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Strain Green's Tensors, Reciprocity, and Their Applications to Seismic Source and Structure Studies

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Abstract Green's function approach is widely used in modeling seismic waveform. The representation theorem expresses the wave field as the inner product of the moment tensor and the spatial gradients of the Green's tensor. Standard practice in waveform calculations has been to compute the Green's tensors first and then obtain their gradients by numerical differentiation. The reciprocity of the Green's tensor enables us to express the wave field explicitly in terms of the strain Green's tensor, a third-order tensor composed of the spatial gradients of the Green's tensor elements. We propose here to use the strain Green's tensors rather than the Green's tensors themselves in computing the waveforms. By bypassing the need for Green's tensors and directly using the strain Green's tensors, we can improve the computational efficiency in waveform modeling while eliminating the possible errors from numerical differentiation. The strain Green's tensor elements are also directly related to the partial derivatives of the waveforms with respect to moment tensor elements and structural parameters. Through the inversion of the focal mechanisms of 27 small events in the Los Angeles region, we demonstrate the effectiveness of the strain Green's tensor database approach in quickly recovering source parameters based on realistic 3D models. We show that the same database can also be used to improve the efficiency and accuracy in computing the Fréchet kernels for tomography inversions.

Introduction

Seismic waveforms of finite frequency carry a great deal of information on earthquake sources and earth structure. Successful and efficient waveform modeling in source and structural studies are important in ensuring the quality of measurements and are therefore instrumental in improving the resolution and reliability of the resulting source and structural models. Recent developments in both waveform modeling techniques and high-performance computational facilities have also helped seismologists to pursue the waveform approach at relatively short periods in ever more complex structural environments (e.g., Olsen, 1994; Graves, 1996; Komatitsch *et al.*, 2004). The applications of these accurate waveform-modeling methods have allowed the use of 3D reference models to be used in both source studies (Liu *et al.*, 2004; Chen *et al.*, 2005) and in constructing the sensitivity kernels (Zhao *et al.*, 2005; Q. Liu and J. Tromp, unpublished manuscript, 2006) for structural inversions.

Numerical waveform-modeling tools provide the wavefield solutions caused by earthquake sources that are either point or distributed moment tensors. They can also simulate the Green's functions, the wave fields from unit impulsive point forces that are also very useful in various kinds of waveform-based source and structural studies. Green's functions are particularly helpful in improving the computational efficiency when there is a need to calculate the waveforms for the same source-station pair again and again since they use the same Green's functions. Furthermore, when waveforms at the same station from multiple sources are needed, a joint use of the complete set of the Green's functions from the station, that is, the Green's tensor, and its reciprocity property can reduce the required computations even more drastically (Eisner and Clayton, 2001; Graves and Wald, 2001).

In calculating waveforms from earthquake sources described by moment tensors, it is in fact the spatial gradients of the Green's tensors that are directly used. Currently, the standard approach (Eisner and Clayton, 2001; Graves and Wald, 2001) obtains the Green's tensors from numerical simulations and then uses numerical differentiation to compute the spatial gradients of the Green's tensors. In the present study we extend this treatment and propose an approach in which we use the reciprocity and express the wave field explicitly in terms of the strain Green's tensors, a third-order tensor composed of the spatial gradients of the Green's tensor elements. By bypassing the need for the Green's tensors and their numerical differentiation, waveform calculation