The excitation of the Earth's polar motion

SEMINAR **II**

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鍾霨詠

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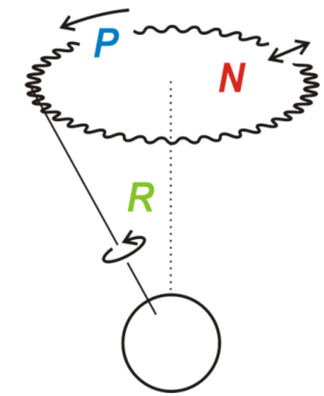
References

Earth's Rotation

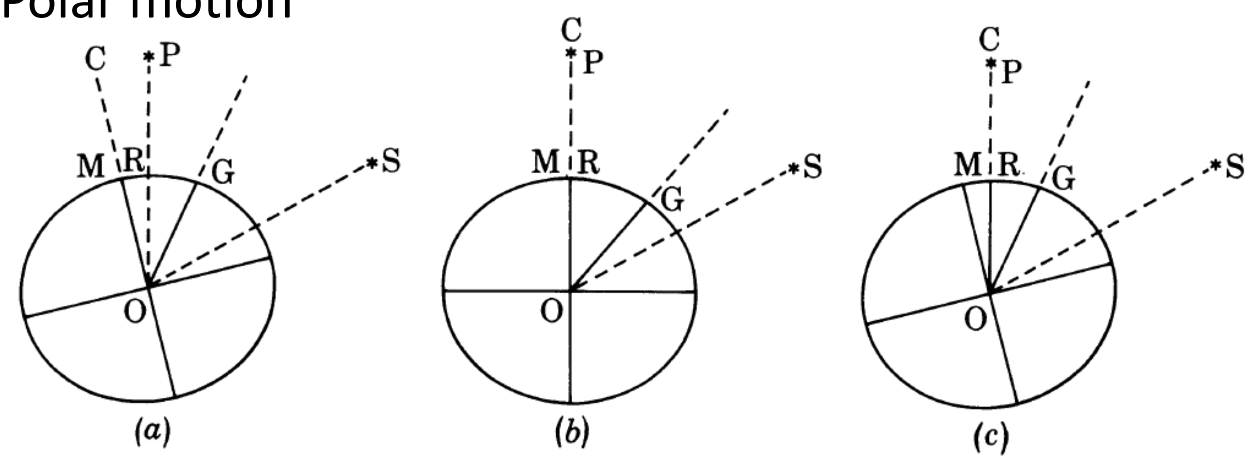
Astronomical (external torques → angular momentum change)

Precession *P*
Nutations *N*
Librations
Tidal Braking
Milankovitz Cycles



Geophysical (internal forces → no angular momentum change)

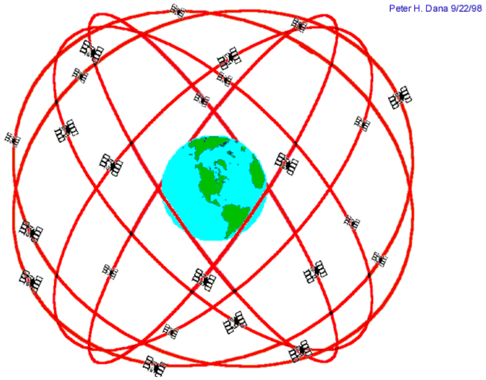
Length-of-day (LOD) change
Polar motion

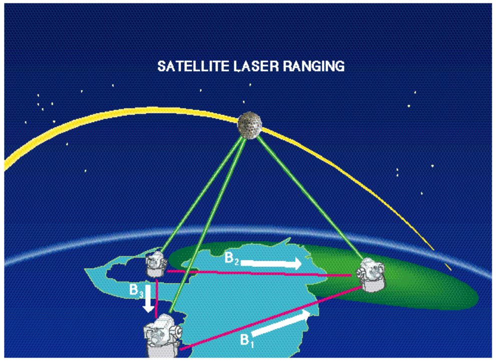


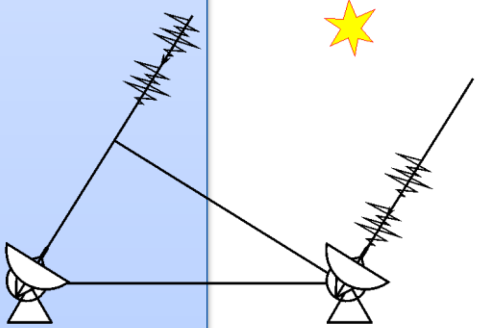
P, Polaris; **S**, star; **M**, geographical (body-fixed) pole; **G**, observatory; **0**, centre of the Earth; **R**, rotation pole; **C**, North Celestial Pole

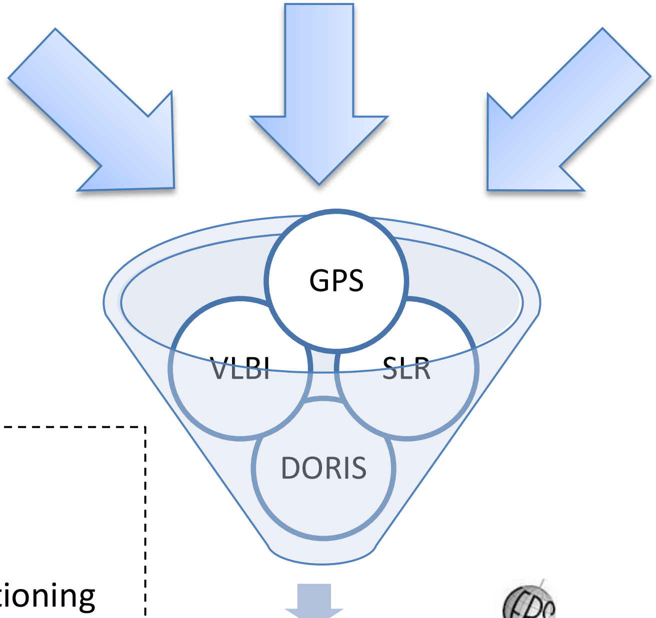
Earth's Rotation

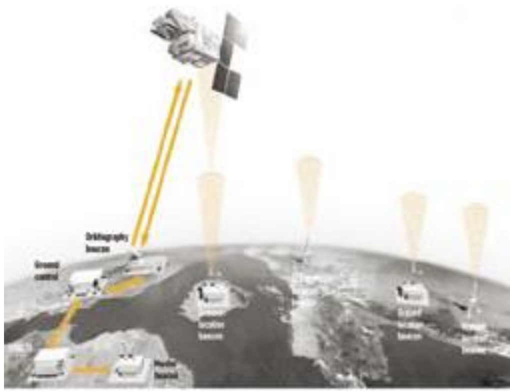
Nowadays, four main geodetic techniques are used to compute accurate coordinates: the GPS, VLBI, SLR, and DORIS.









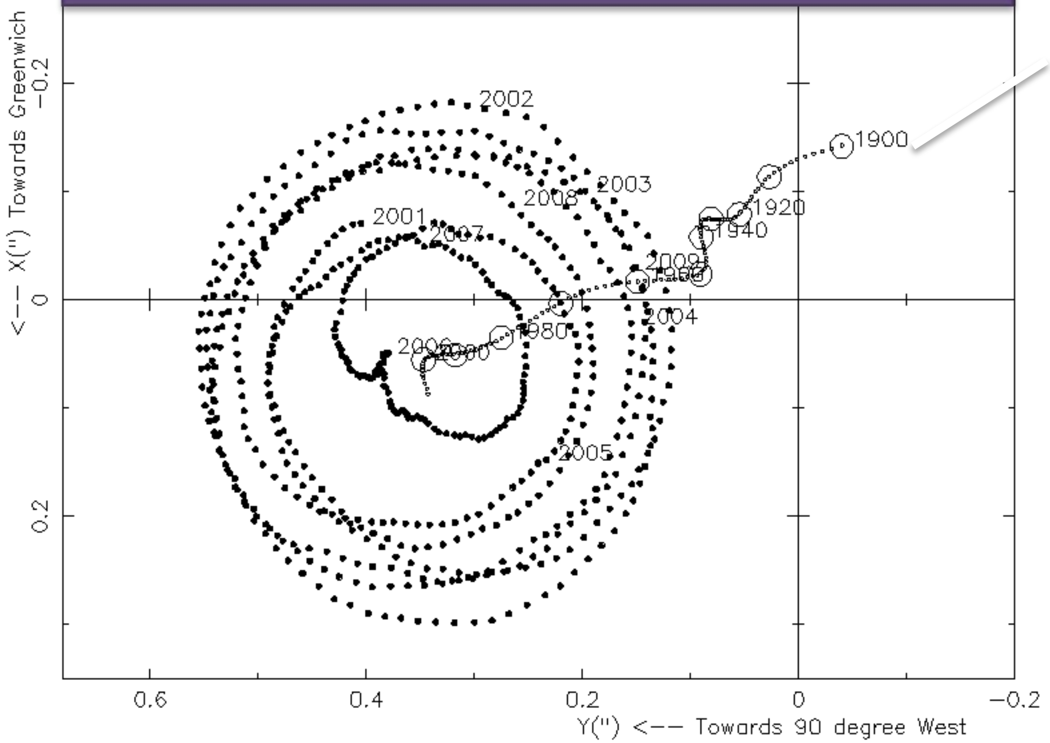


ICRF-ITRF

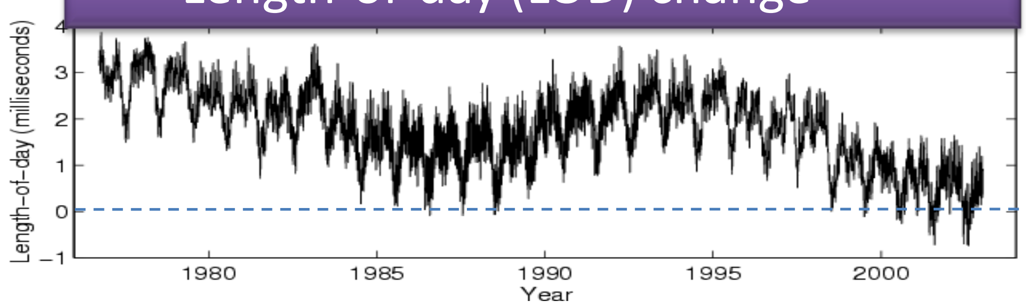
SLR	Satellite Laser Ranging
VLBI	Very-Long-Baseline Interferometry
GPS	Global Positioning System
DORIS	Doppler Orbitography and Radiopositioning Integrated by Satellite
ITRF	International Terrestrial Reference Frame
ICRF	International Celestial Reference Frame

Earth's Rotation

Polar motion

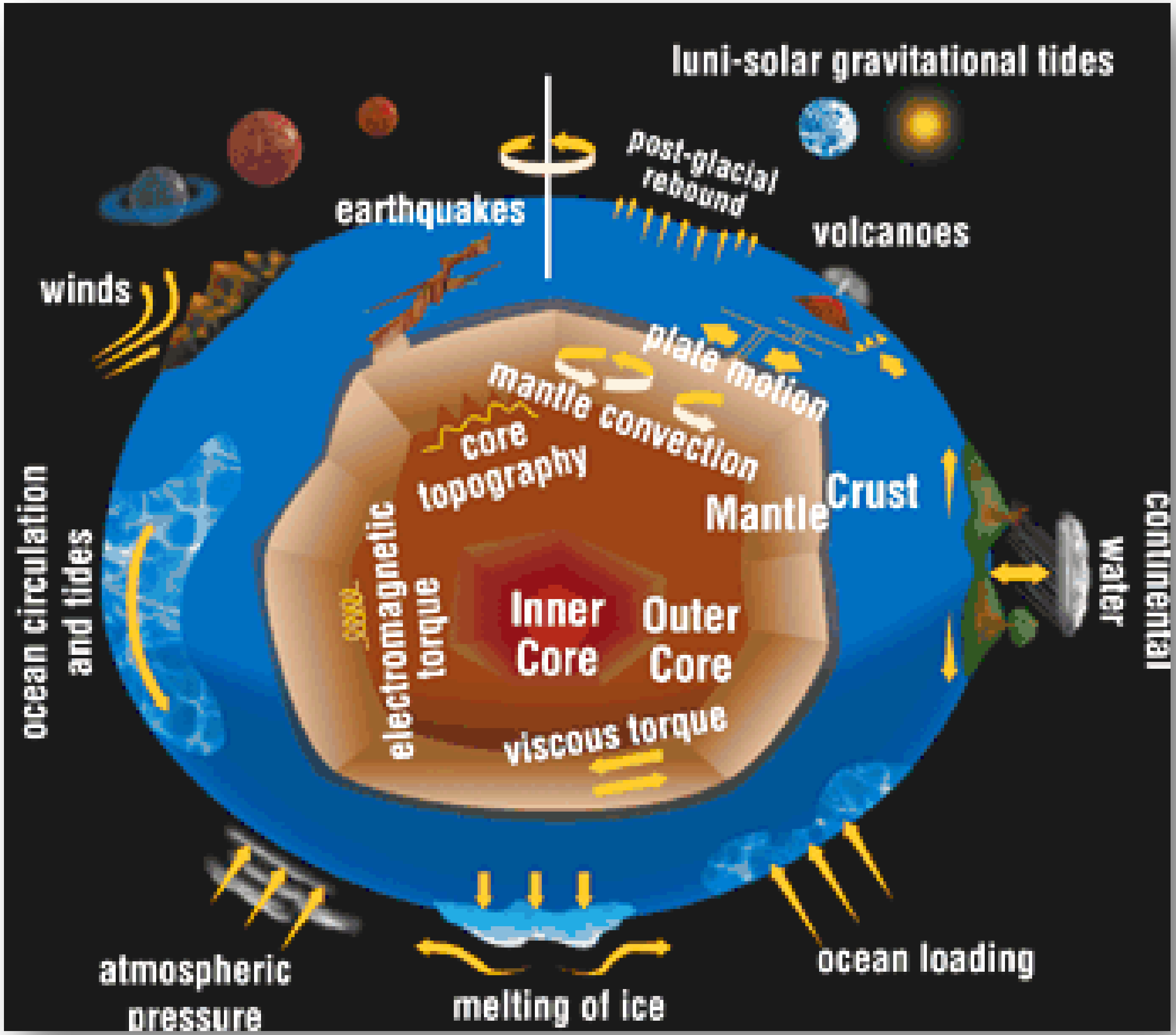


Length-of-day (LOD) change



Earth's Rotation

Earth system



Meteo-
rological

What's in the atmosphere?

ENSO

Air mass

QBO

Hadley circulation

monsoon

Trade winds

Mid-latitude jet-streams

Ferrel cells

ENSO El Niño-Southern Oscillation
QBO Quasi-biennial oscillation

Meteo-
rological

How is their coupling ?

All components of angular momentum would be zero if that surface were **perfectly spherical** and **perfectly slippery**, for then normal stresses would exert no couple and tangential (frictional) stresses would be absent. In practice both 'topographic' and **frictional stresses** are present, with **normal pressure forces** acting on the Earth's equatorial bulge playing a major role in the coupling associated with the changes in excitation that manifest themselves in the observed polar motion.

A brief summary

Meteo-
rological

Atmospheric excitation

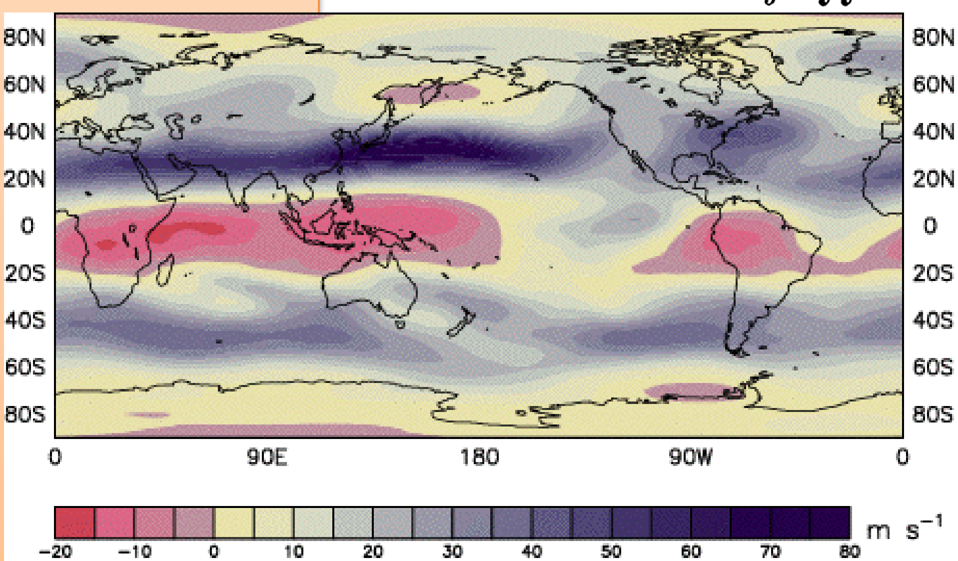
$$\chi_1 = \chi_1^{P} + \chi_1^{W} = \frac{-1.00\bar{R}^4}{(C-A)g}\oiint p_s \sin\phi\,\cos^2\phi\,\cos\lambda\,\mathrm{d}\lambda\,\mathrm{d}\phi$$

$$-\frac{1.43\bar{R}^3}{\Omega(C-A)g}\oiiint (u\sin\phi\,\cos\phi\,\cos\lambda - v\cos\phi\,\sin\lambda)\,\mathrm{d}\lambda\,\mathrm{d}\phi\,\mathrm{d}p,$$

$$\chi_2 = \chi_2^{P} + \chi_2^{W} = \frac{-1.00\bar{R}^4}{(\mathrm{C}-\mathrm{A})g}\oiint p_s \sin\phi\,\cos^2\phi\,\sin\lambda\,\mathrm{d}\lambda\,\mathrm{d}\phi$$

$$-\frac{1.43\bar{R}^3}{\Omega(C-A)g}\oiiint (u\sin\phi\,\cos\phi\,\sin\lambda + v\cos\phi\,\cos\lambda)\,\mathrm{d}\lambda\,\mathrm{d}\phi\,\mathrm{d}p,$$

$$\chi_3 = \chi_3^{P} + \chi_3^{W} = \frac{0.70\bar{R}^4}{Cg}\oiint p_s \cos^3\phi\,\mathrm{d}\lambda\,\mathrm{d}\phi + \frac{\bar{R}^3}{C\Omega g}\oiiint u\cos^2\phi\,\mathrm{d}\lambda\,\mathrm{d}\phi\,\mathrm{d}p.$$



Zonal wind field near jet stream levels (sampled here for January 1997), the primary contributor to atmospheric angular momentum and length of day changes.

(u, v, w)	Eulerian flow velocity vector
Ps	surface pressure
A, C	principal moments of inertia of a spheroid
R	mean radius of the Earth
Ω	mean angular velocity of the Earth
(λ, ϕ, r)	spherical polar coordinates

Dynamics of the earth's rotation

1

$$\mathrm{d}H_i/\mathrm{d}t + \epsilon_{ijk}\omega_j H_k = L_i,$$

※ Euler's dynamical equations

$$H_i(t) = I_{ij}(t)\,\omega_j(t) + h_i(t).$$

$$I_{ij} \equiv \int_V \rho(x_k x_k \delta_{ij} - x_i x_j)\,\mathrm{d}V$$

$$h_i \equiv \int_V \rho\epsilon_{ijk} x_j u_k\,\mathrm{d}V$$

2

$$\mathrm{d}(I_{ij}\omega_j + h_i)/\mathrm{d}t + \epsilon_{ijk}\omega_j(I_{kl}\omega_l + h_k) = L_i.$$

$$I_{ij}(t) = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{pmatrix} + \Delta I_{ij}(t).$$

$$(\omega_1, \omega_2, \omega_3) = (m_1, m_2, 1 + m_3)\,\Omega,$$

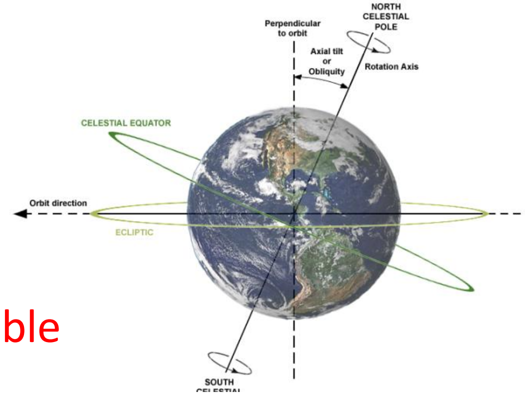
※ ignoring products of small quantities

$$|\Delta I_{ij}| \ll C, \quad |h_i| \ll \Omega C, \quad |m_i| = O(10^{-7}), \quad \dot{m}_i \ll \Omega$$

ω angular velocity
L_i externally applied torque
H_i absolute angular momentum
ϵ_{ijk} 'alternating' tensor
x_i body-fixed axes
I_{ij} (variable) inertia tensor
ρ density
δ_{ij} Kronecker delta
h_i angular momentum
(u,v,w) Eulerian flow velocity vector
A,C principal moments of inertia of a spheroid
m_i direction cosines of rotation axis
Ω mean angular velocity of the Earth

3

$$\left.\begin{aligned}\dot{m}_1/\sigma_r + m_2 &= \psi_2,\\ \dot{m}_2/\sigma_r - m_1 &= -\psi_1,\\ \dot{m}_3 &= \psi_3.\end{aligned}\right\}$$



$$\sigma_r \equiv (C-A)\,\Omega/A$$ ※ Chandler wobble

$$\left.\begin{aligned}\psi_1 &= (\Omega^2\Delta I_{13} + \Omega\Delta\dot{I}_{23} + \Omega h_1 + \dot{h}_2 - L_2)/\Omega^2(C-A),\\ \psi_2 &= (\Omega^2\Delta I_{23} - \Omega\Delta\dot{I}_{13} + \Omega h_2 - \dot{h}_1 + L_1)/\Omega^2(C-A),\\ \psi_3 &= \left(-\Omega^2\Delta I_{33} - \Omega h_3 + \Omega\int_0^t L_3\,\mathrm{d}t'\right)\Big/\Omega^2 C,\end{aligned}\right\}$$

4

$$\left[\begin{aligned}\boldsymbol{m}(t) &= \mathrm{e}^{\mathrm{i}\lambda_r t}\left((\boldsymbol{m}0) - \mathrm{i}\sigma_r\int_0^t \boldsymbol{\psi}(\tau)\,\mathrm{e}^{-\mathrm{i}\sigma_r\tau}\,\mathrm{d}\tau\right)\\ m_3 &= \psi_3 + \text{constant},\end{aligned}\right.$$

σr rigid-body free wobble frequency

Ψ non- homogeneous forcing terms

$$\Delta\boldsymbol{I} \equiv \Delta I_{13} + \mathrm{i}\Delta I_{23}$$
$$\boldsymbol{h} \equiv h_1 + \mathrm{i}h_2$$
$$\boldsymbol{L} \equiv L_1 + \mathrm{i}L_2.$$

$$\left[\begin{aligned}\boldsymbol{m} &\equiv m_1 + \mathrm{i}m_2,\\ \boldsymbol{\psi} &\equiv \psi_1 + \mathrm{i}\psi_2 = [\Omega^2\Delta\boldsymbol{I} - \mathrm{i}\Omega\Delta\dot{\boldsymbol{I}} + \Omega\boldsymbol{h} - \mathrm{i}\dot{\boldsymbol{h}} + \mathrm{i}\boldsymbol{L}]/\Omega^2(C-A)\end{aligned}\right.$$

$$\Omega C(1+m_3)+\Omega\Delta I_{33}+h_3=\text{constant}.$$

※ absence of external torques (Li = 0)

$$m_3=(\omega_3-\Omega)/\Omega$$

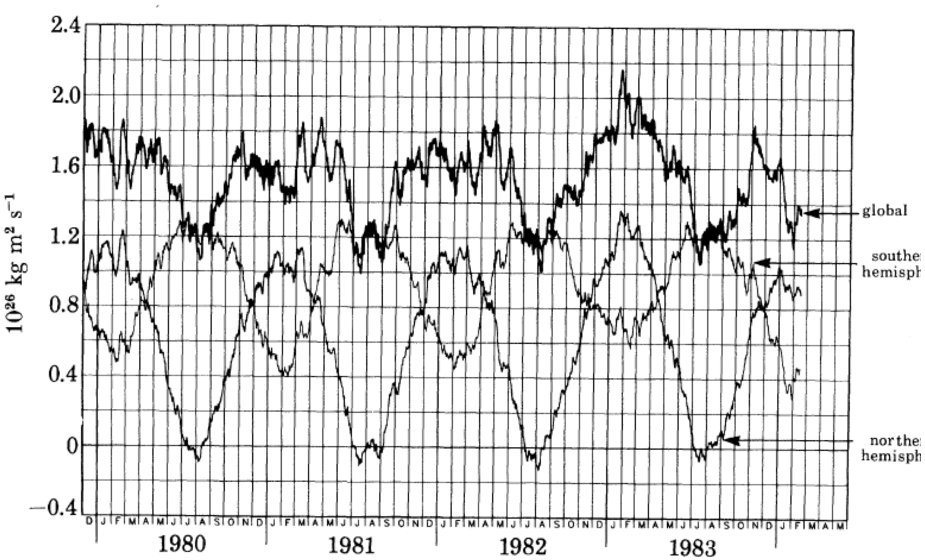
$$\Lambda=2\pi/\omega_3 \quad \Lambda_0=2\pi/\Omega.$$

$$m_3=-\Delta\Lambda/\Lambda_0$$

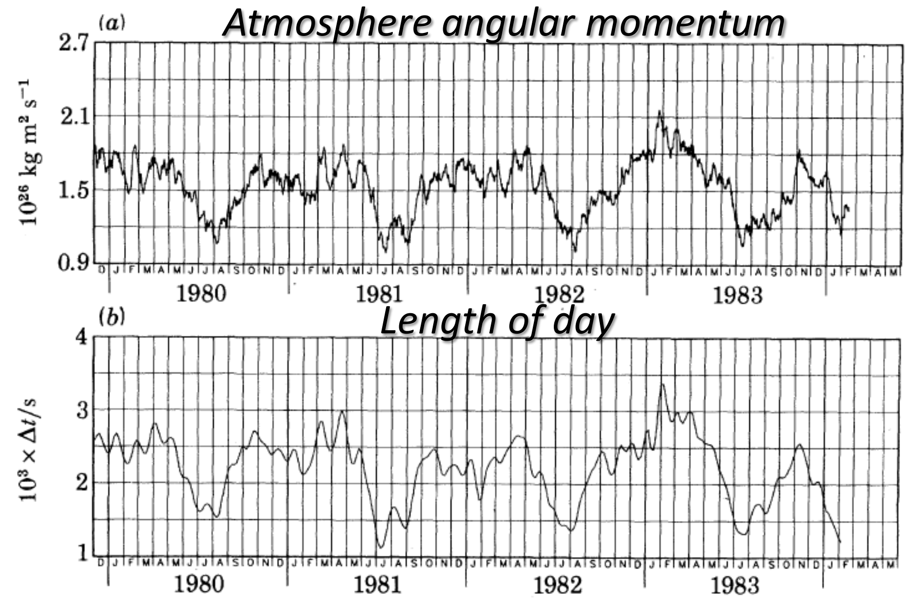
$\Delta\Lambda$ difference of length-of-day from its mean value

Atmosphere angular momentum

1 December 1979--15 February 1984



※ European Centre for Medium-Range Weather Forecasts (E.C.M.W.F.)



※ Bureau International de l'Heure (B.I.H.)

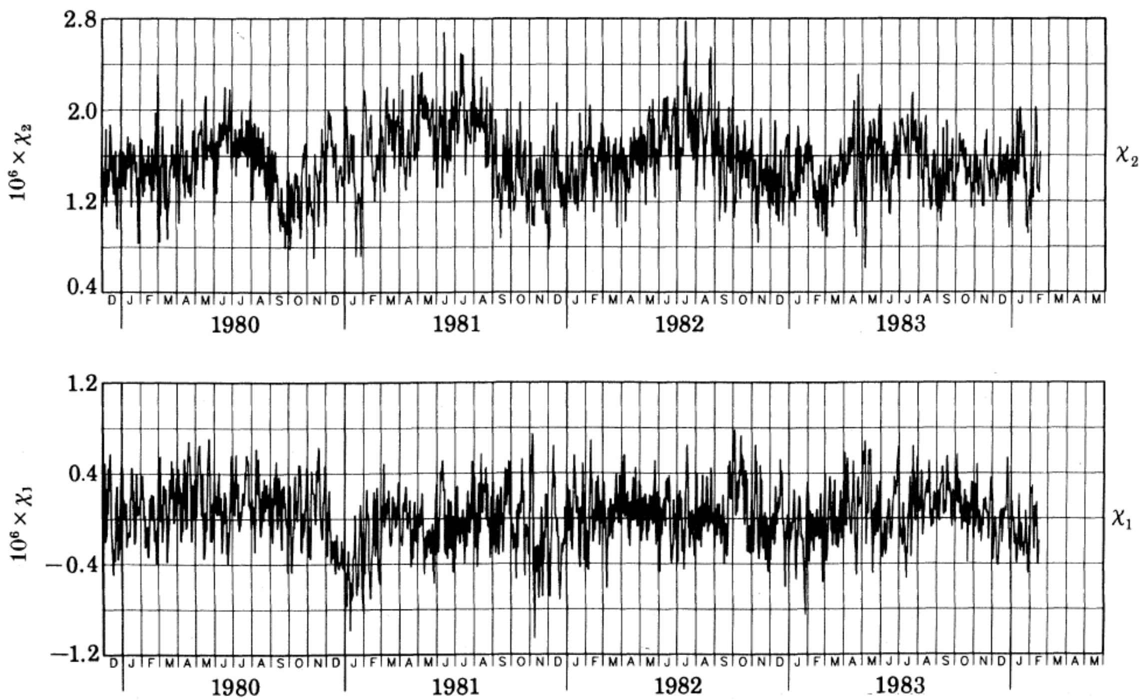
$\tilde{\chi}$ dimensionless atmospheric 'effective angular momentum'

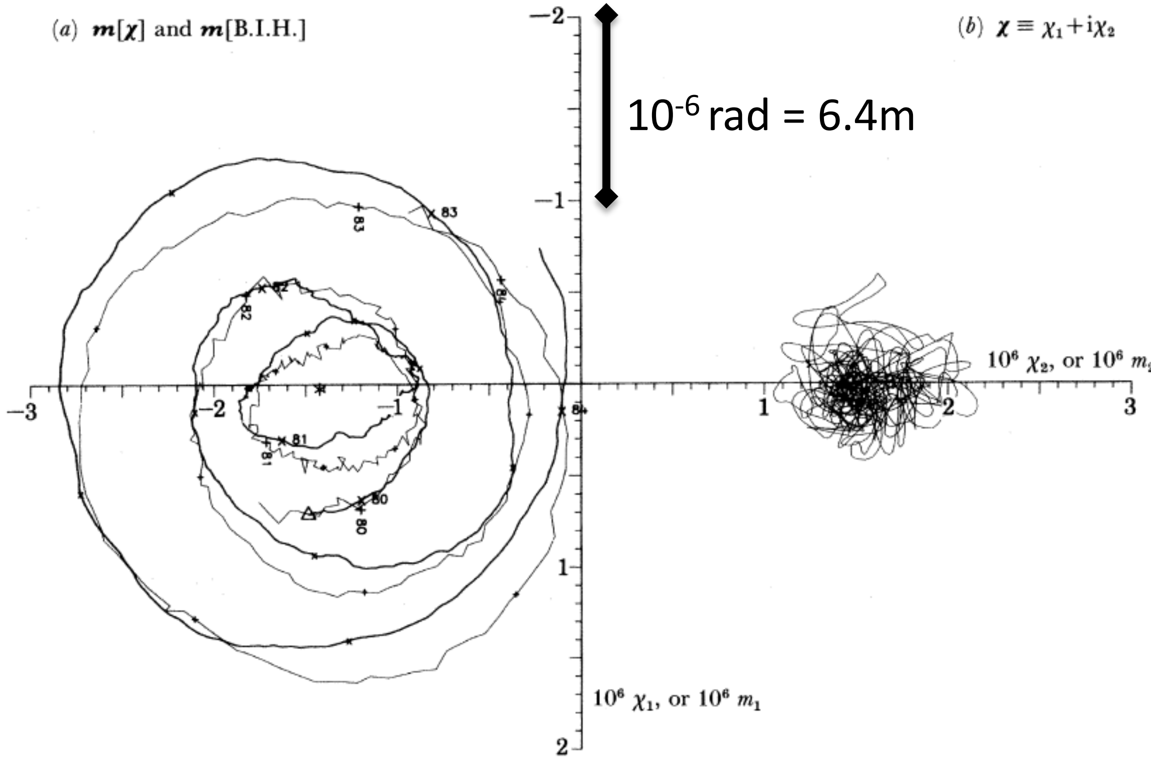
$$\tilde{\boldsymbol{\chi}} \equiv \tilde{\chi}_1 + \mathrm{i}\tilde{\chi}_2 = (\Omega \Delta \boldsymbol{I} + \boldsymbol{h})/\Omega(C-A)$$

$$\boldsymbol{m}(t) = \mathrm{e}^{\mathrm{i}\sigma_{\mathrm{r}}t}\left[\boldsymbol{m}(0) - \mathrm{i}\sigma_{\mathrm{r}}(1+\sigma_{\mathrm{r}}/\Omega)\int_0^t \tilde{\boldsymbol{\chi}}(\tau)\,\mathrm{e}^{-\mathrm{i}\sigma_{\mathrm{r}}\tau}\,\mathrm{d}\tau\right] - (\sigma_{\mathrm{r}}/\Omega)\,[\tilde{\boldsymbol{\chi}}(t) - \mathrm{e}^{\mathrm{i}\sigma_{\mathrm{r}}t}\tilde{\boldsymbol{\chi}}(0)].$$

Atmosphere angular momentum

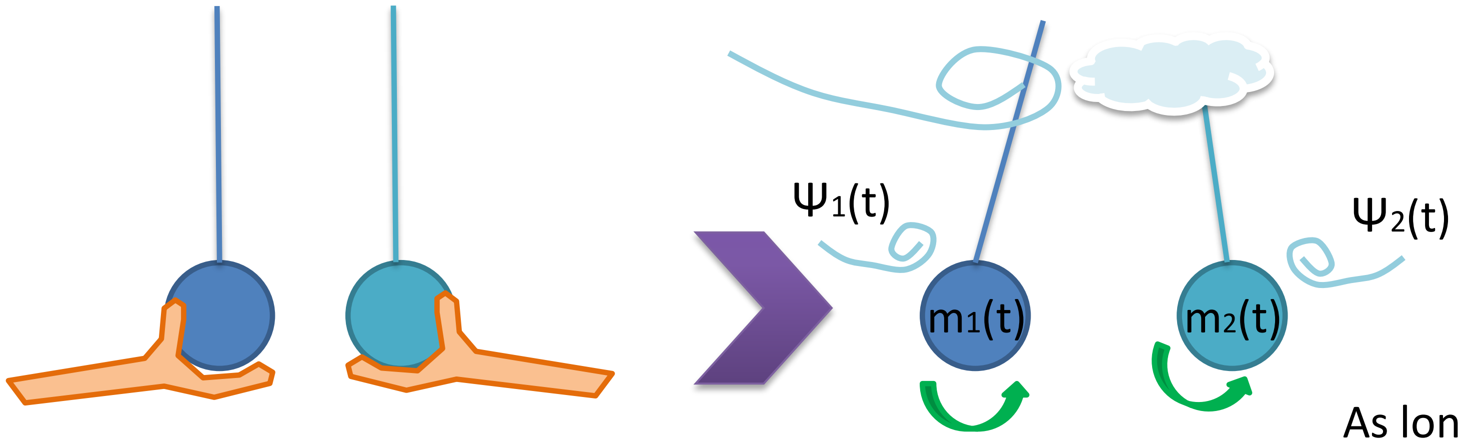
1 December 1979--15 February 1984



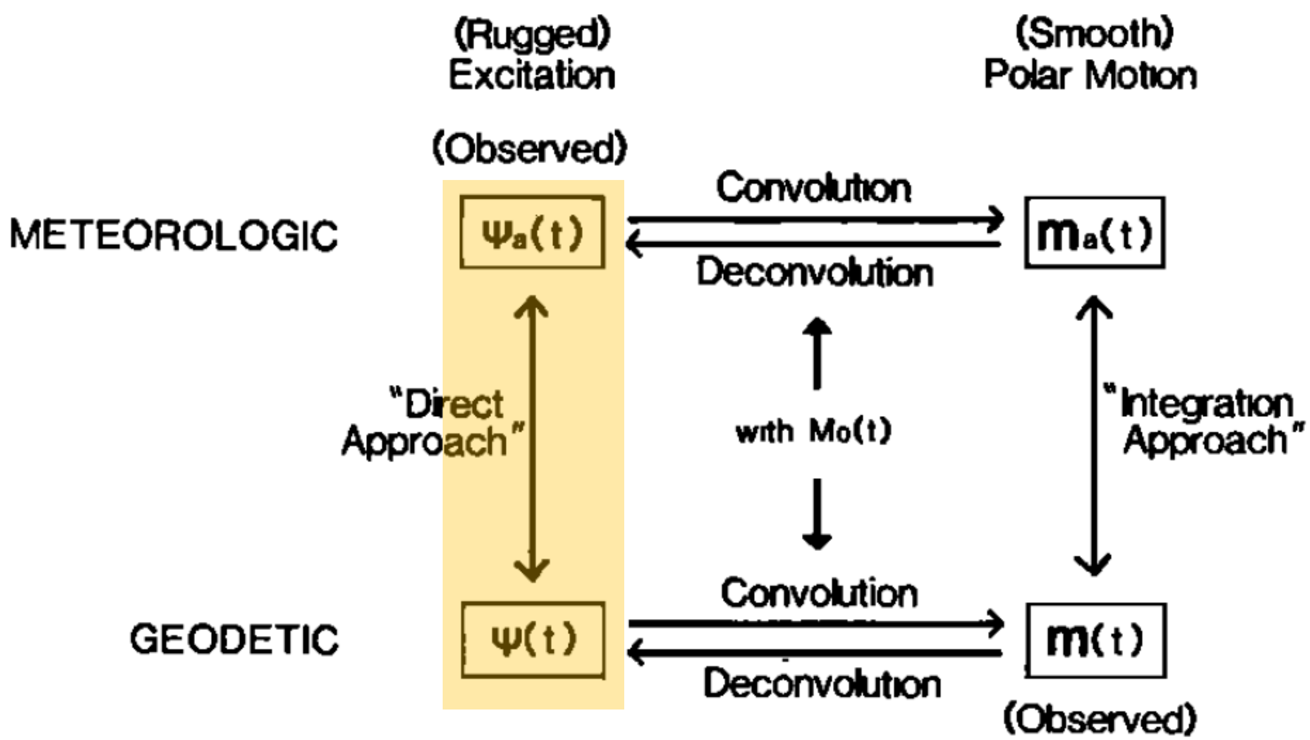


Our evaluation of the atmospheric equatorial effective angular momentum functions from meteorological data over an interval corresponding to 3.6 Chandlerian periods indicates that **atmospheric excitation alone was sufficient to account for the observed polar motion over that interval. There is apparently no need to invoke substantial excitation either by the fluid core of the Earth, or movements in the mantle associated with earthquakes, of which, admittedly, there were no major instances during the interval covered by our study.**

The conclusion
is unjustified !!



$$m(t) = m(0)exp(i\sigma t) + \psi(t) * m_o(t).$$



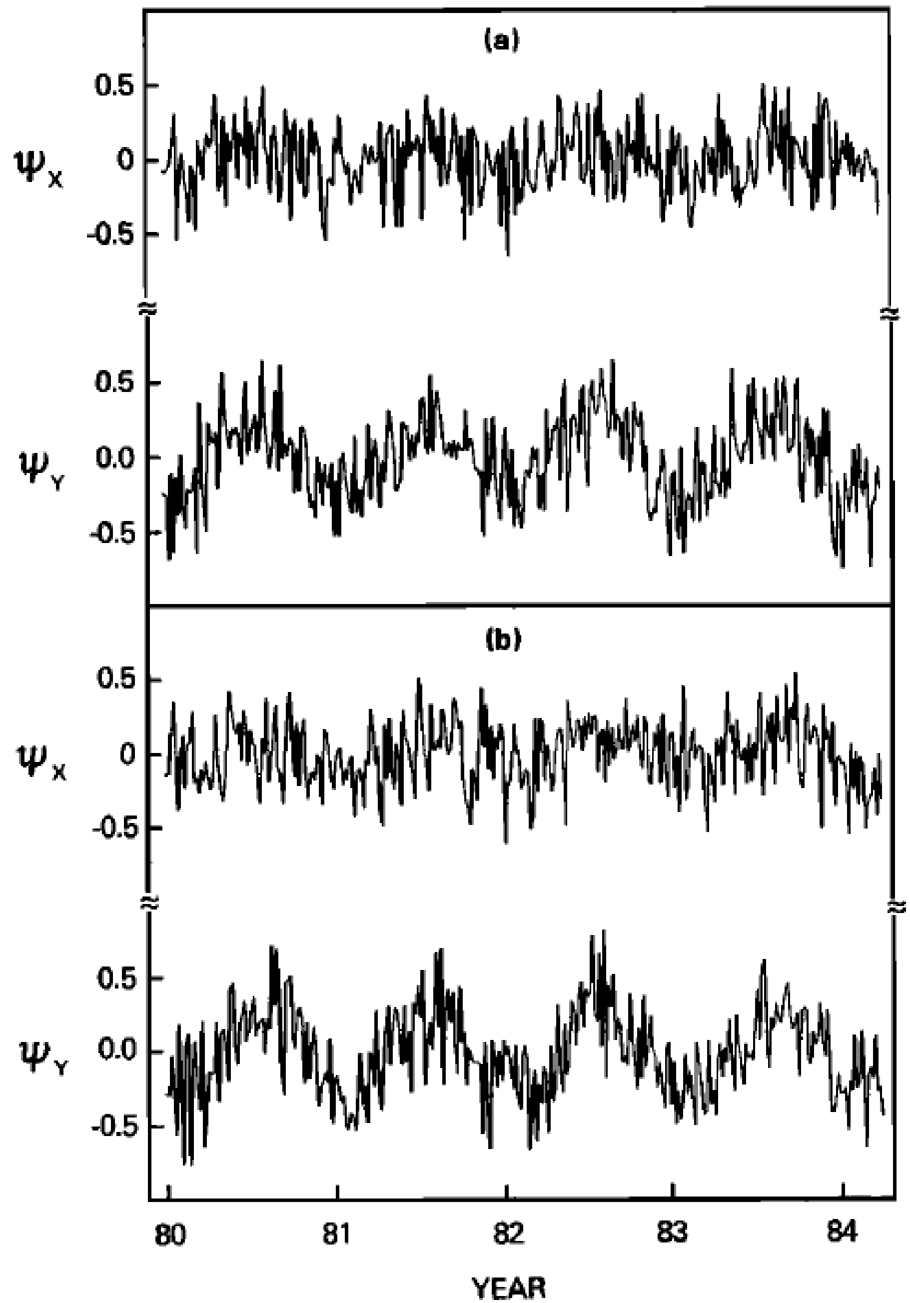
As long as the initial conditions are the same, we will always have $m1(t) \fallingdotseq m2(t)$ regardless of how different Ψ 1(t) and Ψ2 (t) are.

Hypothetical excitation

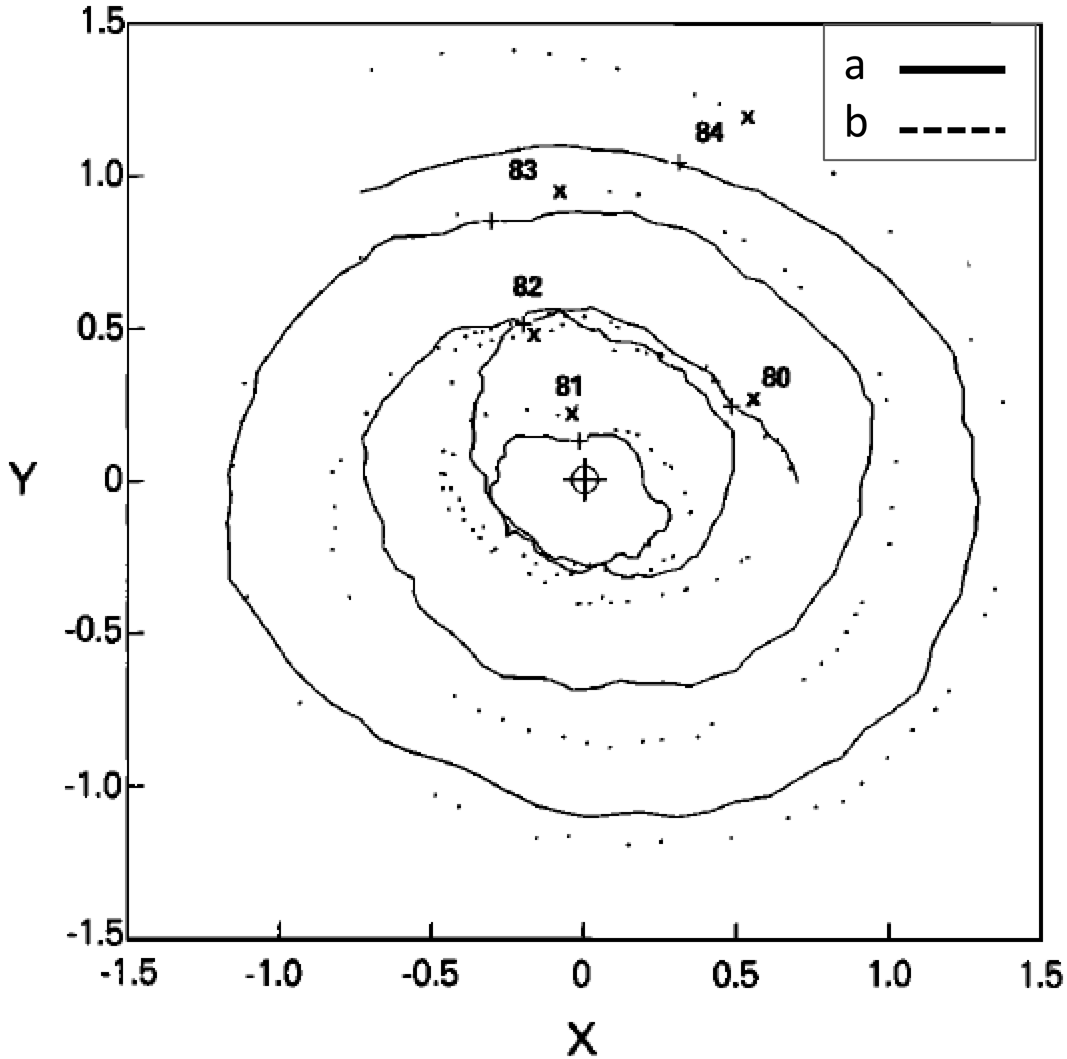
$$\psi_x(t) = A_x \cos(\omega t + \theta_x) + N_x(t),$$

$$\psi_y(t) = A_y \cos(\omega t + \theta_y) + N_y(t).$$

- Nx(t) and Ny(t) denote two computer-generated, zero-mean, Gaussian random series with standard deviations Sx and Sy.
- ω=2π/365days
- 5-day intervals



$$m(t) = m(0)exp(i\sigma t) + \psi(t) * m_o(t).$$



M (T) → Deconvelution → X (T) / Ψ(T)

(1a) $$X(t) = \frac{i}{\sigma_c} \frac{dM(t)}{dt} + M(t).$$

(1b) $$\frac{\sigma_c}{\sigma_c - 2\pi f}$$

※ $\sigma_c = 2\pi F_c(1 + i/2Q_c)$

Fc *Chandler's frequency* 0.843 cycle/yr

Qc the dissipation factor 100

(2a) $$X_t = \frac{i}{\sigma_c T} M_t - \exp(i\sigma_c T) M_{t-T}$$

(2b) $$\frac{-i\sigma_c T}{1 - \exp[i(\sigma_c - 2\pi f)T]}$$

※ $f = \sigma_c / 2\pi$

↓

$Z = \exp(-2\pi i f T)$

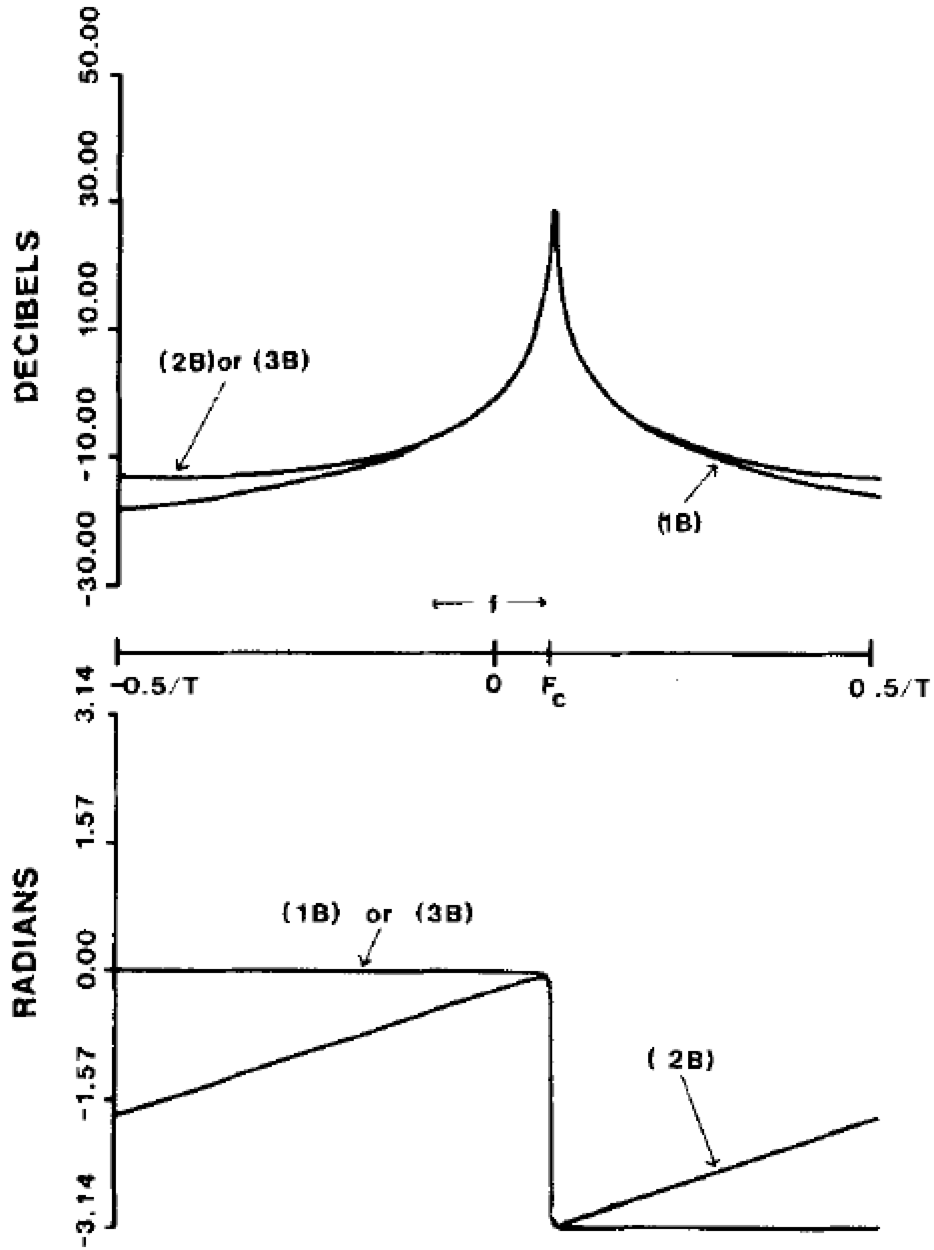
↓

$\exp(i\sigma_c T)$

(3a) $$X_t = \frac{i \exp(-i\pi F_c T)}{\sigma_c T} [M_{(t+T/2)} - \exp(i\sigma_c T) M_{(t-T/2)}]$$

(3b) $$\frac{-i\sigma_c T \exp[i\pi(F_c - f)T]}{1 - \exp[i(\sigma_c - 2\pi f)T]}$$

※ $\exp[i\pi(F_c - f)T]$


DECIBELS
50.00
30.00
10.00
-10.00
-30.00
(2B)or (3B)
(1B)
f
-0.5/T
0
F_c
0 .5/T
RADIANS
3.14
1.57
0.00
-1.57
-3.14
(1B) or (3B)
(2B)

M (T) → Deconvelution → X (T) / Ψ(T)

(1a) $$X(t) = \frac{i}{\sigma_c} \frac{dM(t)}{dt} + M(t).$$

(1b) $$\frac{\sigma_c}{\sigma_c - 2\pi f}$$

※ $\sigma_c = 2\pi F_c(1 + i/2Q_c)$

Fc *Chandler's frequency* 0.843 cycle/yr

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(2a) $$X_t = \frac{i}{\sigma_c T} \quad M_t - \exp(i\sigma_c T) M_{t-T}$$

(2b) $$\frac{-i\sigma_c T}{1 - \exp[i(\sigma_c - 2\pi f)T]}$$

※ $f = \sigma_c/2\pi$

↓

$Z = \exp(-2\pi i f T)$

↓

$\exp(i\sigma_c T)$

(3a) $$X_t = \frac{i \exp(-i\pi F_c T)}{\sigma_c T} \quad [M_{(t+T/2)} - \exp(i\sigma_c T) M_{(t-T/2)}]$$

(3b) $$\frac{-i\sigma_c T \exp[i\pi(F_c - f)T]}{1 - \exp[i(\sigma_c - 2\pi f)T]}$$

※ $\exp[i\pi(F_c - f)T]$

(4a)

$$X_t = \frac{i\exp(-i\pi F_c T)}{2\sigma_c T} [\![M_{t+T} + [1-\exp(i\sigma_c T)]\, M_t - \exp(i\sigma_c T)\, M_{t-T}]\!]$$

(4b)

$$\frac{-2i\sigma_c T \exp[i\pi(F_c - 2f)T]}{1+[1-\exp(i\sigma_c T)]\exp(-2\pi i f T) - \exp[i(\sigma_c - 4\pi f)T]}$$

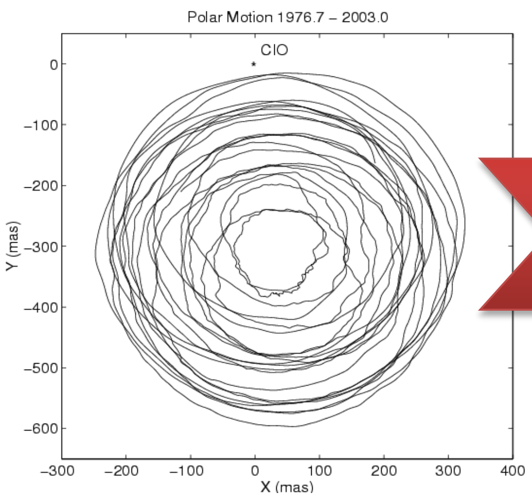
(5a)

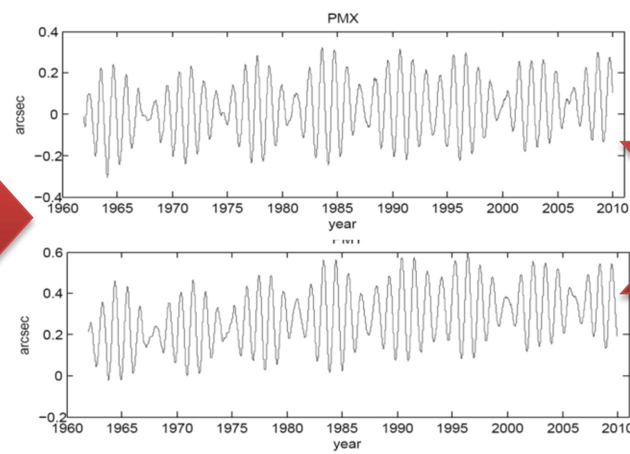
$$M_t = \frac{-i\sigma_c T\exp(i\pi F_c T)}{2}[X_t + X_{t-T}] + \exp(i\sigma_c T) M_{t-T}$$

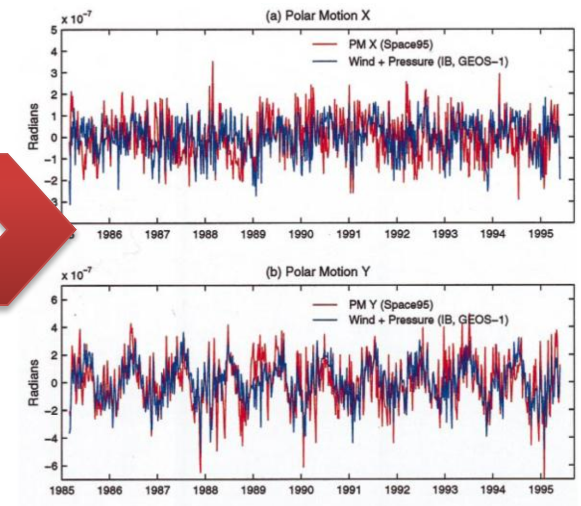
(5b)

$$\frac{-i\sigma_c T\exp(i\pi F_c T)[1+\exp(-2\pi i f T)]}{2(1-\exp[i(\sigma_c - 2\pi f)T])}$$

Direct approach !







Conclusions

- Atmospheric angular momentum fluctuations are of interest also to geophysicists and astronomers concerned with the structure and dynamics of the Earth, who must make allowances for the meteorological contribution to the variable rotation of the solid Earth when dealing with effects due to 'non-meteorological' processes.

- If we want to compare a geophysically observed excitation function with the excitation function deduced (via deconvolution) from the polar motion observation, we should do so directly (the "direct approach").

Conclusions

- It would be useful for estimating X_t from M_t since the transfer function would then be the reciprocal of (4b) and thus would attenuate Nyquist frequency variations, a desirable feature if the M_t values are corrupted by noise.

- However, despite decades of effort by many investigators, the major excitations source(s) for the Chandler wobble still remain a mystery.

Thanks for your attention!