Wavelet Spectral Analysis of the Earth's Orbital Variations and Paleoclimatic Cycles

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ABSTRACT

The wavelet time-frequency spectral analysis is applied to geological records that are proxies of paleoclimatic variations: $\delta^{18}O$ in sedimentary cores, atmospheric CO_2 concentration, and a loess magnetic susceptibility stratigraphy within the past million years. These spectra are compared with those for the astronomically predicted variations of the earth's orbital eccentricity, obliquity, precession, and their resultant variations of the incoming insolation. The latter has been known to be unable to explain the characteristics of the observed 100-kyr paleoclimatic cycles. Based on similarities between the wavelet spectra of the orbital variations and paleoclimatic cycles, the authors introduce a signal–noise resonance theory to understand the dynamics of climate response to the orbital forcing. It is shown that the observed 100-kyr cycles are mainly caused by the period variation in the obliquity, which amplifies the small orbital forcing. But the observed flickers within these cycles are induced by the amplitude variation of obliquity and precession, which are two major components of the Milankovitch insolation deviations.

1. Introduction

It is known that conventional astronomical theory for climate change cannot explain the magnitude of the 100-kyr paleoclimate cycles found in geological records. This quandary has called into question our fundamental understanding of how the climate responds to external or internal forcing (Broecker 1992; Ludwig et al. 1992; Winograd et al. 1992; Imbrie et al. 1993; Beauford 1994). Recently, new astronomical concepts (Liu 1992, 1995) and wavelet spectral methods (Chao and Naito 1995; Bolton et al. 1995; Lau and Weng 1995; Weng and Lau 1994) have been proposed for orbital and climate time series studies. In this paper, we compute the wavelet spectrum of the earth's obliquity to explain the relationships between the observed paleoclimatic cycles and the earth's orbital variations. Our purpose is to see how much of the observed climate variability can be accounted for by considering the temporal modulation in total insolation due to lengthening or shortening of the obliquity cyclic period.

2. Wavelet time-frequency spectral analysis

The wavelet transform of the time signal $f(t)$ is defined as

$$W_\psi(f)(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right) dt, \qquad (1)$$

where $\psi(t)$ is the basic wavelet, a is a dilation/compression-scale factor that determines the characteristics frequency, and b represents the translation in time. Here we follow the scheme of Chao and Naito (1995) and select the Morlet wavelet (Morlet et al. 1982), which is a zero-mean, normalized, Gaussian-enveloped complex sinusoid:

$$\psi\left(\frac{t-b}{a}\right) = \frac{1}{\sqrt{2a\pi}} \exp\left[-\frac{(t-b)^2}{2a^2}\right] \exp\left[\frac{i\omega(t-b)}{a}\right]. \qquad (2)$$

The parameter ω, which determines the number of oscillations of the wavelet, is chosen to be $\omega = \pi (2/\ln 2)^{1/2} \cong 5.336$.

The wavelet spectral amplitude is customarily displayed in the time-frequency domain, that is, varying b and a. We shall examine only the real part of the amplitude, which, because of its symmetric nature, gives the amplitude undulation with the appropriate polarity and phase with respect to time. In contrast, the imaginary part, being antisymmetric, gives the amplitude undulation as well but imparts a 90° phase shift time. Alternatively, one can show the modulus of the wavelet transform; but the polarity and phase information becomes absent. Thus, amplitude peaks and troughs in

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