4/11/00

"Writing" math

1. Rules for incorporating symbols and equations in prose

- normal grammar
- normal punctuation
- define symbols

2. Examples

3. Exceptions

Try to write math like real English. Follow the same rules.

eq in text vs separate line.

At small Reynolds numbers, the discharge Q of a fluid through a straight circular tube of length l is given by

$$Q = \frac{\pi\rho r^4}{8\mu l}\Delta P\,, \tag{6}$$

where ΔP is the pressure difference across the tube, and ρ is the fluid density. For a tube containing bubbles, the discharge becomes

$$Q = \frac{\pi\rho r^4}{8\mu l^*}\cdot\left[\Delta P - \sum_{\text{bubbles}} \Delta P_{\text{b}}(Q)\right]\,, \tag{7}$$

where l^* is the length of the tube minus the lengths of the bubbles in it, and $\Delta P_{\text{b}}(Q)$ is the pressure drop across one bubble, given by (3). Note that ΔP_{b} depends on Ca. Ca, in turn, depends on the average speed of the bubble in the tube. The speed of the bubbles u is related by

$$v = (1 - w)\cdot u \tag{8}$$

to the average speed of the suspending (wetting) fluid v, which is given by

$$v = \frac{Q}{\rho\pi r^2}\,. \tag{9}$$

ΔP_{b} depends thus on Q, and so does w. Substituting (8) and (9) into (1) yields

$$w(1 - w)^{2/3} = \tau \tag{10}$$

Anything complicated gets its own line.
Anything to which you refer later gets own line.

In the low Reynolds number limit the flow in each fluid domain satisfies the Stokes and continuity equations

$$\nabla \cdot \boldsymbol{T} = \mu \nabla^2 \boldsymbol{u} - \nabla p + \rho \boldsymbol{g} = 0 \quad \text{and} \quad \nabla \cdot \boldsymbol{u} = 0, \tag{2.1}$$

in text

where $\boldsymbol{u}$ is the fluid velocity, p is the fluid pressure, $\boldsymbol{T}$ is the stress tensor defined to incorporate the body force $\rho \boldsymbol{g}$, i.e. $\boldsymbol{T} = -p\boldsymbol{I} + \mu[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T] + \rho \boldsymbol{g} \cdot \boldsymbol{x}\boldsymbol{I}$, and μ is the fluid viscosity. As the stress tensor $\boldsymbol{T}$ is defined to be divergence free, the body force thus appears in the boundary conditions, equations (2.4) and (2.5) below. We denote the fluid domains by subscripts 1 and 2 for drops 1 and 2, respectively, and by the subscript *ext* for the external fluid.

We require that the velocity decays to zero far from the drops,

$$\boldsymbol{u}_{ext} \to 0 \quad \text{as} \quad |\boldsymbol{x}| \to \infty \quad ? \tag{2.2}$$

and that the velocity is continuous across all interfaces,

$$\boldsymbol{u}_1 = \boldsymbol{u}_{ext} \text{ on } S_I \quad \text{and} \quad \boldsymbol{u}_2 = \boldsymbol{u}_{ext} \text{ on } S_{II}, \tag{2.3}$$

where S_I is the surface bounding drop 1, and S_{II} is the surface bounding drop 2. The stress jump $[\![\boldsymbol{n} \cdot \boldsymbol{T}]\!]$ across an interface is balanced by the density contrast and interfacial tension forces, which depend on the local curvature $\nabla_s \cdot \boldsymbol{n}$ of the interface:

and ?

$$[\![\boldsymbol{n} \cdot \boldsymbol{T}^I]\!] = \boldsymbol{n} \cdot \boldsymbol{T}^I_{ext} - \boldsymbol{n} \cdot \boldsymbol{T}^I_1 = \sigma\, (\nabla_s \cdot \boldsymbol{n})\boldsymbol{n} + \boldsymbol{n}\Delta\rho\, (\boldsymbol{g} \cdot \boldsymbol{x}) \quad \text{on } S_I, \tag{2.4}$$

$$[\![\boldsymbol{n} \cdot \boldsymbol{T}^{II}]\!] = \boldsymbol{n} \cdot \boldsymbol{T}^{II}_{ext} - \boldsymbol{n} \cdot \boldsymbol{T}^{II}_2 = \sigma\, (\nabla_s \cdot \boldsymbol{n})\boldsymbol{n} + \boldsymbol{n}\Delta\rho\, (\boldsymbol{g} \cdot \boldsymbol{x}) \quad \text{on } S_{II}, \tag{2.5}$$

in text

The velocity field created by a translating drop, described by equations (2.2) and (2.3), creates velocity gradients which deform nearby drops. Let ϵ_i measure the small distortion away from a spherical shape, and let $\boxed{\mathscr{B}_i = \Delta\rho g a_i^2/\sigma}$ be the Bond number for drop i. The far-field velocity gradient generated by drop 1 in the vicinity of drop 2 is $O(U_1 a_1/d^2)$. A balance of viscous stresses, $O(\mu U_1 a_1/d^2)$, by the interfacial tension stresses of drop 2, $O(\epsilon_2 \sigma/a_2)$, which tend to keep the drop nearly spherical, leads to a small shape distortion of drop 2 with magnitude

$$\epsilon_2 = O\left(\frac{a_1^3 a_2 \Delta\rho g}{\sigma d^2}\right) = O\left(\mathscr{B}_2 \frac{a_1}{a_2}\left(\frac{a_1}{d}\right)^2\right). \tag{2.6}$$

A similar analysis for drop 1 gives

$$\epsilon_1 = O\left(\frac{a_1 a_2^3 \Delta\rho g}{\sigma d^2}\right) = O\left(\mathscr{B}_1 \frac{a_2}{a_1}\left(\frac{a_2}{d}\right)^2\right). \tag{2.7}$$

Since the magnitude of deformation is $O(\mathscr{B}a^2/d^2)$, it follows that the correction to the rise speed is $O(U^{(0)}\mathscr{B}a^2/d^2)$, where $U^{(0)}$ is the Hadamard–Rybczyński rise speed. Also, the relative magnitude of distortion of the two drops is

$$\frac{\epsilon_1}{\epsilon_2} = \left(\frac{a_2}{a_1}\right)^2. \tag{2.8}$$

Thus, (surprisingly) the smaller drop will be more deformed than the larger drop, as seen in the experiments shown in figure 1.

boundary value problem assumes the form (see Appendix C)

$$\begin{aligned}\nabla\cdot\boldsymbol{T}^{(2)}&=0,\quad r>1,\qquad \nabla\cdot\hat{\boldsymbol{T}}^{(2)}=0,\quad r<1,\\ \nabla\cdot\boldsymbol{u}^{(2)}&=0,\qquad\qquad\quad\; \nabla\cdot\hat{\boldsymbol{u}}^{(2)}=0,\end{aligned}\tag{5.18}$$

$$\boldsymbol{u}^{(2)}-\hat{\boldsymbol{u}}^{(2)}=\boldsymbol{A}(\boldsymbol{n}_o,f)\quad\text{on}\quad r=1,\tag{5.19}$$

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$$\boldsymbol{u}^{(2)}\cdot\boldsymbol{n}_o=B(\boldsymbol{n}_o,f)+\frac{\boldsymbol{U}^{(2)}\cdot\boldsymbol{n}_o}{U^{(0)}}\quad\text{on}\quad r=1,\tag{5.20}$$

$$\boldsymbol{n}_o\cdot\boldsymbol{T}^{(2)}-\lambda\boldsymbol{n}_o\cdot\hat{\boldsymbol{T}}^{(2)}=\boldsymbol{C}(\boldsymbol{n}_o,f)\quad\text{on}\quad r=1,\tag{5.21}$$

where ^ denotes variables inside the drop. The dimensionless functions $\boldsymbol{A}$, B and $\boldsymbol{C}$ are derived in Appendix C, and depend on the detailed drop shape $f(\theta,\phi)$.

We can then use the Reciprocal theorem (equation (C35) in Appendix C) to obtain the second-order velocity correction

$$\boldsymbol{U}^{(2)}=-\frac{U^{(0)}}{4\pi(2+3\lambda)}\int_S\{[(1+2\lambda)\boldsymbol{I}+\boldsymbol{n}_o\boldsymbol{n}_o]\cdot\boldsymbol{C}(\boldsymbol{n}_o,f)+3\lambda\,[\boldsymbol{I}-3\boldsymbol{n}_o\boldsymbol{n}_o]\cdot\boldsymbol{A}(\boldsymbol{n}_o,f)+3(2+3\lambda)B(\boldsymbol{n}_o,f)\boldsymbol{n}_o\}\,\mathrm{d}S,\tag{5.22}$$

where S denotes a spherical drop surface. Evaluating the integrals we find

$$\boldsymbol{U}_i^{(2)}=(-1)^i c(\lambda)\boldsymbol{U}_i^{(0)}\cdot\hat{\boldsymbol{d}}\left(\boldsymbol{e}_z\cdot\hat{\boldsymbol{d}}\right)\left[\boldsymbol{e}_z-3\boldsymbol{e}_z\cdot\hat{\boldsymbol{d}}\hat{\boldsymbol{d}}\right],\tag{5.23}$$

where $\boldsymbol{e}_z$ is a unit vector in the vertical direction and

$$c(\lambda)=\frac{(16+19\lambda)(8-\lambda+3\lambda^2)}{240(1+\lambda)^2(2+3\lambda)}.\tag{5.24}$$

Note that $c=4/15$ for a bubble, $c=19/240$ for a rigid particle, and c has a minimum value for $\lambda\approx2.64$. The function $c(\lambda)$ is plotted in figure 20. The condition that the horizontal separation between the drops decreases (see equation (5.17)) is given by $\mathscr{B}(a_2^3/a_1d^2)\boldsymbol{U}_1^{(2)}\cdot\boldsymbol{e}_x-\mathscr{B}(a_1a_2/d^2)\boldsymbol{U}_2^{(2)}\cdot\boldsymbol{e}_x+\Delta U_H<0$, i.e.

$$\mathscr{B}>\frac{d}{a_1}\frac{2+3\lambda}{24(1+\lambda)c(\lambda)\cos^2\beta}\left[\left(\frac{a_1}{a_2}\right)^3-1\right].\tag{5.25}$$

5.4. *Translational velocity from the Reciprocal theorem*

The above analysis suggests a small $O(\mathscr{B}a^2/d^2)$ correction to the local description of the flow field owing to the drop's deformation. Since a/d and $\mathscr{B}$ are independent parameters we seek the next-order correction to the velocity field, $\mathscr{B}(a/d)^2\boldsymbol{u}^{(2)}(\boldsymbol{r})$, satisfying the Stokes equations both inside and outside the drop. This analysis corresponds to the translation of an isolated slightly deformed drop in an otherwise quiescent fluid. Thus, we seek solutions for the approximate translational velocity in the form

$$\boldsymbol{U}(a/d,\mathscr{B})=\boldsymbol{U}^{(0)}+\left(\frac{a}{d}\right)\boldsymbol{U}^{(1)}+\mathscr{B}\left(\frac{a}{d}\right)^2\boldsymbol{U}^{(2)}+O\left(U^{(0)}\frac{a^3}{d^3}\right).\tag{5.17}$$

Provided $\mathscr{B}>O(d/a)$ the dominant correction to the migration velocity arises from the third term on the right-hand side. Furthermore, so long as $\mathscr{B}>O(a/d)$, shape modifications are at least as important as the $O(a/d)^3$ corrections calculated using the method of reflections for spherical drops.

Substitution of (5.10) into (5.7) yields:

$$K_k = \frac{\prod_i m_{ik}^{\nu_{ik}} \gamma_{ik}^{\nu_{ik}} (\Sigma n_l)^b}{(\lambda_k n_k)^b} \tag{5.11}$$

There is one mass action equation such as (5.11) for each currently saturated pure mineral or solid solution end member. Recall that for pure minerals, equation (5.11) reduces to equation (5.7), with the a_k in the denominator of (5.7) set to unity.

The mass action equation for gas species g in a mixed gas is:

$$K_g = \frac{\prod_i m_{ig}^{\nu_{ig}} \gamma_{ig}^{\nu_{ig}}}{f_g} = \frac{Q_g}{f_g} \tag{5.12}$$

in which f_g is the fugacity of gas species g, and Q_g is the product in the numerator of the equation, defined here for use below. The fugacity f_g in equation (5.12) can be expressed:

$$f_g = \phi_g P n_g / \Sigma n_l \tag{5.13}$$

in which ϕ_g is the fugacity coefficient for gas species g, n_g is the number of moles of species g in the gas phase, P is fluid pressure, and the summation in the denominator is

Geochemistry