GEOPHYSICS, VOL. XXXI, NO. 2 (APRIL, 1966), PP. 362–371 4 FIGS., 1 TABLE

THE GRAVITATIONAL ATTRACTION OF A RIGHT RECTANGULAR PRISM†

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The derivation of a closed expression is presented to calculate the vertical component of the gravitational attraction of a right rectangular prism, with sides parallel to the coordinate axis. As any configuration can be expressed as the sum of prisms of various sizes and densities, the computation of the total gravitational effect of bodies of arbitrary shapes at any point outside of or on the boundary of the bodies is straightforward. To calculate the gravitational effect of the "unit" building element a subroutine called Prism has been developed, tested, and incorporated, in one program to calculate terrain corrections, and in another program for three-dimensional analysis of a gravity field.

I. INTRODUCTION

A number of papers have been published on methods of computing the gravitational attraction of simple forms such as the sphere, cylinder, ellipsoid, and prism. For most of these cases, only approximate expressions have been obtained, such that there are restrictions limiting the validity of the expressions near the computation point. In this paper a closed expression is developed for the gravitational attraction of a prism which is valid for any point outside of or on the boundary of the prism. It is possible to describe any arbitrary configuration in terms of building blocks composed of prisms of various dimensions and densities and, hence, to compute the vertical component of the gravitational attraction of any given mass distribution at arbitrarily selected points.

II. THE ATTRACTION OF A PRISM

The magnitude of the attraction of an elementary mass on a unit mass at distance r is given by:

$$\Delta F = G\rho \frac{\Delta v}{r^2}, \tag{1}$$

where G is the gravitational constant, ρ the density and Δv the volume element.

If the angle enclosed by r and the vertical axis is denoted by γ, then the vertical component of the attraction of a body can be obtained by integrating $\Delta F \cos\gamma$ over the volume, i.e.,

$$F_z = G\rho \int_V \frac{dv}{r^2} \cos\gamma = G\rho \int_V \frac{z dz}{r^3}. \tag{2}$$

The problem is simply to carry out this integration for a prism.

Using the cartesian coordinate system shown in Figure 1, (2) becomes:

$$F_z = G\rho \int_{x_1}^{x_2} dx \int_{y_1}^{y_2} dy \cdot \int_{z_1}^{z_2} \frac{z dz}{\sqrt{(x^2+y^2+z^2)^3}}. \tag{3}$$

Carrying out the integration with respect to z and without substituting the limits, one finds:

$$I_1 = \int \frac{z dz}{\sqrt{(x^2+y^2+z^2)^3}} = -\frac{1}{\sqrt{x^2+y^2+z^2}}. \tag{4}$$

Integrating (4) with respect to y gives:

$$I_2 = \int I_1 dy = \int \frac{dy}{\sqrt{x^2+y^2+z^2}} = \ln(y + \sqrt{x^2+y^2+z^2}). \tag{5}$$

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