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Autoregressive Harmonic Analysis of the Earth's Polar Motion Using Homogeneous International Latitude Service Data

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The homogeneous set of 80-year-long (1900–1979) International Latitude Service (ILS) polar motion data is analyzed using the autoregressive method (Chao and Gilbert, 1980), which resolves and produces estimates for the complex frequency (or frequency and Q) and complex amplitude (or amplitude and phase) of each harmonic component in the data. Principal conclusions of this analysis are that (1) the ILS data support the multiple-component hypothesis of the Chandler wobble (it is found that the Chandler wobble can be adequately modeled as a linear combination of four (coherent) harmonic components, each of which represents a steady, nearly circular, prograde motion, a behavior that is inconsistent with the hypothesis of a single Chandler period excited in a temporally and/or spatially random fashion), (2) the four-component Chandler wobble model "explains" the apparent phase reversal during 1920–1940 and the pre-1950 empirical period-amplitude relation, (3) the annual wobble is shown to be rather stationary over the years both in amplitude and in phase, and no evidence is found to support the large variations reported by earlier investigations, (4) the Markowitz wobble is found to be marginally retrograde and appears to have a complicated behavior which cannot be resolved because of the shortness of the data set.

1. INTRODUCTION

The earth's rotational axis does not remain fixed relative to the body of the earth. Instead, the intersection of the axis with the surface of the earth (i.e., the pole) traces out a quasi-periodic path about some (slowly drifting) mean position on a scale ≤ 0.3 arc sec = 10 m. This motion is known as the polar motion of the earth. The polar motion in principle consists of a number of components arising from various dynamical processes [see, e.g., *Rochester*, 1973]. The most prominent ones (with amplitude above the 0.01 arc sec level, say) in the International Latitude Service (ILS) polar motion data include the annual wobble, the 14-month Chandler wobble, a "Markowitz wobble" with a period of about 30 years (first reported by *Markowitz* [1960] as having a period of 24 years), and a linear secular drift (or the polar wander) [see, e.g., *Wilson and Vicente*, 1980]. The present paper is aimed principally at a numerical analysis (rather than a geophysical interpretation) of the ILS data by means of the "autoregressive" method [*Chao and Gilbert*, 1980], which has been successfully applied to the analysis of data of earth's normal modes of free oscillations [*Chao and Gilbert*, 1980; *Masters and Gilbert*, 1983]. All of the four said motions will be studied, with emphasis on the Chandler wobble, whose mysterious behavior has defied an unambiguous analysis and has aroused a great deal of controversy.

One of the fundamental questions concerning the Chandler wobble is the following: Is the observed Chandler wobble the manifestation of a single decaying sinusoid excited more or less randomly in space-time by some yet unidentified source(s) or is it the consequence of a beating phenomenon of more than one component having close periods? In this paper, we show that the homogeneous set of 80-year ILS polar motion data is consistent with the multiple-component hypothesis. In fact, it is found that the ILS Chandler wobble can be adequately modeled as a linear combination of as many as four (coherent) sinusoidal components. However, first, let us be warned of the pitfalls in power spectral analysis, the technique commonly used in treating data of a periodic nature, such as the polar motion.

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2. SOME NOTES ON THE USE OF POWER SPECTRA AND DATA WINDOWS

Take a noisy time series containing a number of sinusoidal components (whether pure, decaying, or growing) that have different signal amplitudes and are very close together in frequency, with frequency difference of the order of 1/(total length), say. Suppose we hope to analyze each and every component. A common practice to obtain the frequency estimates is to calculate a discrete Fourier power spectrum (DFPS) and identify the spectral peaks. Unfortunately, in the present case, the DFPS is a "dangerous" frequency estimator. For one thing, if the DFPS is obtained through a fast Fourier transform (FFT) algorithm, the frequency resolution is generally inadequate to give satisfactory frequency estimates and may even lead to completely erroneous conclusions (for an example, see *Buland and Gilbert* [1978]). This can be easily remedied by directly interpolating around the spectral peaks using discrete Fourier transform (DFT) or, often more efficiently, by padding zeros at the end of the series (normally several times as long as the series itself) and then running the FFT to get a denser DFPS. However, a more serious problem arises from spectral leakage (manifesting itself as side bands around the spectral peaks) which, in turn, is caused by the end effects (or, equivalently, by the finiteness of the record length) of the time series. This interference can, and usually does, considerably bias the DFPS frequency estimates (for an example, see *Dahlen* [1982]) and may even introduce spurious peaks.

The latter problem can be greatly reduced by the technique of time-domain windowing, which will play an important role in our study to follow. A time-domain window, in a nutshell, is a bell-shaped taper to be multiplied into the time series under consideration prior to DFT in order to reduce spectral leakage. However, it does so at the expense of the sharpness of the spectral peaks; so a good window represents some optimum compromise (for a review, see, for example, *Harris* [1978]). Because of this, the windowed DFPS is, at its best, an unbiased but still poor estimator for frequencies when a high resolution is required.

The interference, enforced by the broadening of spectral peaks due to the finite length and window (if used), makes the DFPS an even worse estimator for the amplitude and the quality factor Q ($2Q$ = (frequency)/(exponential decay rate)) of the components. Finally, it should be pointed out that any power spectrum is completely void of any information about the phases and is unable to distinguish between positive and negative Q (because both have the same effect on the spectral width).

The spectral peak in a maximum entropy power spectrum (MEPS) is sometimes used as a high-resolution frequency estimator in polar motion studies [see, e.g., *Smylie et al.*, 1973; *Wells and Chinnery*, 1973; *Currie*, 1974]. One major difficulty, however, lies in the somewhat ad hoc choice of the length of the prediction error filter; a short filter offers no better resolution than the conventional Fourier technique, whereas a long filter often leads to instabilities [see *Chen and Stegen*, 1974; *Currie*, 1974]. In addition, *Chen and Stegen* [1974] have shown that MEPS, when used on short records, yields biased frequency estimates, depending on the initial phase of the record. Moreover, the Q and the amplitudes are not directly related to the MEPS, and MEPS contains no phase information.

To avoid the problems suffered by DFPS and MEPS, we shall use the "autoregressive" (AR) method developed by *Chao and Gilbert* [1980]. For easy reference, we now give a brief review of the AR method.

3. A Brief Review of the Autoregressive (AR) Method

The AR method, in essence, is a Prony's method formulated in the frequency domain. It is designed to estimate the complex frequency (or, equivalently, period and Q) and the complex amplitude (or, equivalently, amplitude and phase) of each complex sinusoidal function from a discrete time series consisting of a number of such components. The time series can be written as

$$x(n) = \sum_{j=1}^{M} [A_j \exp(in\sigma_j) + A_j^* \exp(-in\sigma_j^*)] \qquad (1)$$

$$n = 1, 2, 3, \cdots, N$$

where N is the number of data points, M is the number of components, $\{A_j\}$ and $\{\sigma_j\}$ are, respectively, the complex amplitude and the complex frequency of the jth component; $\{A_j\}$ and $\{\sigma_j\}$ are all unknowns to be estimated from the data $\{x(n); n = 1, 2, \cdots, N\}$. The nonlinear problem of estimating $\{\sigma_j\}$ is rendered linear by transforming (1) into the following autoregressive form:

$$x(n) = \sum_{i=1}^{2M} S_i x(n-i) \qquad n = 2M+1, \cdots, N \qquad (2)$$

with a new set of unknowns to be determined: the $2M$ real AR coefficients $\{S_i; i = 1, 2, \cdots, 2M\}$. Once the latter is found (see below), the set of M complex frequencies $\{\sigma_j; j = 1, 2, \cdots, M\}$ can be recovered by finding the $2M$ poles of the AR series (2) via the following $2M$th degree polynomial equation in complex variable Z:

$$Z^{2M} - S_1 Z^{2M-1} - S_2 Z^{2M-2} - \cdots - S_2$$
$$= \sum_{j=1}^{M} [Z - \exp(i\sigma_j)][Z - \exp(-i\sigma_j^*)] \qquad (3)$$

Equation (3) is from Prony [see, e.g., *Froberg*, 1969] and can be easily solved numerically using, for example, Bairstow's method.

To find $\{S_i\}$, instead of doing, say, a least squares time domain estimation directly from (2), we taper all time series on both sides of (2) by a Hanning window and then Fourier transform (2) at a few frequency points in a narrow frequency band containing the target single component or a few interfering components whose complex frequencies are to be estimated. Then, considering only these components, we can obtain the least squares solutions for the S_i thereof by means of, say, singular value decomposition. The advantage of doing so in the frequency domain is twofold.

1. We have "decomposed" (2) into small, independent subsystems each of which corresponds to one component ($M = 1$) or a small number of interfering components ($M \le 3$, say) regardless of how many components are actually present in the time series (which, in many cases, is unknown).

2. The use of tapering windows and the fact that Fourier transformation "concentrates" the information of a sinusoidal component into a narrow frequency band greatly improve our estimates at a given signal-to-noise ratio. However, we should emphasize here that the uncertainty in the Q estimate is, in general, $2Q$ times larger than that in the frequency estimate [*Chao and Gilbert*, 1980].

After obtaining the complex frequency estimates of a given component as described above, we can go back to (1) and do (linear) least squares estimation for its complex amplitude A_j. For the same reasons as above, we do this in the frequency domain.

Now that we have obtained both the complex frequency and the complex amplitude of a particular component, we can subtract that component from the time series and hence remove the corresponding spectral peak from the power spectrum (for an example, see Figure 3). In general, the quality of our AR estimates for a given component is directly proportional to the signal-to-noise ratio of the component, and formulae given by *Chao and Gilbert* [1980] will be used to assess the uncertainties in the estimates. Finally, we point out that the AR method is robust, fast, and above all, has high frequency-resolving power.

4. Data Analysis

The polar motion data set that we use in this study is the homogeneous ILS data set reduced by *Yumi and Yokoyama* [1980]. It spans 80 years: 1900–1979; that makes it the only homogeneous data set in existence which is long enough to resolve the fine structure in the Chandler wobble. It consists of two monthly time series: the X direction motion along the Greenwich meridian and the Y direction motion along the 90°E longitude. Note that we have reversed the astronomical Y direction so that the coordinate system used here is a right-handed one (when viewed from above the north pole).

4.1. *Pretreatment of the Data*

Prior to our AR analysis of the polar wobbles, we first subtract from both series a mean and a linear trend estimated by a least squares fit. This procedure is simply a measure of "cleaning up" the data. It yields, as a by-product, an estimate for the secular drift of the pole: 3.52 arc sec × 10^{-3}/yr in the direction 79.4°W. This estimate, unsophisticated as it is, agrees well with earlier investigations (see the summary by *Lambeck* [1980, pp. 90–91], and also *Dickman* [1981]). The two resultant zero-mean, trendless series will be designated ILS-X and

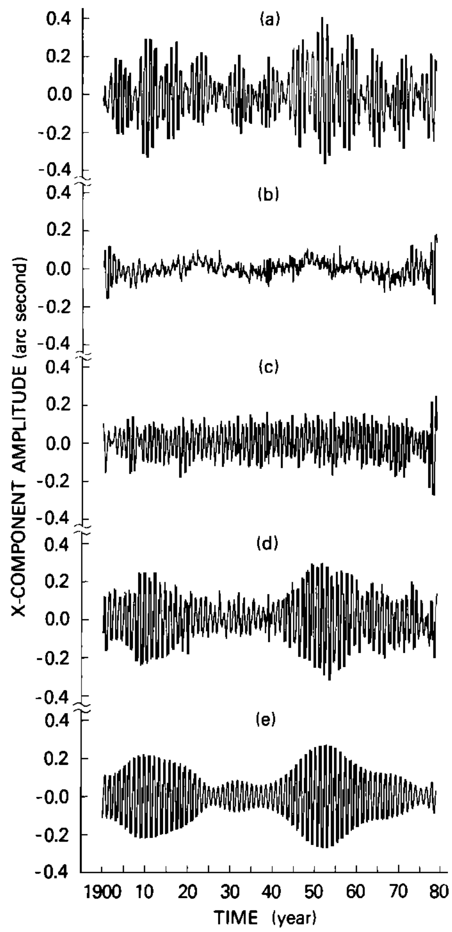

Fig. 1. X component of (a) the zero-mean, trendless series, ILS-X (see section 4.1); (b) Markowitz wobble, ILS-X-MW (see section 5.1); (c) annual wobble, ILS-X-AW (see section 5.2); (d) Chandler wobble, ILS-X-CW (see section 5.3); (e) the four-component synthetic Chandler wobble (see Table 3).

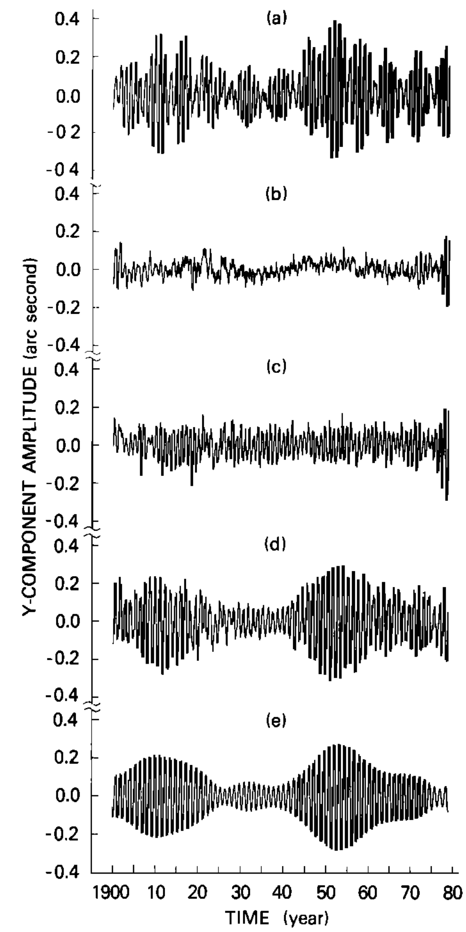

Fig. 2. Same as Figure 1, but for the Y component.

ILS-Y, and are shown in Figures 1a and 2a, respectively. The (Hanning windowed) Fourier power spectra, heavily interpolated (by means of zero padding of the time series prior to an FFT), of ILS-X and ILS-Y are given in Figure 3. Seen rising well above the noise level are the spectral peaks belonging to the Markowitz wobble (WM), the annual wobble (AW), and the Chandler wobble (CW).

4.2. *Autoregressive (AR) Analysis of the Polar Wobbles*

1. We perform a single-component AR analysis for the Markowitz wobble on ILS-X and ILS-Y; the resultant estimates of complex frequency and complex amplitude (the latter referred to the epoch 1900) are listed in Table 1.

2. Similarly, we obtain single-component AR estimates for the annual wobble, resulting in Table 2.

3. For the Chandler wobble, Figure 3 clearly warns of complicated structures; it shows (at least) four spectral peaks associated with the Chandler wobble, barely resolvable visibly. This is a situation to which the AR method is particularly suited and useful. It is our experience that for noisy data containing signals of considerably different signal-to-noise ratios (as in the present case), AR estimates deteriorate when given too many degrees of freedom. Therefore, as a first step, only two components near the central period (referred to as components II and III) are tried and subtracted. Now that the two components II and III are removed, a remaining set of two components (referred to as I and IV) are revealed in the spectrum. We then reverse the role of the two sets and iterate this procedure to ensure that each set of two-component estimates is essentially free from contaminations from the other two components. Figures 4 and 5 show the result of the said subtraction procedures for ILS-X and ILS-Y, respectively (the annual wobble has been removed beforehand according to Table 2). The results converge after only one iteration; they are listed in Table 3.

Two things should be noted here.

1. As mentioned earlier in section 3, AR method yields estimates for harmonic component(s) in a narrow frequency band, independent of components that reside in other disjoint

TABLE 1. AR Estimates for the Markowitz Wobble

	Period, years	Q	Amplitude (0.001 arc sec)	Phase, deg
ILS-X	29.6 ± 1.1	$>15, < -11$	24.6 ± 4.4	197 ± 10
ILS-Y	31.7 ± 0.9	$>25, < -12$	23.0 ± 3.2	242 ± 8

TABLE 2. AR Estimates for the Annual Wobble

	Period, days	Q	Amplitude (0.001 arc sec)	Phase, deg
ILS-X	$365.20 \pm .08$	$-930 \sim -3700$	88.4 ± 2.5	108 ± 2
ILS-Y	$365.10 \pm .10$	$800 \sim 4600$	84.0 ± 3.0	10 ± 2

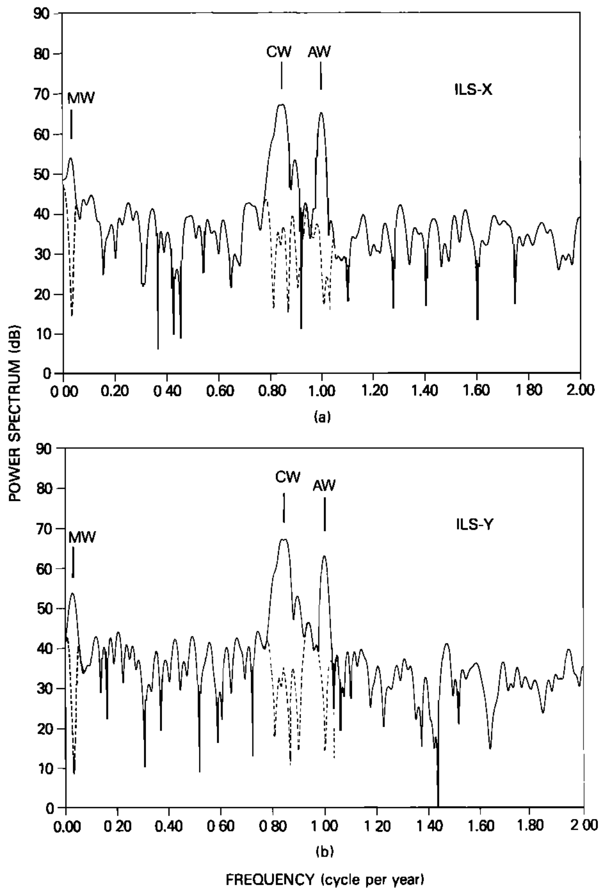

Fig. 3. Hanning-windowed Fourier power spectrum of (*a*) ILS-X (see Figure 1*a*) and (*b*) ILS-Y (see Figure 2*a*). The dashed line indicates the spectrum after removal (by subtraction according to Tables 1, 2, and 3, respectively) of the Markowitz wobble (MW), annual wobble (AW), and Chandler wobble (CW).

frequency bands. In the present case, the spectral peaks of the three wobbles are indeed well separated thanks to the long record length (see Figure 3). This means that there is no significant interference (in the frequency domain) between wobbles, and our estimates presented in Tables 1–3 have not been contaminated by the presence of other wobble terms. This, however, is not an essential requirement. For instance, if the record length is such that the annual Chandler interference is significant, we can simply perform a simultaneous estimation for both wobbles in a similar manner as in analysis 3 above. The removal of the wobble terms, by subtracting them from ILS-X and ILS-Y according to Tables 1–3, is shown in Figure 3 (as dashed lines).

2. In Tables 1–3 we have also included the standard deviations for the estimates, calculated according to formulae given by *Chao and Gilbert* [1980]. That some of the Q estimates are associated with large uncertainties is inevitable in situations where the record length is much shorter than the decay, or growth, time of the component in question. Note that a Q estimate such as ">15, <-11" means that the amplitude of the component, if decaying, decays less rapidly than $Q = 15$, or, if growing, grows less rapidly than $Q = -11$.

5. Motion of the Polar Wobbles

In this section, we give a detailed discussion of the foregoing results as presented in Tables 1–3.

5.1. *Markowitz Wobble*

If we subtract the annual and the Chandler wobbles (according to Tables 2 and 3, respectively) from both ILS-X and ILS-Y, we get ILS-X-MW and ILS-Y-MW, the series in which the Markowitz wobble is the sole significant signal. They are shown in Figures 1*b* and 2*b*. The predominant feature, clearly, is a random noise, but the 30-year periodicity is now visible. The "excursion" near both ends is the consequence of some misfit due, in turn, to the usage in the AR analysis of the tapering window which tends to deemphasize the end values of the time series in the fitting procedure.

Figure 6*a* gives a visual presentation of the results in Table 1 by showing an extrapolated five-cycle pole path of the Markowitz wobble. Note that the Q used in Figure 6 correspond to the mean rate of decay (or growth) (that is, the mean of $1/Q_1$ and $1/Q_2$, where Q_1 and Q_2 are the upper and lower bounds on Q as listed in Tables 1–3). For instance, in the present case for the Markowitz wobble, $Q = -88$ for ILS-X, and $Q = -43$ for ILS-Y. It is marginally retrograde (ILS-X-MW lags ILS-Y-MW by $45 \pm 18°$), in agreement with *Dickman*'s [1981] analysis. The wandering about of the Markowitz pole path (as opposed to a steady, repetitive one) is due mainly to the large difference in the two period estimates. However, this difference may not indeed be substantial when we take into consideration the uncertainties associated with the period estimates. In any event, in light of the fact that the ILS data contain only about $2\frac{1}{2}$ cycles of the motion, all we can claim is that Table 1 (and hence Figure 6*a*) only reflects certain features of a possibly complicated motion during a relatively short span of time. Note that our analysis shows little, if any, indication of a rapidly decaying Markowitz amplitude as reported by *Vicente and Currie* [1976]: Our Q estimates for the Markowitz wobble have absolute values considerably higher than their values. *Vicente and Currie* [1976] obtained their Q values based on the spectral width in the MEPS, an estimator that cannot distinguish between decaying and growing amplitudes (see section 2). In fact, our results do favor the latter, implying that the Markowitz wobble has been excited during the past 80 years.

5.2. *Annual Wobble*

The period estimates for the annual wobble agree well with the theoretical value of 365.25 days; and the Q estimates are

TABLE 3*a*. AR Estimates for the Chandler Wobble From ILS-X

Component	Period, days	Q	Amplitude (0.001 arc sec)	Phase, deg
I	$406.45 \pm .29$	$>500, < -1080$	26.7 ± 2.4	216 ± 5
II	$426.00 \pm .08$	$711 \pm 27\%$	119.9 ± 2.7	273 ± 1
III	$437.46 \pm .09$	$-189 \pm 8\%$	57.3 ± 1.3	342 ± 1
IV	$452.73 \pm .27$	$180 \pm 22\%$	68.6 ± 4.3	150 ± 4

TABLE 3*b*. AR Estimates for the Chandler Wobble From ILS-Y

Component	Period, days	Q	Amplitude (0.001 arc sec)	Phase deg
I	$406.85 \pm .42$	$-210 \sim -1930$	17.0 ± 2.1	132 ± 7
II	$426.15 \pm .10$	$703 \pm 34\%$	119.5 ± 3.4	185 ± 2
III	$437.43 \pm .11$	$-184 \pm 10\%$	57.2 ± 1.6	253 ± 2
IV	$452.39 \pm .33$	$220 \sim 600$	56.9 ± 4.5	54 ± 5

optimum representations (in the least squares sense) of the behavior of the annual term in the 80-year span according to ILS observations.

We can reveal the annual wobble by subtracting the Markowitz and the Chandler wobbles (according to Tables 1 and 3, respectively) from both ILS-X and ILS-Y. The resultant series, designated ILS-X-AW and ILS-Y-AW, are shown in Figures 1*c* and 2*c*. It is seen that the annual wobble, although fluctuating from year to year, appears to be fairly steady on a longer time scale (except for the excursion near both ends which, as stated above, is an artifact). This is also manifested by the high absolute values of the Q estimates. Figure 6*b* gives a five-cycle pole path of the annual wobble according to Table 2. ILS-X-AW leads ILS-Y-AW by $98 \pm 4°$, giving an almost purely prograde motion.

Brillinger [1973] and *Dickman* [1981] have studied the time variations in the amplitude and phase of the polar wobbles at some chosen nominal frequency by means of complex demodulation. We shall now do the same for the observed annual wobble.

Following *Dickman* [1981], the complex demodulate $\Gamma_f(t)$ at frequency f of a complex time series $z(t)$ is defined as

$$\Gamma_f(t) = \frac{1}{2L+1} \sum_{|t-u| \le L} z(u) \exp(-i2\pi f u) \qquad (4)$$

Thus, $|\Gamma_f(t)|$ is a "running amplitude" and $\arg(\Gamma_f(t))$ a "running phase" of $z(t)$ at frequency f, averaged over a $(2L+1)$-point long segment of the time series. We choose $L = 48$ so that the averaging length is about 10% of the total, or 8 years.

Now let f correspond to the theoretical period of the annual wobble, 365.25 days (positive for prograde motions), and let

$$z(t) = (\text{ILS-X-AW}) + i(\text{ILS-Y-AW}) \qquad (5)$$

The resultant complex demodulate is shown in Figure 7. It confirms the earlier observation that the amplitude and phase of the annual wobble are rather stationary over the years (except, again, for the artifact near the ends). Thus, thanks to the "cleanness" of our annual series resulted from the successful removal of the Chandler components, we found no evidence for the large variations reported in the literature (see, for example, *Rochester* [1973] and *Lambeck* [1980]; also compare Figure 7 with *Dickman*'s [1981] Figure 3). This, of course, is indicative of a more or less stationary annual excitation function.

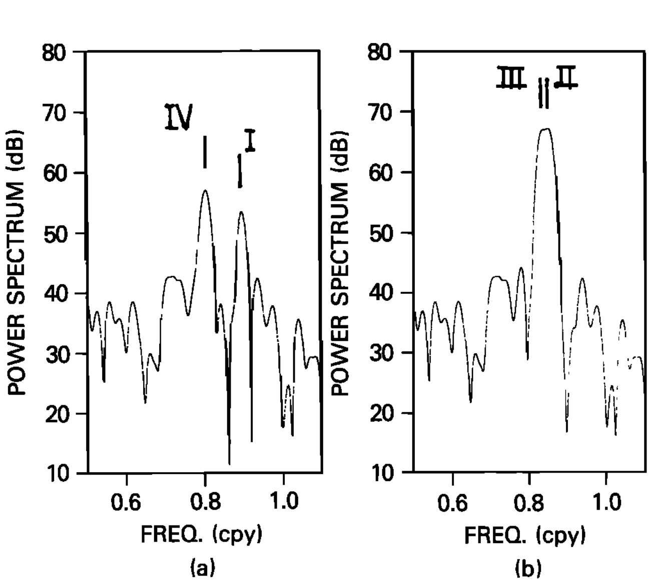

Fig. 4. Hanning-windowed Fourier power spectrum showing the removal of (*a*) components II and III of Chandler wobble and (*b*) components I and IV of Chandler wobble from ILS-X-CW (see Table 3*a*).

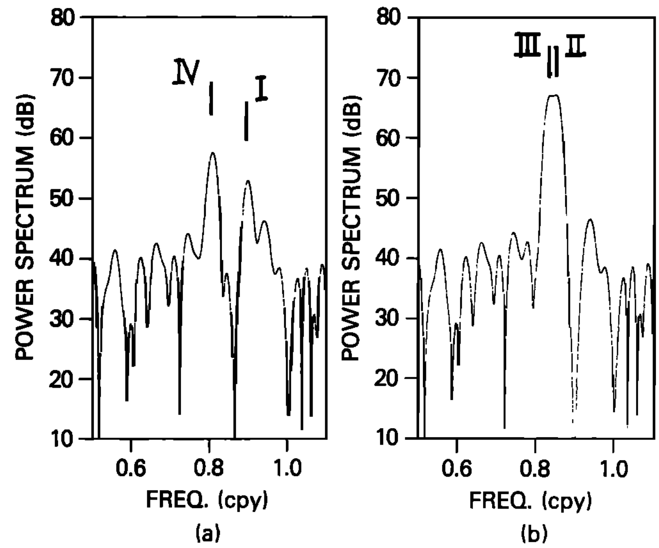

Fig. 5. Same as Figure 4, but for ILS-Y-CW (see Table 3*b*).

5.3. *Chandler Wobble*

By subtracting the Markowitz and the annual wobbles according to Tables 1 and 2 from ILS-X and ILS-Y, we obtain the two Chandler series, ILS-X-CW and ILS-Y-CW, shown in Figures 1*d* and 2*d*. Notice in Table 3 the close agreement between the corresponding estimates for period Q and amplitude obtained independently from ILS-X and ILS-Y. Note also that for all four components, ILS-X-CW leads ILS-Y-CW by an estimated phase angle close to 90°; in fact, it is seen that 90° is within one standard deviation of all four estimated phase differences. It seems that this coherent behavior of the four components is not consistent with the single-component hypothesis [see, e.g., *Munk and MacDonald*, 1960; *Lambeck*, 1980], which considers the Chandler wobble as a single component with a fixed period and an amplitude modulated by a sequence of temporally and/or spatially random excitations. The steady, nearly circular, prograde path of the pole associated with each component alone is shown in Figures 6*c*–6*f*.

Figures 1*e* and 2*e* give the "synthetic" Chandler series, that is, the linear combination of the four components according to Table 3. Compare them with the observations, Figures 1*d* and 2*d*. The resemblance is, of course, not surprising, for it is simply a time-domain presentation of the successful removal (see Figure 3) of the spectral peaks belonging to the four Chandler components.

The complex demodulate of the complex Chandler series

$$z(t) = (\text{ILS-X-CW}) + i(\text{ILS-Y-CW}) \qquad (6)$$

at a "center Chandler period" of 432.00 days is shown as solid lines in Figure 8, which are practically the same as those obtained by *Dickman* [1981] (except for some unimportant phase conversions). Also shown in Figure 8 as dashed lines is the complex demodulate, similarly obtained, of the four-component synthetic Chandler series. Again, the close agree-

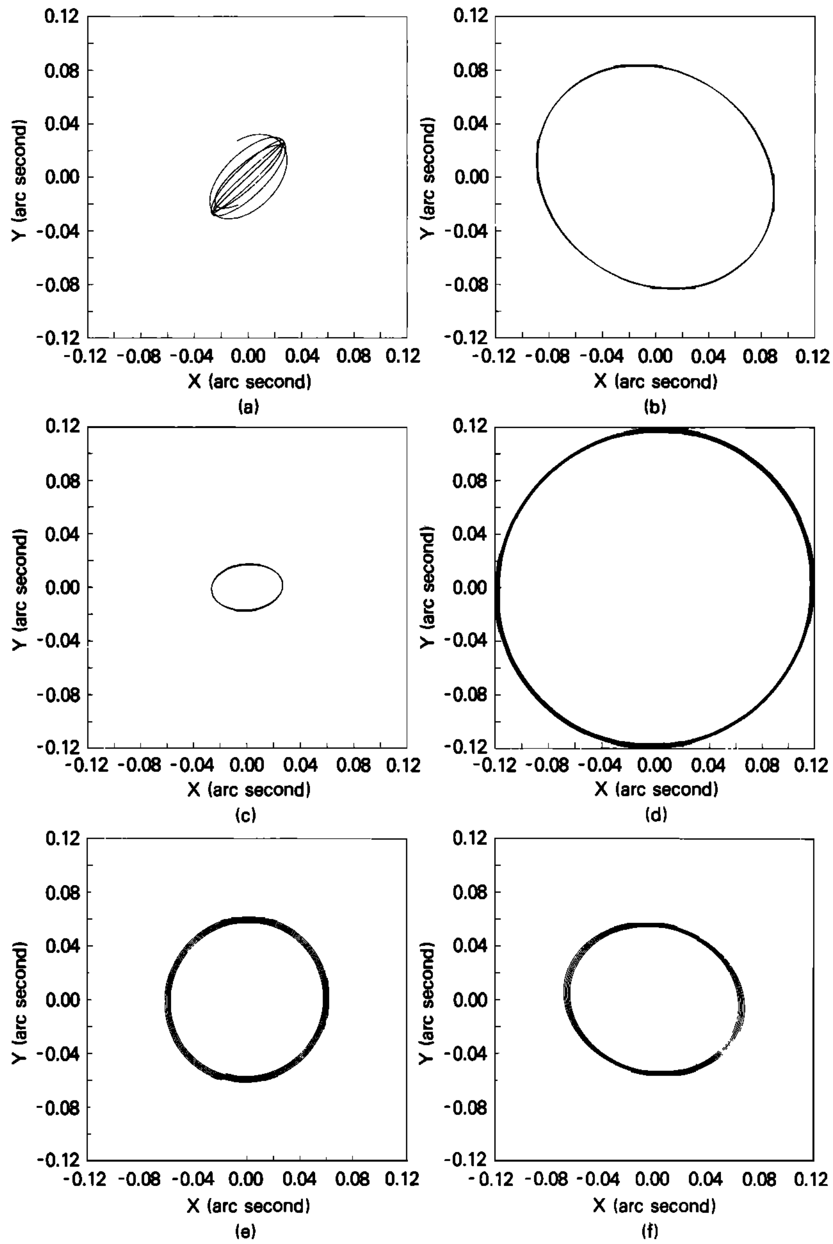

Fig. 6. Five-cycle pole path of (*a*) Markowitz wobble (see Table 1), (*b*) annual wobble (see Table 2), and (*c*)–(*f*) components I–IV of Chandler wobble (See Table 3).

ment (except near the ends) is to be expected. The amplitude variation (Figure 8*a*) is, of course, nothing but a smoothed version of the envelope of the Chandler series. The phase variation (Figure 8*b*), on the other hand, in effect reveals the cumulative deviation of the zero crossings (say) of the Chandler series from the zero crossings that a single component with period 432.00 days would have. The nearly 180° change in phase during 1920–1940 has been pointed out as suggestive of a phase reversal due to some sudden change in the (unknown) excitation mechanism [see, e.g., *Fedorov and Yatskiv*, 1965; *Guinot*, 1972]. However, our analysis here "resolves" the mystery by showing that the apparent phase variation of the Chandler wobble is an inevitable consequence of the multiple-component nature of the Chandler wobble, a view advocated also by *McCarthy* [1974].

Next, we shall examine the empirical period-amplitude relation proposed by *Melchior* [1957] [see also *Munk and MacDonald*, 1960, p. 151]: the period and amplitude of the Chandler wobble (at least up to the year 1950) are, to a certain degree, proportional. Here we shall employ the concept of instantaneous period (IP) and instantaneous amplitude (IA) [*Munk and MacDonald*, 1960], see Figure 9. Figure 10 gives the IP and IA sequences for the synthetic Chandler X series obtained numerically (those for the Y series are not shown because they are practically identical). It clearly shows a high positive correlation between IP and IA prior to 1950; the correlation coefficient is found to be 0.73 (or 0.76 for the Y series). Thus, again, our four-component model offers an "explanation" to the period-amplitude relation.

After 1950, however, the IP-IA correlation diminished drastically and IP has remained fairly constant at around 435 days (Figure 10*b*). In fact, we do not see the large period variations during 1960–1975 reported by *Graber* [1976] [see also *Carter*, 1981], who used the maximum entropy method (MEM) on

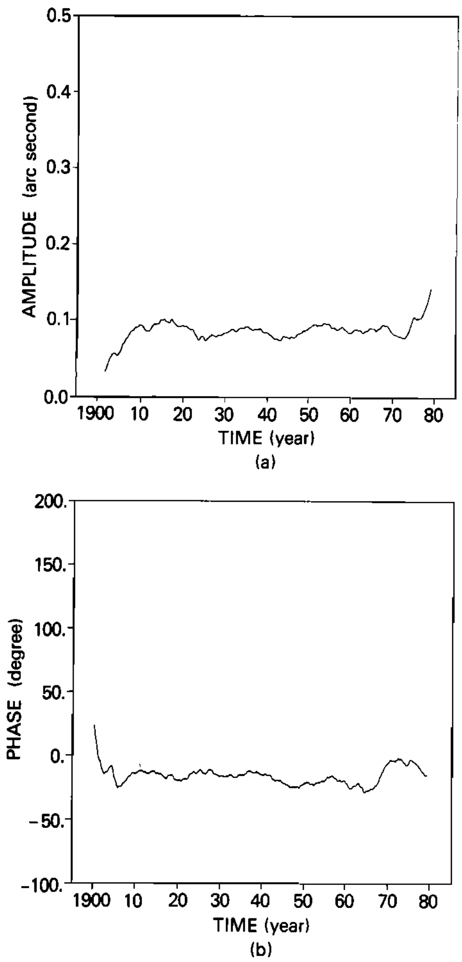

Fig. 7. Complex demodulate of the annual wobble (Figures 1*c* and 2*c*) at period = 365.25 days: (*a*) amplitude and (*b*) phase.

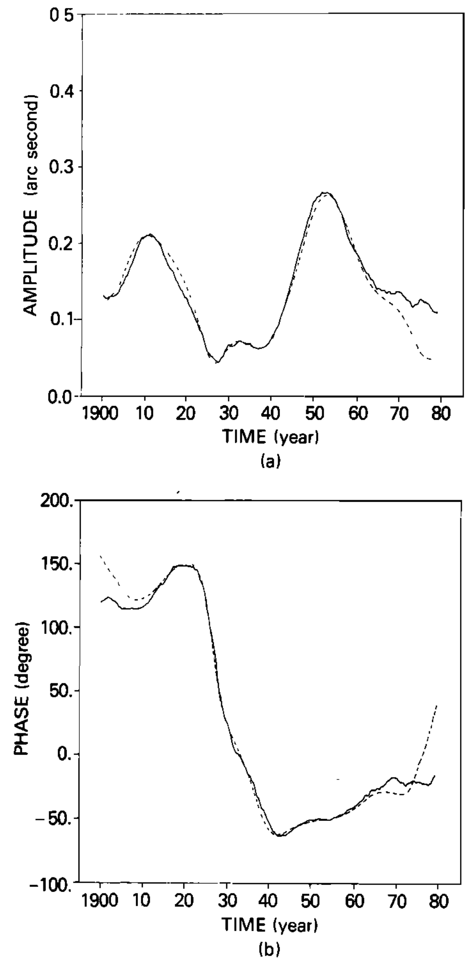

Fig. 8. Complex demodulate of the Chandler wobble at period = 432.00 days: (*a*) amplitude and (*b*) phase. The solid lines are for ILS-X-CW and ILS-Y-CW (Figures 1*d* and 2*d*), the dashed lines are for the four-component synthetic Chandler series (Figures 1*e* and 2*e*).

overlapping 2.9-year segments of the International Polar Motion Service (IPMS) data. This discrepancy may have resulted from the method that Graber used; as pointed out earlier in section 2, MEM tends to give biased frequency estimates as used on such short records.

6. Further Discussions

1. The ILS data set has been known for its systematic errors due to a lack of observational continuity and uniformity as well as alterations in reduction procedures over the years [see, e.g., *Lambeck*, 1980]. Some of the noise is bound to find its way into the (rereduced) homogeneous data set used in our study. Indeed, a look at Figures 1 and 2 shows that the mean noise level is about as high as one third of the signal level of the annual wobble, or a quarter of that of the Chandler wobble. However, this does not immediately condemn the data set. As stated in section 3, the Fourier transformation "concentrates" a periodic component into a narrow frequency band, while it "spreads" a random noise over the whole spectrum. This has resulted in the 26–28 dB signal-to-noise ratio for the annual term, and 15–30 dB signal-to-noise ratio for the Chandler components (see Figure 3). In fact, these are the values used to obtain the standard deviations quoted in Tables 1–3.

2. The multiple-component model for the Chandler wobble is certainly not a new invention. *Chandler* [1901*a*, *b*] [see also *Mulholland and Carter*, 1982], through pre-1900 observations, derived two periods for the then newly found motion: 428.5 days and 436.0 days; they most likely correspond to component II and III of our Table 3. Modern investigators, using post-1900 ILS data, have constantly been haunted by the (at least apparent) presence of multiple periods as well [e.g., *Yashkov*, 1965; *Colombo and Shapiro*, 1968; *Pedersen and Rochester*, 1972]. Our present study certainly advocates this hypothesis, hopefully on a stronger ground. *Gaposch-*

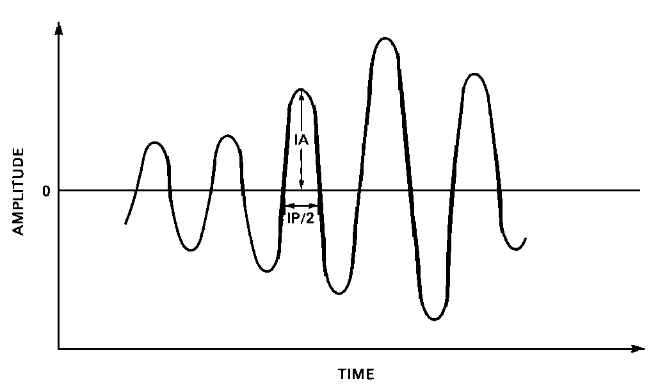

Fig. 9. Schematic diagram illustrating the meaning of IA (instantaneous amplitude) and IP (instantaneous period).

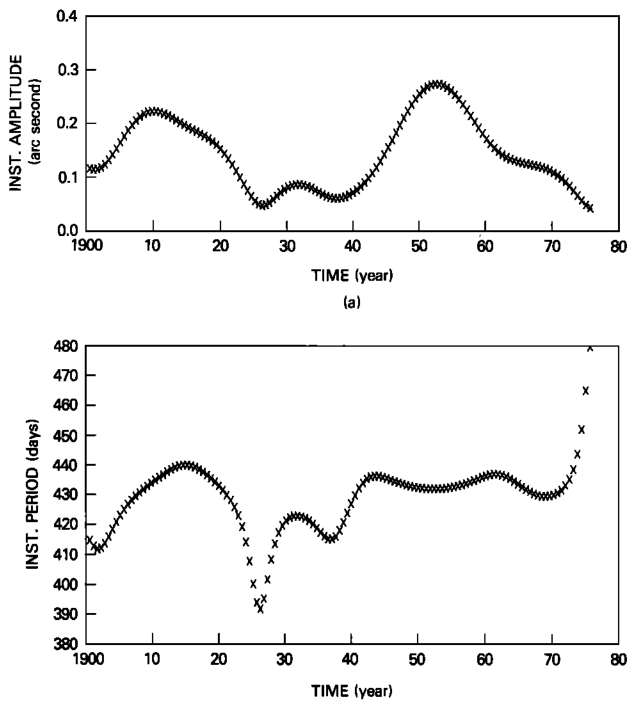

Fig. 10. The sequence of (*a*) IA and (*b*) IP of the *X* component of the four-component synthetic Chandler series.

kin [1972], using a least squares analysis, has also identified in the polar motion from 1846 to 1970 four periodic terms: 406.44, 426.67, 437.20, and 452.89 days. The rather striking agreement between his results (for 1846–1970) and Table 3 (for 1900–1979) provides a partially independent evidence for the four-component behavior. On the other hand, *Gaposchkin*'s [1972] *Q* estimates, in general, have higher absolute values than our values, and the corresponding amplitude estimates (reduced to the epoch 1900) differ by as much as 40%. This presumably can be attributed to the differences between the two data sets (as well as the methods of analysis) because, unlike the periods, the *Q* and amplitude estimates are indeed sensitive to noises and data inhomogeneities.

3. The origin of the multiple Chandler periods, their existence granted, may be perplexing. Theoretically, an elastic rotating earth model has only one free Chandler period. In fact, the Chandler wobble corresponds to a "singlet" normal mode of free oscillation belonging to an elastic rotating earth [*Chao*, 1983], so the existence of multiple periods is certainly not a consequence of some spectral splitting phenomenon. The answer, then, presumably lies in the existence of inelastic layers in the earth (hydrosphere, asthenosphere, and outer core) and the coupling thereof with the elastic parts of the earth [see, e.g., *Colombo and Shapiro*, 1968; *Zhang*, 1982]. Whether this system is consistent with the observed four-component behavior of the Chandler wobble awaits further investigations.

4. Numerically, the *Q* estimates for the Chandler components listed in Table 3 are, as with the annual wobble, those which achieve an optimum least squares fit to the data over the past 80 years. They ought not to be used, say, to extrapolate the time series in either direction; nor, from a geophysical point of view, should they be interpreted as the energy dissipation factor of the earth at Chandler periods. For one thing, they are quite different from one another, and some of them are negative (meaning growing amplitudes). Rather, they should be taken as reflecting the behavior of the excitation mechanism and, perhaps, the coupling between different components. As a matter of fact, to separate the effect of an unknown excitation function from that of energy dissipation numerically in an objective manner is rather difficult, if not impossible. As a result, the anomalously low *Q* estimates in the literature (as summarized by *Rochester* [1973] and *Lambeck* [1980]) might be questionable. This is a problem that we have not attempted to address in this paper.

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