Eos, Transactions, American Geophysical Union, Vol. 76, No. 16, April 18, 1995 Page 161, 164-165

Wavelet Analysis Provides a New Tool for Studying Earth's Rotation

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The solid Earth's rotation varies slightly with time due to geophysical processes that involve motions and redistributions of mass occurring on or within the Earth, as dictated by the conservation of angular momentum. In particular, these variations (ΔLOD) in the atmosphere in terms of the axial atmospheric anglar momentum (AAM) are the primary cause for nontidal length-of-day variations on timescales of several days to several years [e.g., *Rosen*, 1993]. Here ΔLOD is a convenient measure of Earth's rotational speed relative to the uniform time kept by atomic clocks.

AAM and ΔLOD have many periodic oscillations that are more or less stationary in time, such as the seasonal terms with annual and semiannual periods. ΔLOD, in addition, has large tidal terms due to tidal deformations (see below). These are externally forced oscillations with known fixed periods. The Fourier spectrum is a conventional technique for analyzing such periodicities.

But nonstationary oscillations also abound in AAM and hence are duly reflected in ΔLOD. These oscillations could be internal modes that result from couplings between the atmosphere and oceans that evolve with time in amplitude, period, and/or phase. To reveal nonstationary, "localized" periodicities in a time series, the wavelet time-frequency spectrum has proven a powerful tool. A recently developed mathematical technique, wavelet analysis, can be used to represent functions that are local in time and frequency [*Morlet et al.*, 1982]. It has found applications in a wide variety of fields such as speech and signal analysis, image processing, and geophysics. *Gambis* [1992] and *Abarca del Rio and Cazenave* [1994] have used wavelet transform to study some aspects of the interseasonal Earth rotation variations. Here we employ wavelet analysis on two independent data sets and compare them: the geodetically determined LOD variation and the meteorologically derived AAM variation. Their wavelet spectra are shown in Figure 1 (detailed discussion later). They reveal interesting temporal evolution of quasi-periodic and intermittent oscillations in both LOD and AAM, and demonstrate the time-frequency characteristics of the LOD-AAM correlation.

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Wavelet Time-Frequency Spectrum

The wavelet transform of the time series f(t) is defined as

$$W_\psi(f)(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\, \psi\left(\frac{t-b}{a}\right) dt$$

where $\psi(t)$ is the basic wavelet (or a "wave packet") with effective length that is usually much shorter than the target time series $f(t)$. The variables are a and b: a is the dilation/compression scale factor that determines the characteristic frequency so that varying a gives rise to a "spectrum"; and b is the translation in time so that varying b represents the "sliding" of the wavelet over $f(t)$. The wavelet spectrum is thus customarily displayed in the time-frequency domain, or the a-b space with the horizontal time axis b and the vertical frequency axis a. Orthogonal sets of $\psi(t-b)/a$ (when a varies in powers of 2 or "octave") are often exploited in applications involving inverse transforms and reconstruction of signals. For our application we select the Morlet wavelet [*Morlet et al.*, 1982], which is a normalized, Gaussian-enveloped complex sinusoid with zero mean. It is only nearly orthogonal but offers satisfactory resolution and stability. We choose to examine the real part of the wavelet transform (for real f). It gives the amplitude undulation with the appropriate polarity and phase with respect to time owing to the symmetric nature of the real part of the Morlet wavelet as the kernel in integral (1). In contrast, the imaginary part, being antisymmetric in the kernel, gives the amplitude undulation as well but imparts a 90° phase shift in time. In many applications the modulus—combining real and imaginary parts—is preferred; but then the polarity/phase information, which is important in the present study, becomes absent. We use color contours such that amplitude peaks and troughs in horizontal successions in high contrast colors indicate the presence of strong oscillations in the data, relative to the weaker and less significant "background" of low color contrast.

Some limitations of the wavelet spectrum should be pointed out: Because of the temporal localization of the wavelet, the frequency resolution is limited. We show resolution at a quarter octave; the nonorthogonality of wavelets within an octave prevents much finer resolutions. Further, the limited time span introduces edge effect to the spectrum, which is more severe with longer periods. In our computation, time series values outside our time span are simply assumed to be zero.

ΔLOD Wavelet Spectrum

Figure 2a shows the geodetically determined ΔLOD derived from the "Space93" data set (courtesy of R. S. Gross, 1994). The data are very rich in signal content: the decadal, seasonal, and long-period tidal signals are the most prominent, superimposed on broad-band interannual and intraseasonal variations.

We shall first remove the decadal and the seasonal signals from the LOD series. The decadal fluctuation (including the mean value), believed to reflect fluid core activities, would be beset by edge effects while little further information than that already evident in Figure 2a would be obtained from the spectrum anyway. The seasonal signal, although interesting in its own right, is not to be studied here. We achieve the removal by performing a simultaneous least-squares fit of these signals to the entire LOD series followed by subtraction (decadal signal represented by a fourth degree polynomial and seasonal signals by annual and semiannual sinusoids). A year-by-year empirical removal of the seasonal terms is not desirable here, as signals at nearby periods would be removed because of the limited spectral resolution of the yearly series and the adaptive nature of the fitting procedure.

Figure 1a displays the wavelet spectrum of the resultant nonseasonal ΔLOD series within the frequency range corresponding to periods of 6.25 days to 3200 days. The short-period cutoff was chosen because of the inherent smoothing of the Space93 LOD data at several days. The long-period cutoff was selected because the spectral values at periods longer than that would not be realistic subject to the edge effect.

Toward the top of Figure 2a are the strong long-period tidal signals. Unrelated to the atmosphere, these signals in ΔLOD primarily result from tidal deformations in the solid Earth and are modified by the oceanic tides. The fortnightly zonal tides (primary periods: 13.63, 13.66, and 14.77 days) are rather prominent, with the half-year and 18.6-year modula-

0096/3941/7616/95/161/$01.00.