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A Student's Guide to and Review of Moment Tensors

M. L. Jost and R. B. Herrmann

Department of Earth and Atmospheric Sciences
Saint Louis University
P. O. Box 8099
St. Louis, MO 63156

ABSTRACT

A review of a moment tensor for describing a general seismic point source is presented to show a second order moment tensor can be related to simpler seismic source descriptions such as centers of expansion and double couples. A review of literature is followed by detailed algebraic expansions of the moment tensor into isotropic and deviatoric components. Specific numerical examples are provided in the appendices for use in testing algorithms for moment tensor decomposition.

INTRODUCTION

A major research interest in seismology is the description of the physics of seismic sources. A common approach is the approximation of seismic sources by a model of equivalent forces that correspond to the linear wave equations neglecting non-linear effects in the near source region (Geller, 1976; Aki and Richards, 1980; Kennett, 1983; Bullen and Bolt, 1985). Equivalent forces are defined as producing displacements at the earth's surface that are identical to those from the actual forces of the physical process at the source. The equivalent forces are determined from observed seismograms that contain informations about the source and path and distortions due to the recording. Hence, the principle problem of source studies is the isolation of the source effect by correcting for instrument and path.

The classical method of describing seismic sources, having small dimensions compared to the wavelengths of interest (point source approximation) is by their strength (magnitudes, seismic moment) and their fault plane solution (Honda, 1962; Hirasawa and Stauder, 1965; Herrmann, 1975). Recently, seismic moment tensors have been used routinely for describing seismic point sources (e.g. Kanamori and Given, 1982; Dziewonski and Woodhouse, 1983b; Dziewonski *et al.*, 1983a-c, 1984a-c; Giardini, 1984; Ekström and Dziewonski, 1985; Dziewonski *et al.*, 1985a-d, 1986a-c, 1987a-f; Ekström *et al.*, 1987; Sipkin, 1987; PDE monthly listings published by NEIS). Gilbert (1970) introduced moment tensors for calculating the displacement at the free surface which can be expressed as a sum of moment tensor elements times the corresponding Green's function. An elastodynamic Green's function is a displacement field due to an unidirectional unit impulse, i.e. the Green's function is the impulse response of the medium between source and receiver. The response of the medium to any other time function is the convolution (Arfken, 1985) of that time function with the impulse response. The Green's function depends on source and receiver coordinates, the earth model, and is a tensor (Aki and Richards, 1980). The linearity between the moment tensor and Green's function elements was first used by Gilbert (1973) for calculating moment tensor elements from observations (moment tensor inversion). The concept of seismic moment tensors was further extended by Backus and Mulcahy (1976), and Backus (1977a, b). Moment tensors can be determined from free oscillations of the earth (e.g. Gilbert and Dziewonski, 1975), long-period surface waves (e.g. McCowan, 1976; Mendiguren, 1977; Patton and Aki, 1979; Patton, 1980; Kanamori and Given, 1981, 1982; Romanowicz, 1981; Lay *et al.*, 1982; Nakanishi and Kanamori, 1982, 1984) or long-period body waves (e.g. Stump and Johnson, 1977; Strelitz, 1978, 1980; Ward, 1980a, b; Fitch *et al.*, 1980; Fitch, 1981; Langston, 1981; Dziewonski *et al.*, 1981; Dziewonski and Woodhouse, 1983a, b). Throughout this Student's Guide, we will focus on second-rank, time independent moment tensors (Appendix I). We refer to Dziewonski and Gilbert (1974), Gilbert and Dziewonski (1975), Backus and Mulcahy (1976), Backus (1977a), Stump and Johnson (1977), Strelitz (1980), Sipkin (1982), and Vasco and Johnson (1988) for a description of time dependent moment tensors. Higher order moment tensors are discussed by Backus and Mulcahy (1976), Backus (1977a, b), and Dziewonski and Woodhouse (1983a).

The reason that moment tensors are important is that they completely describe in a first order approximation the equivalent forces of general seismic point sources. The equivalent forces can be correlated to physical source models such as sudden relative displacement at a fault surface (elastic rebound model by H. F. Reid, 1910), rapidly propagating metastable phase transitions (Evison, 1963), sudden volume collapse due to phase transitions, or sudden volume increase due to explosions (Kennett, 1983; Vasco and Johnson, 1988). The equivalent forces representing a sudden displacement on a fault plane form the familiar double couple. The equivalent forces of a sudden change in shear modulus in presence of axial strain are represented by a linear vector dipole (Knopoff and Randall, 1970). In conclusion, a seismic moment tensor is a general concept, describing a variety of seismic source models, the shear dislocation (double couple source) being just one of them.

The equivalent forces can be determined from an analysis of the eigenvalues and eigenvectors of the moment tensor (Appendix I). The sum of the eigenvalues of the moment tensor describes the volume change in the source (isotropic component of the moment tensor). If the