - A_2$$
$$B^+ = B_1 - B_2$$
$$C^+ = -C_1 + C_2$$
$$E^+ = E_2 = E \quad (27)$$

the terms A_1, B_1, C_1 and A_2, B_2, C_2, E_2 being defined by eqn. (26). From eqns. (26) and (27) the two unknown parameters α and δ can be found: combining eqns. (26) and (27) yields

$$\frac{[-A_1\sin(2\alpha) + B_1\cos(2\alpha)]}{[-C_2 - E_2\sin(2\alpha)\cos(2\alpha)]} = \frac{C_1}{[-A_2\sin(2\alpha) + B_2\cos(2\alpha)]} = \tan\delta \quad (28)$$

Assuming that

$$(B_1A_2 + A_1B_2 + C_1E_2)^2 > 4(B_1B_2 - C_1C_2)(A_1A_2 - C_1C_2) \quad (29)$$

then the solution for α is

$$\tan(2\alpha_{1,2}) = \frac{1}{2}\frac{(B_1A_2 + A_1B_2 + C_1E_2)}{(A_1A_2 - C_1C_2)} \pm \left[\frac{1}{4}\frac{(B_1A_2 + A_1B_2 + C_1E_2)^2}{(A_1A_2 - C_1C_2)^2} - \frac{(B_1B_2 - C_1C_2)}{(A_1A_2 - C_1C_2)}\right]^{1/2} \quad (30)$$

and δ can then be obtained from eqn. (28). The subscripts 1 and 2 referring to the two different signs of the root in eqn. (30) describe two different coordinate systems in which the regional skew η is either converted into a minimal phase deviation angle δ or into a maximal phase deviation. For 'physical' reasons, the minimal δ solution is preferred, although from a mathematical point of view both solutions are of the same value.

It should be noted that eqn. (30) yields an unstable result for the regional strike if the regional 1D indicator μ (eqn. (7)) is zero. In that case, $A_1 = B_1 = C_1 = 0$, no phase differences occur in the tensor and the simpler model of a regional 1D Earth (eqn. (3)) is again appropriate. Equation (30) also yields unstable or no results if the regional skew η defined by eqn. (12) is zero. In that case, $C_1 = 0$, as can be seen from eqns. (12) and (26), and therefore no phase deviation angle δ is established by eqn. (28). However, in that case, the regional strike α from eqn. (11), obtained by the method described by Bahr (1988), is already a robust straightforward solution.

The application of the 'phase deviation' method developed in this section must be restricted to those complicated cases where the regional 1D indicator μ and the regional skew η do not vanish. In these cases, eqn. (30) yields a robust estimate of the regional strike.

5. A note on the physical and mathematical decomposition schemes

Other decomposition schemes that do not make assumptions about the physical model (e.g. the dimensionality of the conductivity structure), referred to as 'mathematical decompositions' in the following, have been suggested by Eggers (1982), Spitz (1985), Cevallos (1986), LaTorraca et al. (1986) and Yee and Paulson (1987). The relationship between the parameters of these techniques and the parameters of the decomposition of eqn. (15) has been investigated by Groom and Bailey (1990) by use of synthetic data. The main result of their comparison is that if the true conductivity structure meets the requirements of the superimposition model, the principal impedances obtained by the mathematical decompositions still yield mixtures of the two regional impedances.

A useful application of the mathematical decompositions is, however, restricted to very few cases for a second reason. None of these techniques allows for the occurrence of hidden static shifts which are described by eqns. (14) and (18). The problem of recognition and removal of static shifts has been treated by numerous authors. Their suggestions have included predictions of the distortion by use of model calculations (Berdichevsky and Dmitriev, 1976), geoelectrical mapping of the top layer structure (Kemmerle, 1977), normalization of electromagnetic sounding results by use of spatial Fourier analysis (Berdichevsky and Zhdanov, 1984), comparison of the magnetotelluric impedance with an undistorted magnetic impedance at low frequencies (Larsen, 1977; Bahr and Filloux, 1989), a combination of the magnetotelluric method and the magnetovariational method (Bahr, 1988), and model calculations of voltage differences instead of telluric fields (Poll et al., 1989). It must be pointed out clearly that even the decomposition techniques which are based on the superimposition model do not solve the problem of static shifts. They only include a possible application of one of the static shift removal techniques referred to above on top of the decomposition by an estimation of the static shift parameter D', D'' described in eqn. (14), or g, s described in eqn. (18), respectively. Junge (1988) applied the comparison technique suggested by Larsen (1977) and described by Bahr and Filloux (1989) to electromagnetic fields at six sites in the F.R.G., three of which were situated in a sedimentary basin. He found strong departures of D' and D'' from unity at all sites. In contrast, Bahr and Filloux (1989) found no distortion, or only moderate distortion, at ocean-floor MT sites. It must be concluded that, at least in land magnetotellurics and at low frequencies, static shifts can occur in almost any geological environment.

To provide a complete set of formulae for the evaluation of field data in a later section, two of the mathematical decomposition methods are tested. Eggers' (1982) method yields two principal impedances:

$$\lambda_{1,2} = (Z_{xy} - Z_{yx})/2 \pm \left[\tfrac{1}{4}(Z_{xy} + Z_{yx})^2 - Z_{xx}Z_{yy}\right]^{1/2} = D_2/2 \pm \left[D_2^2/4 - \det(\mathbf{Z})\right]^{1/2} \tag{31}$$

which are eigenvalues of the impedance tensor. The term det(**Z**) is the determinant of **Z**. λ_1 and λ_2 are rotationally invariant. If the conductivity structure of the subsurface is correctly described by the superimposition model, the principal impedances obtained are

$$\lambda^*_{1,2} = \tfrac{1}{2}\left(a_{11}Z_{x'y'} - a_{22}Z_{y'x'}\right) \pm \tfrac{1}{2}\left[\left(a_{11}Z_{x'y'} + a_{22}Z_{y'x'}\right)^2 - 4a_{12}a_{21}Z_{x'y'}Z_{y'x'}\right]^{1/2} \tag{32}$$

In general, they contain mixtures of the regional impedances as found by Groom and Bailey (1990). If no local anomaly occurs or if it is 2D in the (x', y') coordinates of the regional strike, then $(a_{12}=0,\ a_{21}=0)$ and therefore

$$\lambda_1 = a_{11}Z_{x'y'},\ \lambda_2 = a_{22}Z_{y'x'} \tag{33}$$

i.e. the regional impedances are obtained correctly. This is also the case if one of the skew angles (eqn. (17)) vanishes or if both are small and therefore

$$a_{12}a_{21} = 0 \tag{34}$$

LaTorraca et al. (1986) and Cevallos (1986) first suggested evaluating the real eigenvalues r_1, r_2 of a Hermitian tensor that is computed by multiplying **Z** by its complex conjugate transpose $\mathbf{Z}^{T*}$. These eigenvalues are found from

$$\det(\mathbf{Z}^T*\mathbf{Z}) = r_1^2 \cdot r_2^2$$
$$\mathrm{trace}(\mathbf{Z}^T*\mathbf{Z}) = r_1^2 + r_2^2 \tag{35}$$

They provide only rotationally invariant magnitudes of the impedance, but no rotationally invariant phases as do the eigenvalues eqn. (31) of Eggers (1982). Equation (35) leads to

$$r_{1,2}^2 = \mathrm{trace}(\mathrm{Z}^T*\mathrm{Z})/2 \pm \left[\mathrm{trace}(\mathrm{Z}'*\mathrm{Z})^2/4 - \det(\mathrm{Z}'*\mathrm{Z})\right]^{1/2} \tag{36}$$

(Yee and Paulson, 1987). The superimposition model would yield the eigenvalues

$$r_{1,2}^2 = \left(D'^2Z^2_{x'y'} + D''^2Z^2_{y'x'}\right)/2 \pm \left[\left(D'^2Z^2_{x'y'} + D''^2Z^2_{y'x'}\right)^2/4 + \left(a_{21}a_{22} + a_{12}a_{11}\right)Z^2_{y'x'}Z^2_{x'y'}\right]^{1/2} \tag{37}$$

where D' and D'' are defined by eqn. (14). r_1 and r_2 are the shifted regional impedances, if the two skew angles defined by eqn. (13) are identical and therefore

$$a_{12}/a_{22} = -a_{21}/a_{11} \tag{38}$$

This case is treated again in the next section as 'class 4' type of telluric distortion.

6. The seven classes of telluric distortion

This section tries to provide a 'cookbook' for the evaluation of measured impedance tensors, whereby the theoretical concepts which have been reviewed or developed in the previous sections are applied. The investigation starts with the simplest physical model and proceeds to more complex models until an appropriate model has been found. It is restricted to the evaluation of MT data at a single frequency, although the parameters which are to be calculated might be frequency dependent. For example, the subsurface under a particular site appears to be 1D for sufficiently high frequencies. With increasing penetration depth, a high-conductivity structure in the vicinity of the site might generate an inductive anomaly. At very low frequencies, the effect of that structure on the tensor impedance can be considered to be a galvanic anomaly (Haak, 1978). An example for the frequency dependence of the proposed dimensionality parameters is presented in the next section.

The tensor impedance is first converted into the modified impedances given by eqn. (4). If the skew (eqn. (5)) is small,

$$\kappa < 0.1, \tag{39}$$

then the tensor impedance is either undistorted—Cagniard's (1953) model of a layered half-space would be appropriate—or it is described by Swift's (1967) model.

Class 1: the simple 2D anomaly

A rotationally invariant measure of two-dimensionality is

$$\Sigma = \left(D_1^2 + S_2^2\right)/D_2^2 \tag{40}$$

If $\Sigma > 0.1$ the conductivity distribution should be considered to be 2D. Swift's (1967) method of determining the strike α may be applied, and after the coordinate transformation (eqn. (24)) the impedance tensor is of the form

$$\mathbf{Z} = \begin{pmatrix} 0 & Z_{x'y'} \\ Z_{y'x'} & 0 \end{pmatrix} = \begin{pmatrix} 0 & S_2' + D_2' \\ S_2' - D_2' & 0 \end{pmatrix} \tag{41}$$

The parameters of the 2D anomaly are the anisotropy

$$A = \mathrm{Re}\left(Z_{x'y'}/Z_{y'x'}\right) \tag{42}$$

and the phase difference

$$\delta\phi = \mathrm{Im}\left(Z_{x'y'}/Z_{y'x'}\right) \tag{43}$$

They constitute a statement on the size of the 2D anomaly; if

$$\delta\phi \ll A, \tag{44}$$

then this 2D anomaly is purely local.

All 'higher' classes deal with cases in which the skew (eqn. (5)) does not vanish, e.g. $\kappa > 0.1$. In that case, the test eqn. (7) yields a measure of the phase differences in the impedance tensor.

Class 2: the purely local 3D anomaly

If that measure does not exceed the relative error,

$$\mu < dD_2/D_2 \tag{45}$$

or if $\mu < 0.05$, Larsen's (1977) model can be applied to the impedance tensor ($\mathrm{d}D_2/D_2$ is a rotationally invariant measure of data errors). Then one phase

$$\phi = \arg(S_2 - D_1) \tag{46}$$

can be computed. The parameter μ can therefore be considered as a measure of regional one-dimensionality. The condition $\mu = 0$ formally yields a modified skew angle β defined by

$$\mathrm{Re}\ S_1/\mathrm{Re}\ D_2 = \mathrm{Im}\ S_1/\mathrm{Im}\ D_2 = \tan(\beta) = \kappa \tag{47}$$

(Bahr, 1988, eqn. (21)).

For an estimation of the true magnitude of the impedance, additional knowledge of Z_n or of the local conductivity structure is necessary.

All other classes deal with cases in which neither the skew (eqn. (5)) nor the phase difference measure (eqn. (7)) vanish. The application of the test eqn. (12) provides a measure of whether the superimposition model is adequate. Again, the test parameter, here η, can be compared with a rotationally invariant measure of the data errors. The differences between classes 3, 4, 5 and 6 consist only of the amount of local distortion. For very small regional skew, e.g. $\eta < 0.1$, eqn. (11) yields a reliable regional strike (except for a 90° uncertainty). Otherwise, a more robust scheme such as eqn. (30) or the solution of the system of equations (15) after Groom and Bailey (1989) should be applied. In the coordinate system of the regional strike, the skew angles β_1 and β_2 (eqn. (13)), or alternatively twist and shear (eqn. (17)), are considered.

Class 3: a regional 2D anomaly with weak local distortion

This class includes all cases for which

$$\beta_1 < 5^\circ \text{ and } \beta_2 < 20^\circ$$

or

$$\beta_2 < 5^\circ \text{ and } \beta_1 < 20^\circ \tag{48}$$

Because of eqns. (33) and (34), the eigenvalues of Eggers (1982) can be considered as scaled regional impedances.

Class 4: a regional 2D anomaly in rotated coordinates

This class includes cases in which the skew angles are equal:

$$\beta_1 = \beta_2 = \beta \tag{49}$$

and the twist parameter t vanishes, as can be seen from eqn. (17). In the case of extreme anisotropy, e.g. $Z_{x'y'} \ll Z_{y'x'}$, the phase of one regional impedance can be only roughly estimated and the class 4 type structure might then be confused with a class 2 type. In this case, it is sufficient to rotate the impedance tensor according to

$$\mathbf{Z}_\beta = \mathbf{T}_\beta^T \mathbf{Z} \tag{50}$$

by the angle β which was defined for class 2. A similar modified skew angle has been proposed by Groom and Bailey (1989, eqn. (39)). $\mathbf{Z}_\beta$ will then belong to class 1 and therefore no decomposition is necessary. A more likely explanation for the occurrence of the angle β is, however, a maladjustment of the electrodes with respect to the magnetometer (Cox et al., 1980).

Class 5: a regional 2D anomaly with strong local distortion

This class includes all other cases with the exception of the special case considered under class 6. Here a decomposition is necessary and the concepts described in the last two sections should be applied. Equation (11) or (30) yields the regional strike and eqn. (20) or (21) yields the principal phases. It should be noted that eqns. (11) and (12) provide unstable results if μ (cf. eqn. (7)) is very small, in which case class 2 already describes the impedance tensor correctly. Similarly, it does not make sense to apply eqn. (30) if η is very small, e.g. $\eta < 0.05$.

Class 6: a regional 2D anomaly with strong local channeling

This class includes the cases where

$$-\beta_1 + \beta_2 \approx 90° \tag{51}$$

Then the shear parameter

$$e \approx 1 \tag{52}$$

(Groom and Bailey, 1989). In this case, the direction of the electric field does not at all depend on the direction of the magnetic field, except for a sign change. It can easily be shown that the impedance tensor then takes the form of eqn. (9) in any coordinate frame, although the phases of the regional impedances $Z_{x'y'}$ and $Z_{y'x'}$ will vary with the orientation of the chosen coordinate system. The superimposition model then works well for all regional strikes and the straightforward solution (eqn. (11)) for the regional strike becomes unstable. Correct regional phases can be obtained if the regional strike is fixed by some a priori information, e.g. the induction arrows or the regional strike of neighbouring sites which have less strong telluric distortion.

Class 7: a regional 3D anomaly

This class includes those cases where

$$\eta > 0.3$$

and therefore even the regional conductivity distribution is not 2D. The superimposition model is not then appropriate. It may, nevertheless, be tested whether eqn. (30) or the decomposition method of Groom and Bailey (1989) yields a regional strike that coincides with the regional strike of neighbouring sites, or that coincides with the regional strike of the particular site in some other frequency band. If this is the case, then the regional conductivity structure can be considered to be approximately 2D.

7. An example from the German deep drilling site: the frequency dependence of skew parameters

In 1986, Metronix-Geometra carried out magnetotelluric measurements in the target area of the German deep drilling site in the Oberpfalz, Bavaria (Fig. 1). Details of field procedures and data processing have been given by Jensen et al. (1988). Figure 2 shows the period dependence of the skew parameters κ (eqn. (5)) and η (eqn. (12)) of site 002, situated at the place of the 1987 pilot hole. Also displayed are the real part of the induction arrow, the regional strike (eqn. (11)) and the preferred direction of the telluric field, for six periods. The length values refer to the real part of the Schmucker inductive scale length $C = Z/i\omega$.

At the highest frequencies (300 Hz), the conductivity distribution can be considered to be 2D with a north–south strike, as is obvious from the eastward-directed induction arrows. At frequencies around 30 Hz, $\eta \approx 0.25$ and $\kappa \approx 0.5$. Therefore, the regional conductivity distribution in a 2 km range is approximately 2D with even smaller embedded anomalies. Therefore site 2 now belongs to class 5. The NW–SE regional strike in this frequency band is probably caused by a steep graphitized cataclasite zone. These graphites cause,

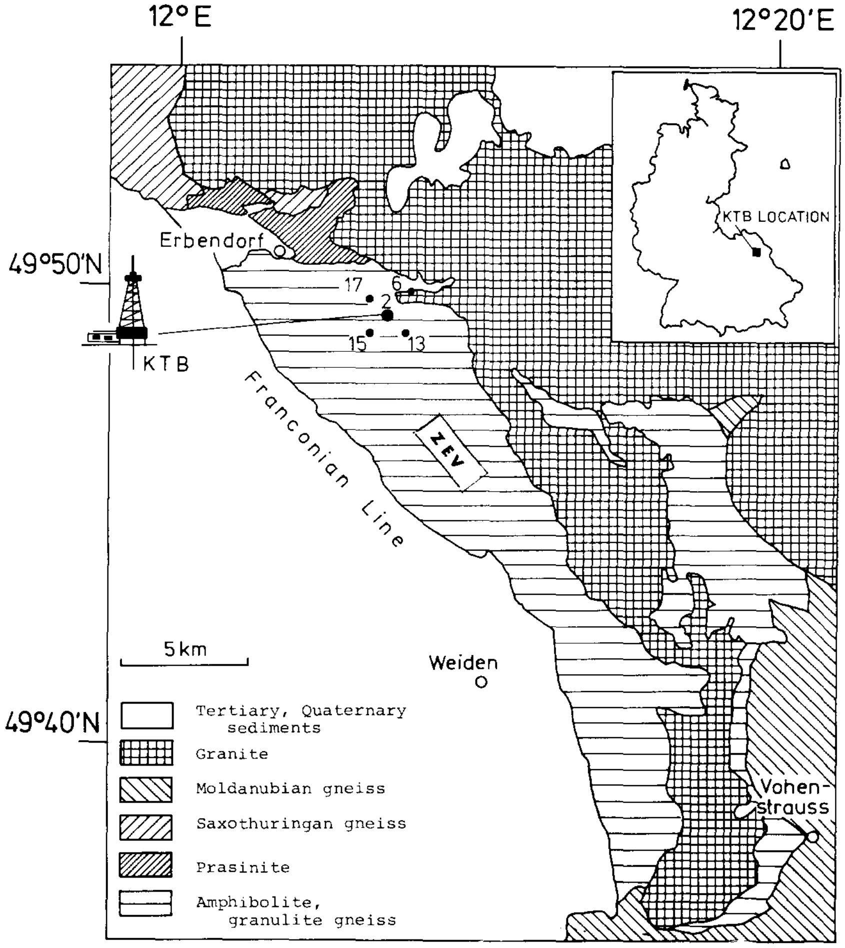

Fig. 1. Field sites 2, 6, 13, 15 and 17 in the vicinity of the German deep drilling project KTB in the Oberpfalz, Bavaria. The NW–SE-striking amphibolite surface petrology between Erbendorf and Vohenstrauss is referred to as 'Zone Erbendorf–Vohenstrauss' (ZEV).

in addition to a conductivity anomaly, a large self-potential anomaly (Haak et al., 1991).

In the period range 0.3–3 s, the inductive scale length is in the 6–13 km range, and the impedance tensor belongs to class 7 as $\eta > 0.4$; the regional conductivity distribution is thus 3D. From the SW–NE direction of the induction arrows as well as from the extension of this structure—visible in the inductive scale length—it can be concluded that this regional conductivity structure is the 'Zone Erbendorf–Vohenstrauss' (ZEV) (see Fig. 1).

At the longest periods, this site belongs to class 5 again; the induction arrows as well as the magnetotelluric regional strike indicate a large-scale east–west-striking structure. It has been found previously with electromagnetic methods (Berktold and Regner, 1984). Nevertheless, the local telluric strike in this low-frequency band serves as a 'frozen' measure of the strike of the induction anomaly in the 0.3–3 s period range.

This leads to an important conclusion: the telluric distortion at long periods is caused by a conductivity structure which is not really 'local' but is of small dimensions (6–13 km) if compared with the inductive scale length at these long periods (60 km). At very short periods this structure is not seen in the impedance tensor. There-

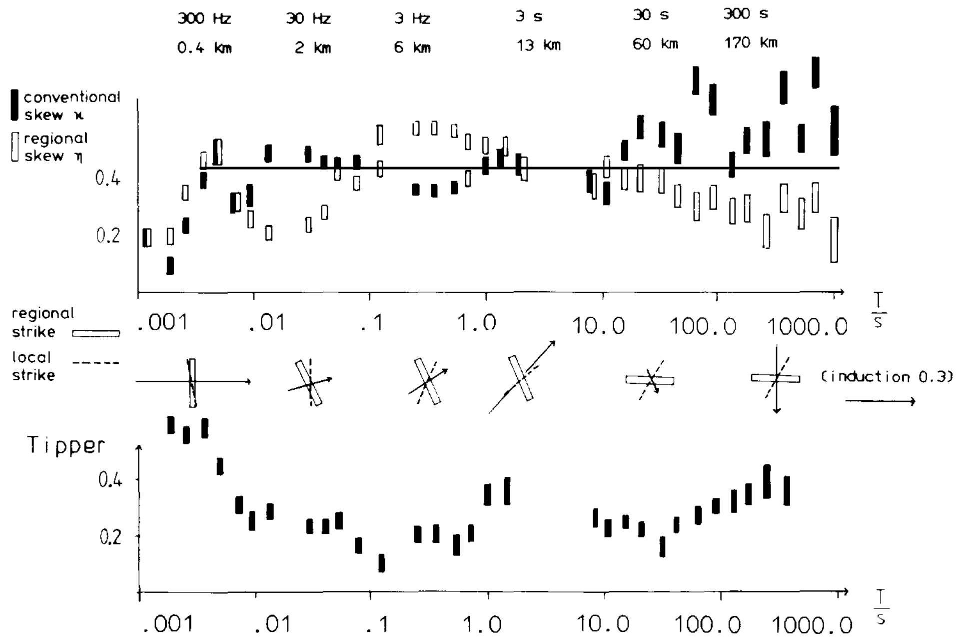

Fig. 2. Skew parameters κ and η for site 2 (see Fig. 1). 'Regional' strike refers to an angle computed by use of eqn. (11) and (at long periods) eqn. (30), whereas 'local' strike refers to an angle obtained by Swift's (1967) rotation analysis. Together with these strike angles the real part of the Schmucker induction arrow is displayed. For further explanation, see text.

fore, the classifications 'local' and 'regional' which are used throughout this paper as well as in earlier contributions (Bahr, 1988; Groom and Bailey, 1989) make sense only for a particular period range. A more general study on the relationship between the period range and the depth of investigation has recently been presented by Spies (1989).

Figure 3 shows the long-period phases of the regional impedances for five sites in the vicinity of the German deep drilling site. The maximum distance between two sites was 3 km (see Fig. 1); therefore the regional phases of all sites could be expected to be similar at long periods. Sites 2 and 15 belong to class 5, sites 13 and 17 belong to class 6, and site 6 belongs to class 7. At all sites, large induction arrows pointing towards the south indicate a strong east–west-striking conductivity contrast (Jensen et al., 1988). Figure 3a shows the phases $\phi_{x'y'}$ and $\phi_{y'x'}$, obtained with the conventional method of strike determination. They vary strongly from site to site and the zero phase $\phi_{y'x'}$ cannot be interpreted. Figure 3b shows the phases computed from eqn. (21) in a regional strike direction that was obtained from eqn. (12) for site 15 or from eqn. (30) for sites 2 and 6. Thereby a regional strike in the range $-20°$ to $0°$ was found. For computations of the phases of sites 13 and 15, the regional strike was fixed at $\alpha = 0$. Although the regional phases of these five sites still scatter slightly, as a result of gross data errors, clearly a decoupling of the two branches belonging to the two telluric vectors is observable. The occurrence of two phases can be interpreted by a 2D east–west-striking conductivity structure, which, in turn, is responsible also for the large induction arrows.

8. Conclusion

There are many types of distortion of telluric fields by near-surface conductivity anomalies, and they may occur in different environments of regional conductivity distribution. Probably no simple formula exists that can be applied in all cases to separate local and regional contributions

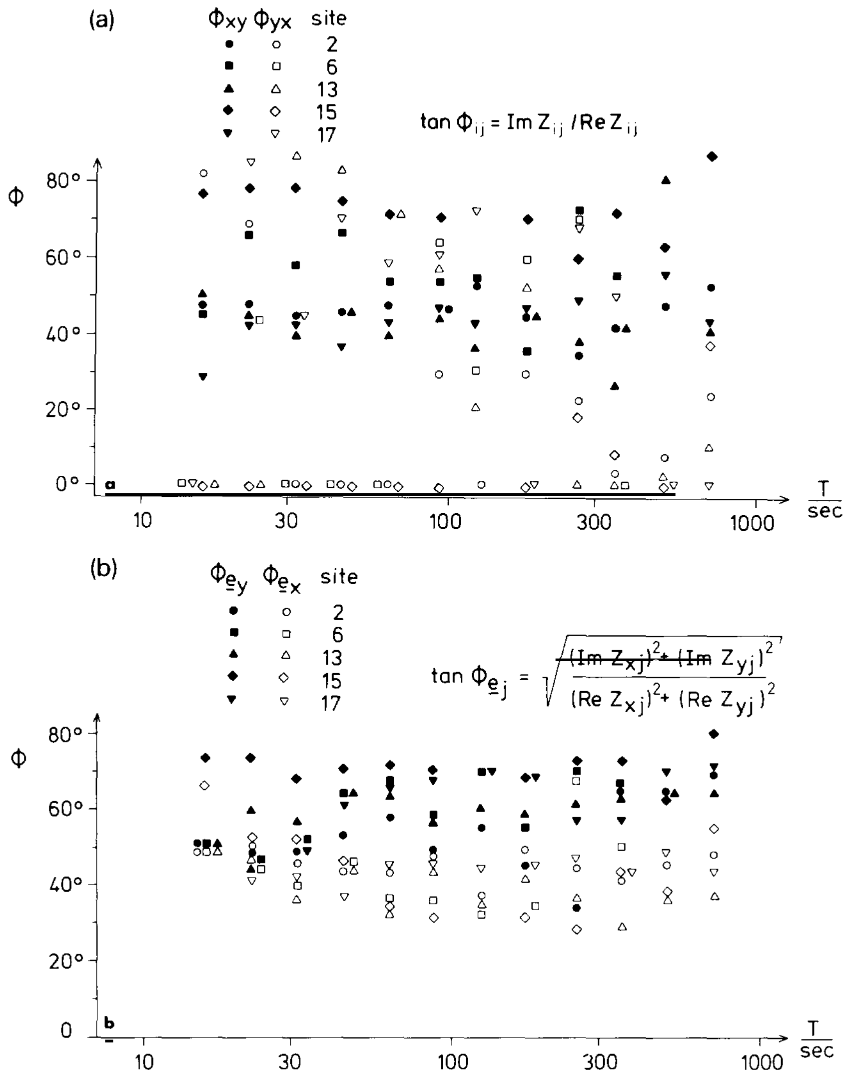

Fig. 3. (a) Long-period phases for five sites obtained in rotated coordinates using Swift's (1967) rotation analysis. (b) Long-period 'regional' phases obtained by use of eqn. (21). For further explanation, see text.

to a measured impedance tensor and to obtain parameters which correctly describe near-surface or regional conductivity structures.

Consequently, this paper describes dimensionality parameters which may be used to check whether a particular magnetotelluric tensor may be described by some general physical model, e.g. a 1D Earth covered by a purely local 3D structure. These parameters refer to a series of models that incorporate an increasing complexity of the general resistivity distribution:

(1) a 1D Earth;

(2) a 2D Earth;

(3) a 1D Earth with an overlying 3D structure (Larsen, 1977);

(4) the superimposition of a 2D Earth and a surficial 3D structure (Bahr, 1988; Groom and Bailey, 1989);

(5) a general model that allows for moderate deviation of the regional structure from the purely 2D case.

In the last, most complicated, model, the solution for the strike of the regional structure is stabilized by introducing an additional 'phase deviation' angle. The phase difference between two elements in the same column of the impedance tensor should be zero in the original superimposition model but is now a measure of regional skew, i.e. of the amount of deviation from the regional 2D case.

The dimensionality parameters for the more complex models (e.g. regional skew) as well as the model parameters then derived for these models (e.g. regional strike) become unstable if a measured impedance tensor already meets the requirements of a previous (less complex) general model. For example, the application of the superimposition model (Bahr, 1988; Groom and Bailey, 1989) does not yield reliable results if the magnetotelluric tensor is described correctly by the less complex Larsen (1977) model. In that case, all of its elements have the same phase, and methods which require two phases in the magnetotelluric tensor will fail. It is therefore recommended that these complexity tests are applied successively, starting with the less complex general models.

The application of the most complicated decomposition methods to field data from the German deep drilling site has indicated a large east–west-striking conductivity anomaly under the Oberpfalz, Bavaria. This anomaly is also observable in the anomalous magnetic fields. To recover its parameters, e.g. regional phases from electric field measurements, a tensor decomposition is necessary.

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