NONLINEAR DYNAMICS OF HARMONIC TREMOR TIME SERIES RECORDED AT SEMERU VOLCANO (INDONESIA)

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OUTLINE

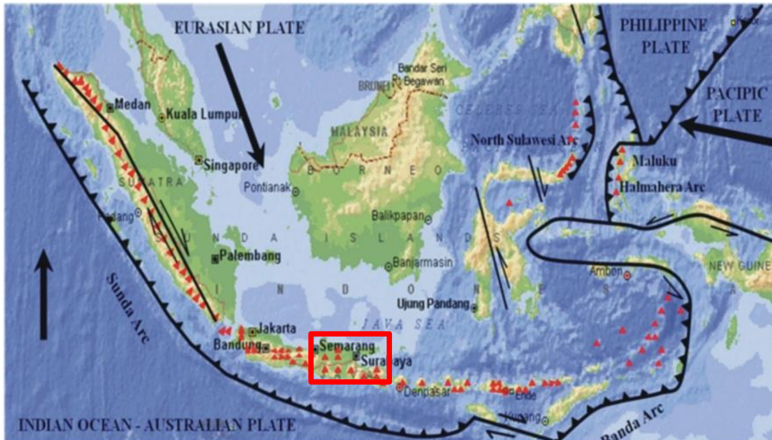
- INTRODUCTION
- METHOD
- DISCUSSION
- RESULT
- CONCLUSION

INTRODUCTION

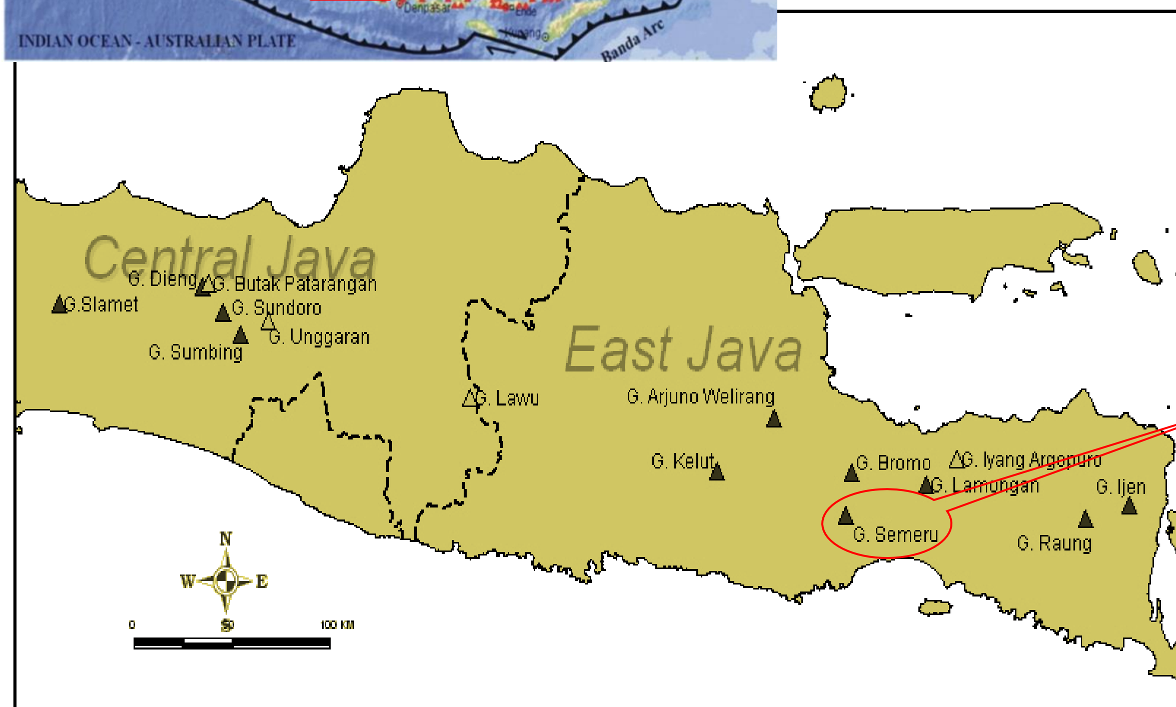
Overview Semeru Volcano and harmonic tremor

Clue of nonlinearity in tremor generation

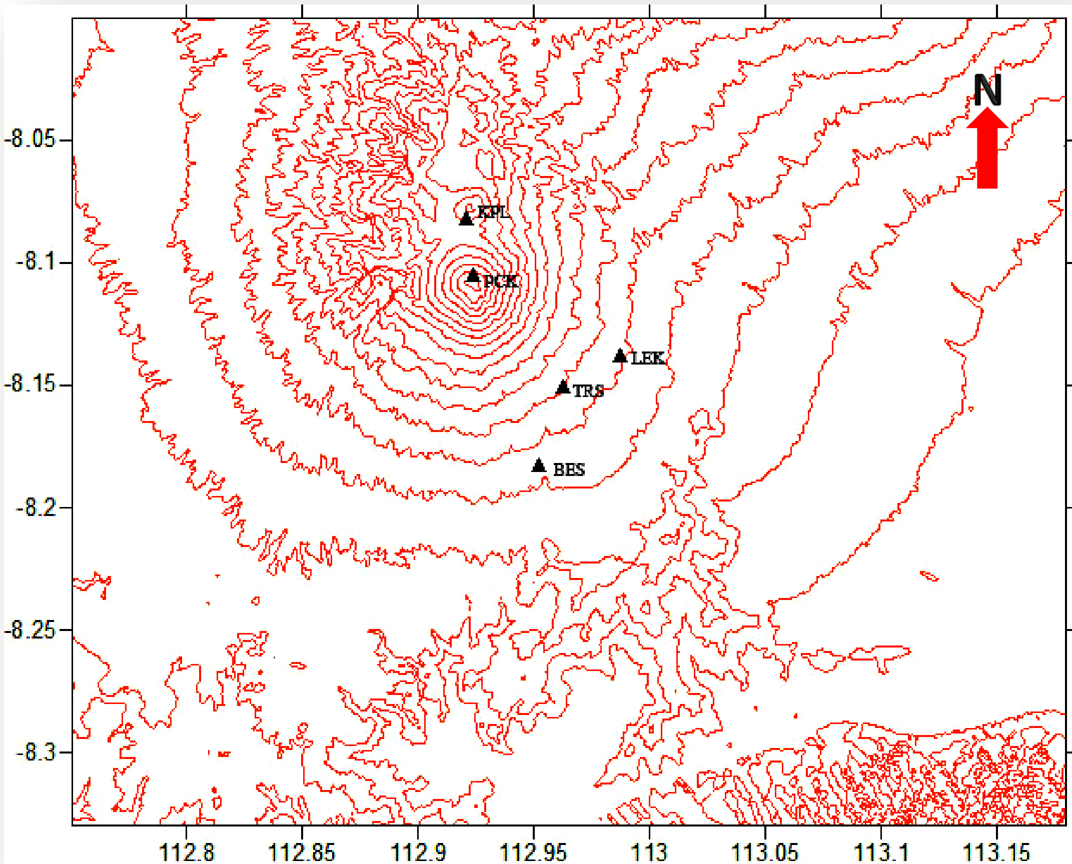
Overview of Semeru Volcano



Location	: East Java, Indonesia
Latitude	: 08˚ 06’ 05” S
Longitude	: 112˚ 50’ 00” E
Altitude	: 3676 m (asl)



Seismic System Monitoring



Instrument : Seismometer 1 componen

L-4C seismograf type PS-2

1 Seismometer in near summit : PCK

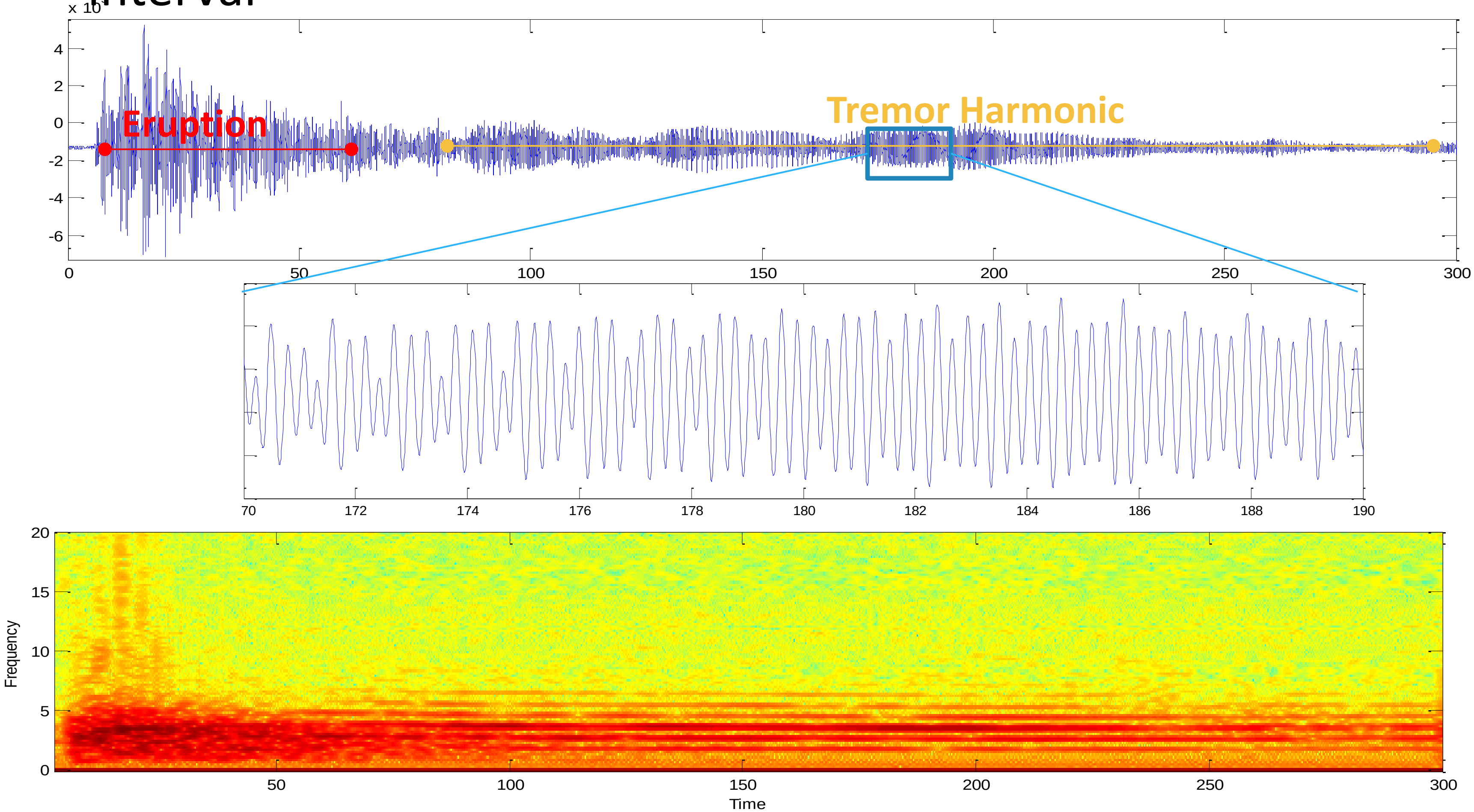
4 seismometer arround body volcano : KPL, LEK, TRS, BES

Tremor in Semeru Volcano

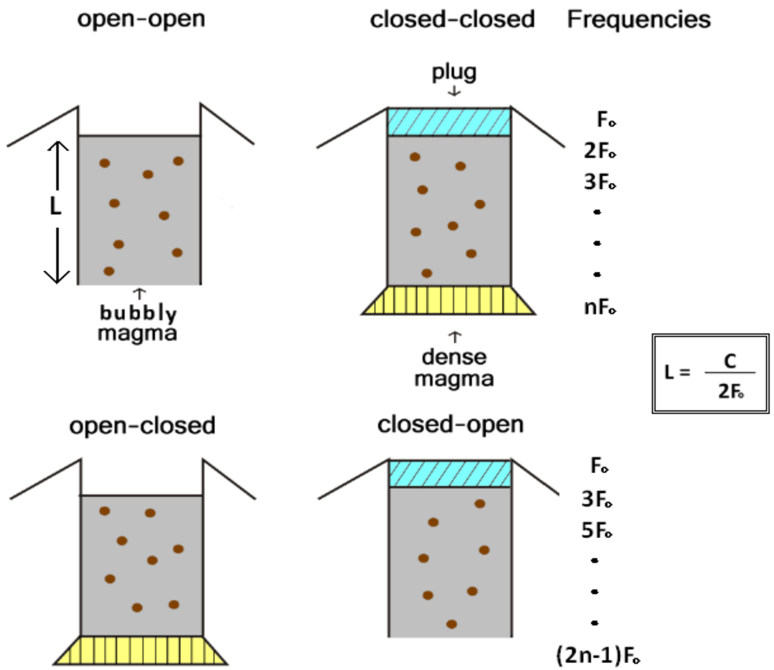
- Tremor : persistent seismic signal that observed near active volcanoes lasting from several minutes up to several days precesing and/or accompanying most volcanic eruption
- Tremor in Semeru devided into two type :
  1. Spasmodic Tremor
  2. Harmonic Tremor

Tremor Harmonic

- Characterized by narrow spectral peaks, composed of fundamental and overtone frequency and its multiple integers interval



Tremor Generation



Any tremor **generation process must necessarily be nonlinear. A time-invariant linear system cannot oscillate spontaneously**, because it's output can contain only frequencies that are present in its input. Tremor is an oscillatory response to a steady (near zero frequency) input (for example, stress, fluid pressure, or heat), so a nonlinear process of some kind is essential to its occurrence. (Julian, 1994, 200)

Aim of Research

- Obtain indication of nonlinear process that might be involved in generating harmonic tremor
- Explain tremor generation in Semeru Volcano based on existing nonlinear model

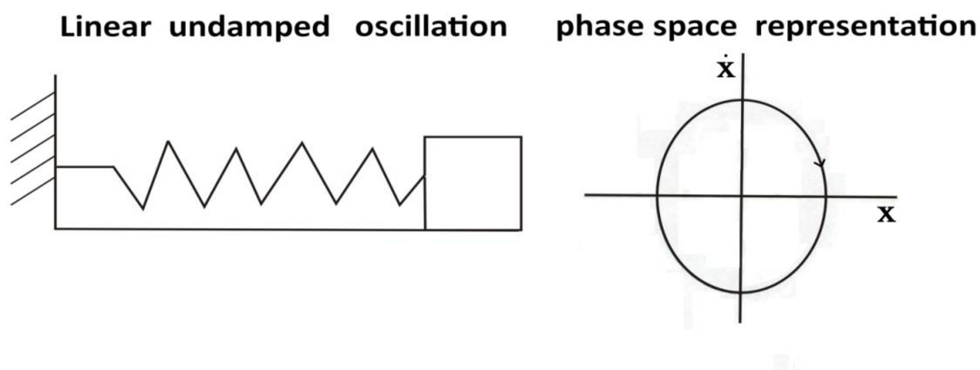
METHOD

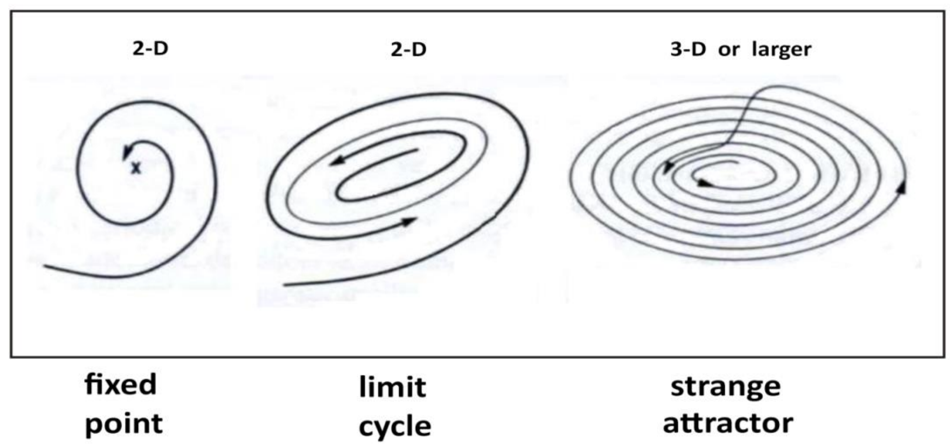
Delay Embedding Theorem (Takens 1981)

Maximal Lyapunov exponent

Describing nonlinear system behavior: *The Phase space*

Phase space is an abstraction where the system is replaced by representation of the space of the possible states (dimension) where system can be in





According to Çambel (1993), "Dynamic systems are attracted to attractors the way fireflies are attracted to light"

Delay Embedding Theorem (Takens 1981)

- “Any time series generated by nonlinear process can be considered as the projection on the real axis of higher dimensional geometrical object that describe the behaviour the system understudy”

$$x = s(t), s(t-\tau), \ldots\ldots, s(t+(m-1)\tau)$$

τ = *delay time*

m =*embedding dimension.*

Delay Time

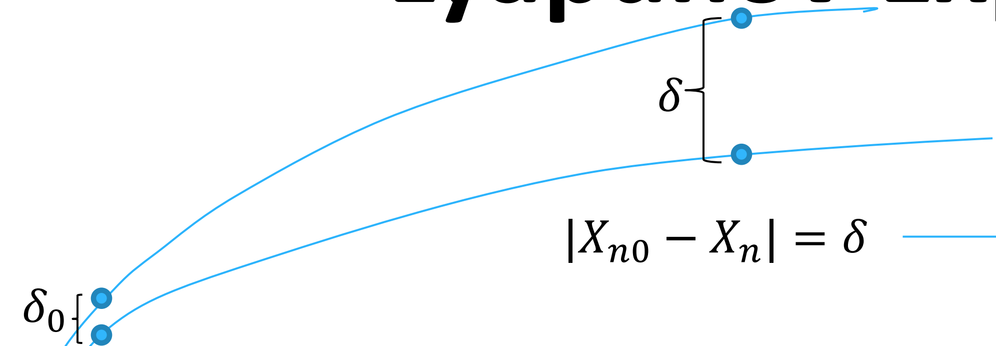
- Choosing the delay time based on criteria :
  - First minimum in value of autocorrelation function that called Averange Mutual Information (AMI) consideras the most suitable τ (delay time)
  - Attractor in phase space portrait shold be unfolded

Embedding Dimension

- Sufficient embedding dimension was selected by applying **flase nearest neighbour method**

The basic idea of nearest neighbour method is to the 2 points are characterized as ‘false’ neighbors (being in the same neighborhood because of the projection and not because of the dynamics)

Lyapunov Exponent



$$|X_{n0} - X_n| = \delta \longrightarrow \delta = \delta_0 e^{\lambda t}$$

$$|X_{n0} - X_n| = \delta_0$$

- Lyapunov exponent describes the ***rate of expansion or contraction of nearby orbits in the phase space.***
- The main interest is focused on the largest of these exponents since it can be calculated relatively easy and it yields evidence for the presence of deterministic chaos in the observed data.

$$S = \frac{1}{N}\sum_{n_0=1}^{N}\left(\ln\frac{1}{|u_{x_{n_0}}|}\sum|x_{n_0} - x_n|\right)$$

$\boldsymbol{U_{X_{no}}}$: number of neighbors found around point X_{n0}

N : mumber of points

Lyapunov Exponent

- A plot of the stretching factor versus N will yield a curve with a linear increase at the beginning, followed by an almost flat region. A least-squares line fitting for the slope of the linear part of the curve should then yield an estimation of the largest Lyapunov exponent
- Possible types of motion and the corresponding largest Lyapunov exponent

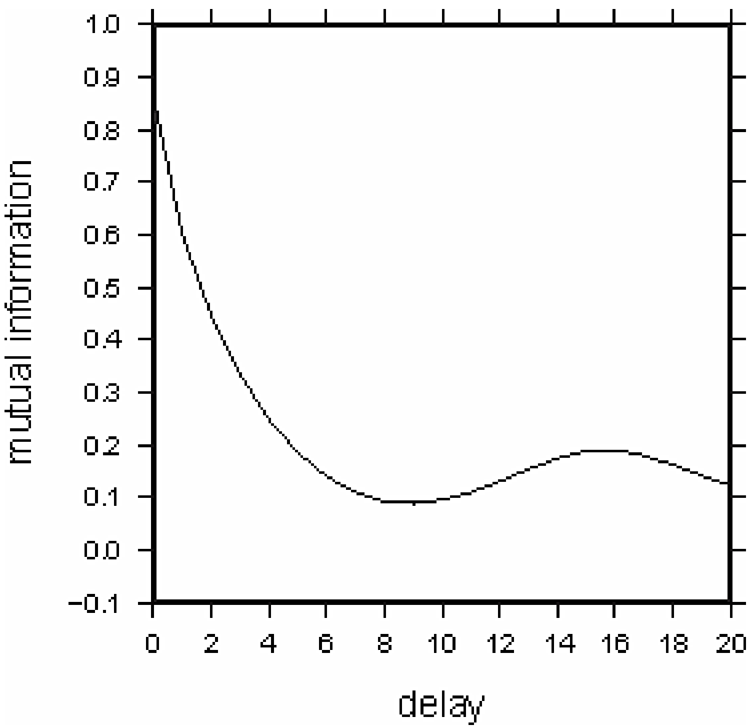
Tipe motion/sistem	Maximal Lyapunov Exponent
Stable fixed point	$\lambda < 0$
Stable limit cycle	$\lambda = 0$
Chaos	$0 < \lambda < \infty$
Noise	$\lambda = \infty$

RESULT AND DISCUSSSION

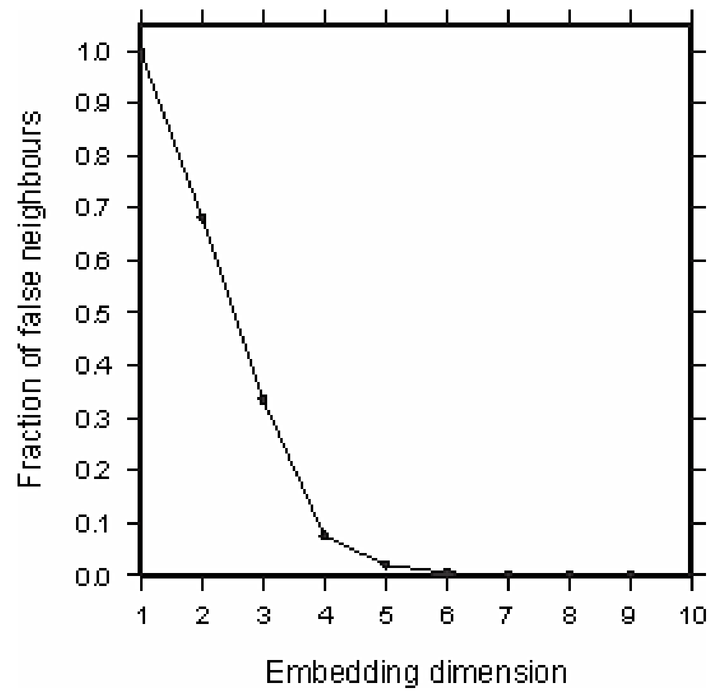
Data Selection

- Data selection was done during December 2009 and 24 episodes of harmonic tremor was selected during from closest station (KPL) from crater (assume that tremor come from the conduit near the crater)

Delay time and Embedding Dimension

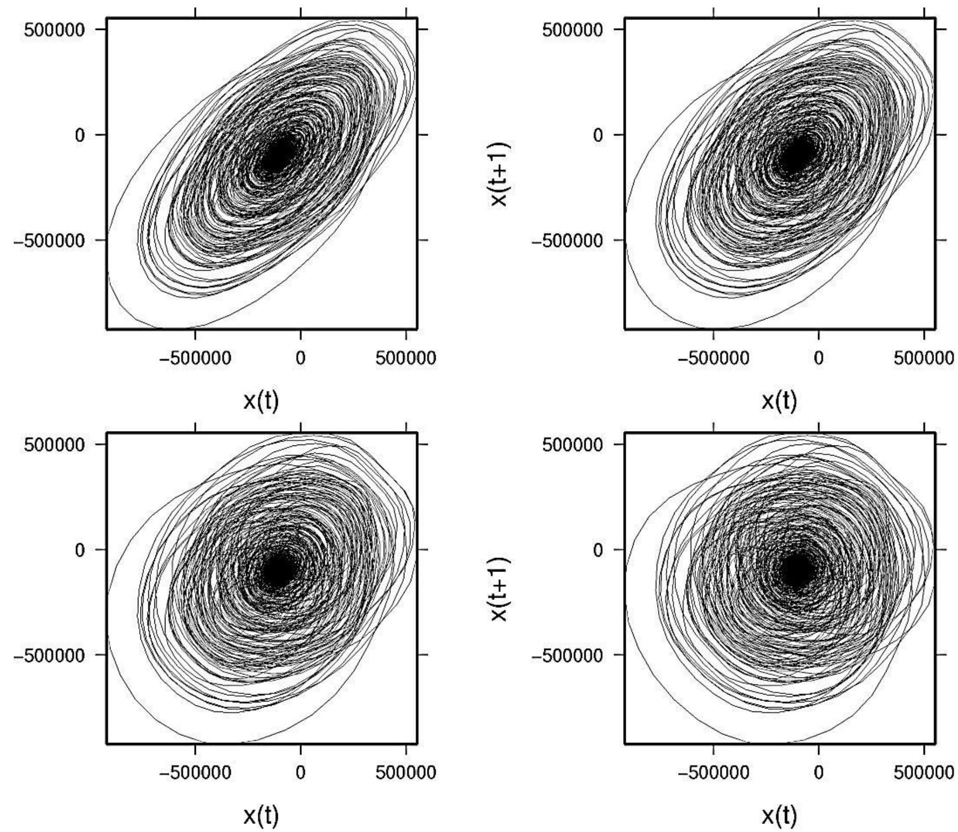


AMI curve reach first minimum of event #25 in **time lag 8**



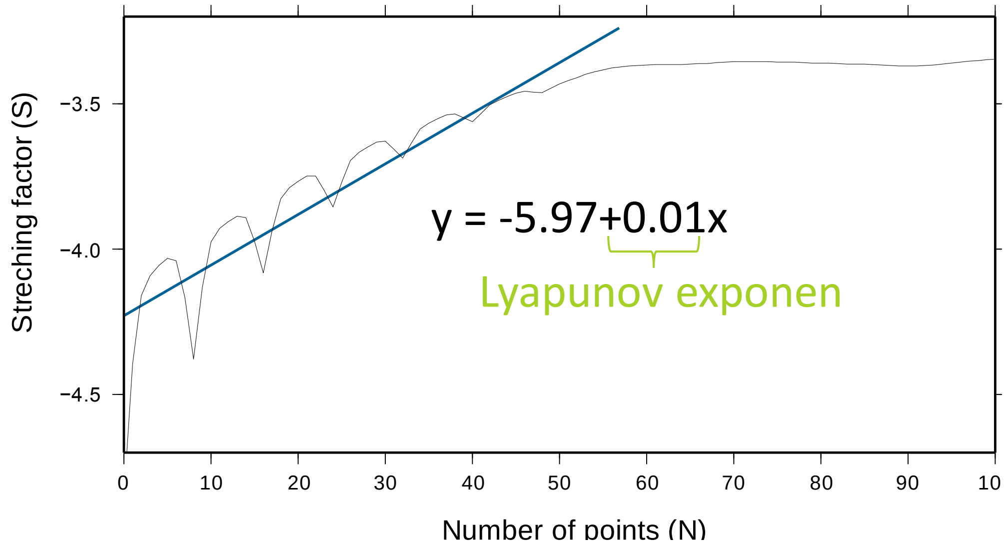
Distribution of false nearest neighbor statistic in dimension 1-10. The embedding **dimension is 6** indicated by very small percentage. (event #25)

No Event	Date	Delay Time	Embedding dimension
1	4-Dec-09	8	7
2	9-Dec-09	8	6
3	11-Dec-09	8	7
4	12-Dec-09	8	7
5	14-Dec-09	7	7
6	14-Dec-09	9	7
7	15-Dec-09	9	5
8	16-Dec-09	9	6
9	17-Dec-09	8	5
10	18-Dec-09	10	6
11	19-Dec-09	10	6
12	20-Dec-09	9	7
13	21-Dec-09	10	5
14	22-Dec-09	9	6
15	23-Dec-09	9	6
16	23-Dec-09	9	5
17	23-Dec-09	8	5
18	23-Dec-09	9	5
19	23-Dec-09	8	5
20	26-Dec-09	11	6
21	26-Dec-09	9	6
22	30-Dec-09	8	7
23	31-Dec-09	8	5
24	31-Dec-09	8	6



Two dimension phase portrait of event #25, reconstructed for τ=5 (upper left), τ=6 (upper right), τ=7 (lower left), **τ=8 (lower right)**

Largest Lyapunov Exponent

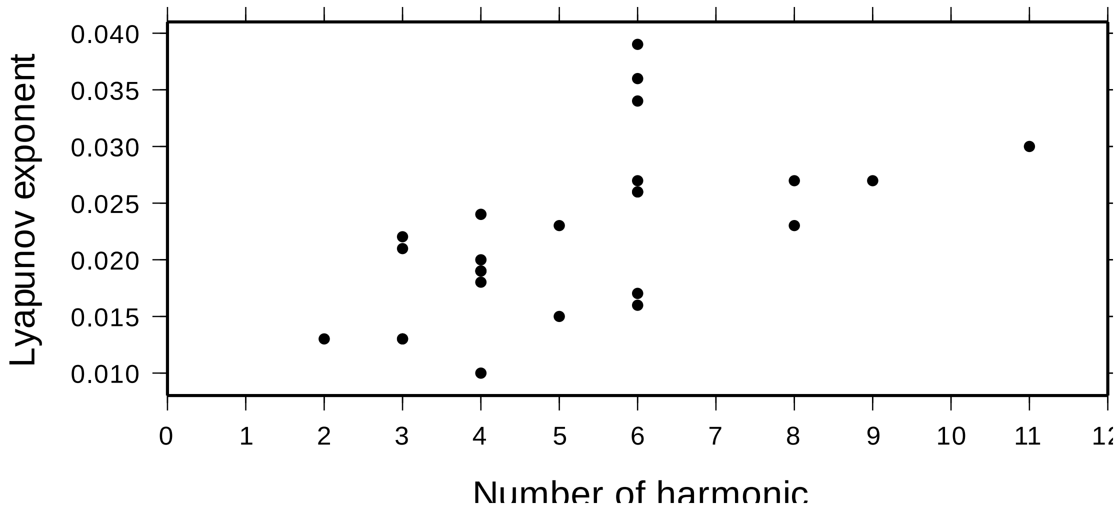


Largest Lyapunov exponent calculated after a plot of stretching factor versus number of point yielded a curve with linear increase at the beginning and almost flat region in the end.

A least square fit for the slope of the line corresponds to the largest Lyapunov exponent.

No Event	Date	Lyapunov Exponen	Number of Harmonic
1	4-Dec-09	0.013	3
2	9-Dec-09	0.024	4
3	11-Dec-09	0.013	2
4	12-Dec-09	0.019	4
5	14-Dec-09	0.020	4
6	14-Dec-09	0.013	3
7	15-Dec-09	0.027	8
8	16-Dec-09	0.021	3
9	17-Dec-09	0.030	11
10	18-Dec-09	0.017	6
11	19-Dec-09	0.015	5
12	20-Dec-09	0.018	4
13	21-Dec-09	0.026	6
14	22-Dec-09	0.023	8
15	23-Dec-09	0.026	7
16	23-Dec-09	0.023	5
17	23-Dec-09	0.034	6
18	23-Dec-09	0.027	9
19	23-Dec-09	0.039	6
20	26-Dec-09	0.013	4
21	26-Dec-09	0.019	4
22	30-Dec-09	0.016	6
23	31-Dec-09	0.036	6
24	31-Dec-09	0.027	8

Correlation between Lyapunov exponent and number of harmonic



Linear correlation of Lyapunov exponent and number of harmonics can be assumed as the addition of “disturbances" in the physical system.

Possible soure model based on nonlinear method

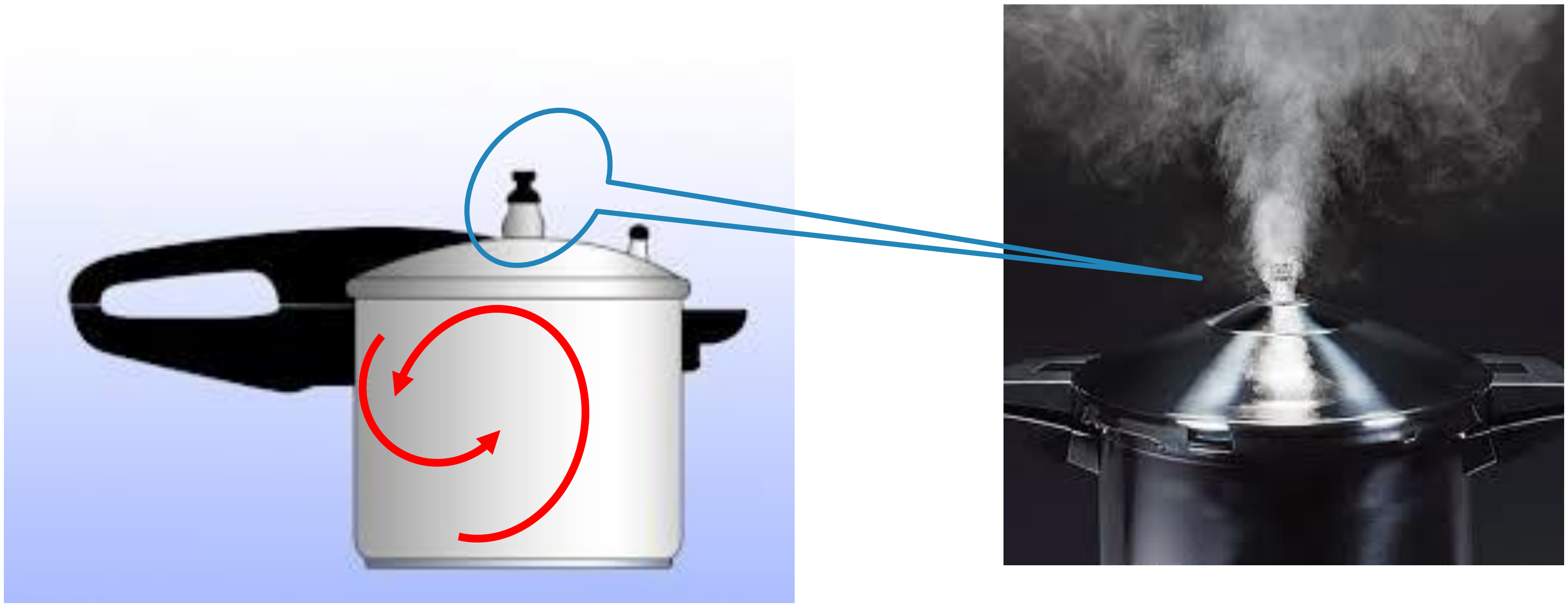
We consider some models of source mechanism to explain Semeru tremor :

(a) harmonic tremor may be modeled as a **pressure cooker mechanism** with a lava dome in the crater acting as a plug (Less and Bolton, 1998)

(b**)** **fluid flow induced oscillations** can explain the occurrence of harmonic tremor in the Semeru volcano if the fluid is gas (Julian, 1994)

(c) **oscillation or wagging magma column model** (Jellinek and Bercovici, 2011)

Pressure Cooker Mechanism Model
(Lees and Bolton, 1998)



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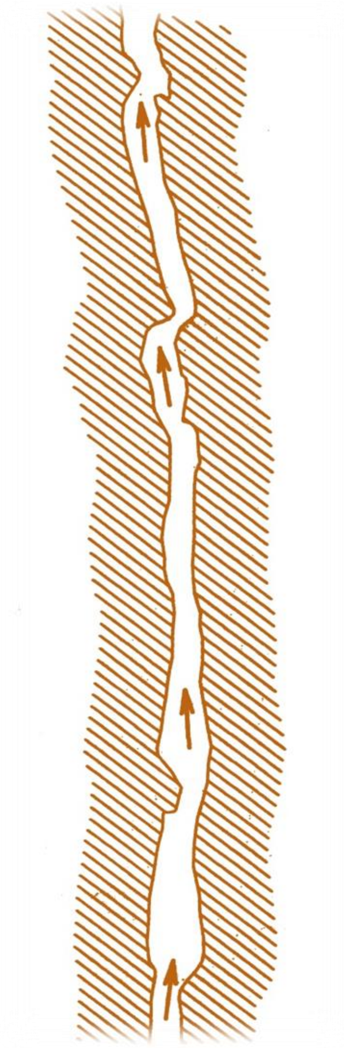
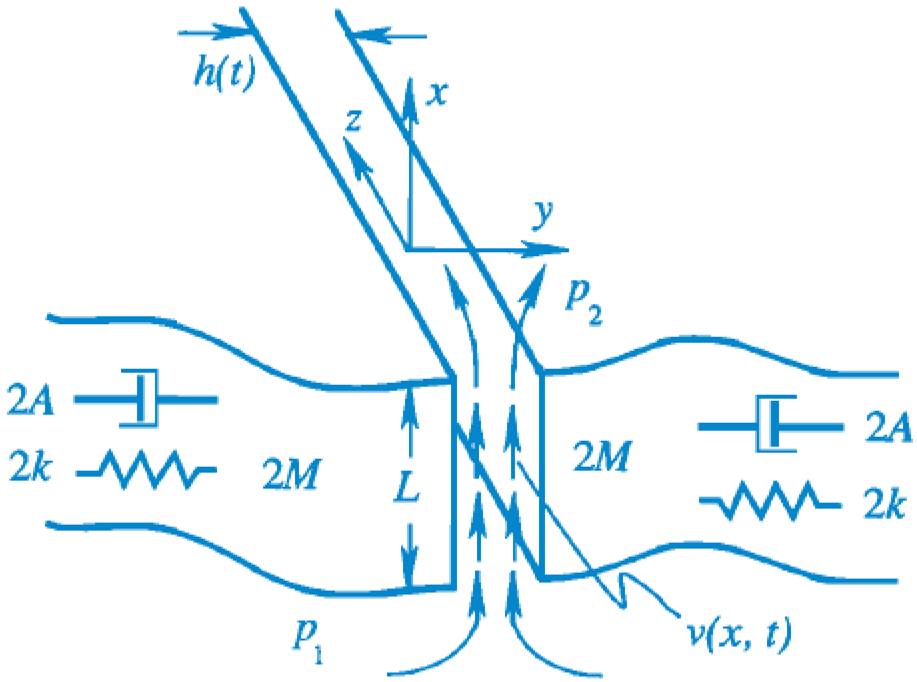
Fluid Flow Induced Oscillations Model (Julian, 1994)





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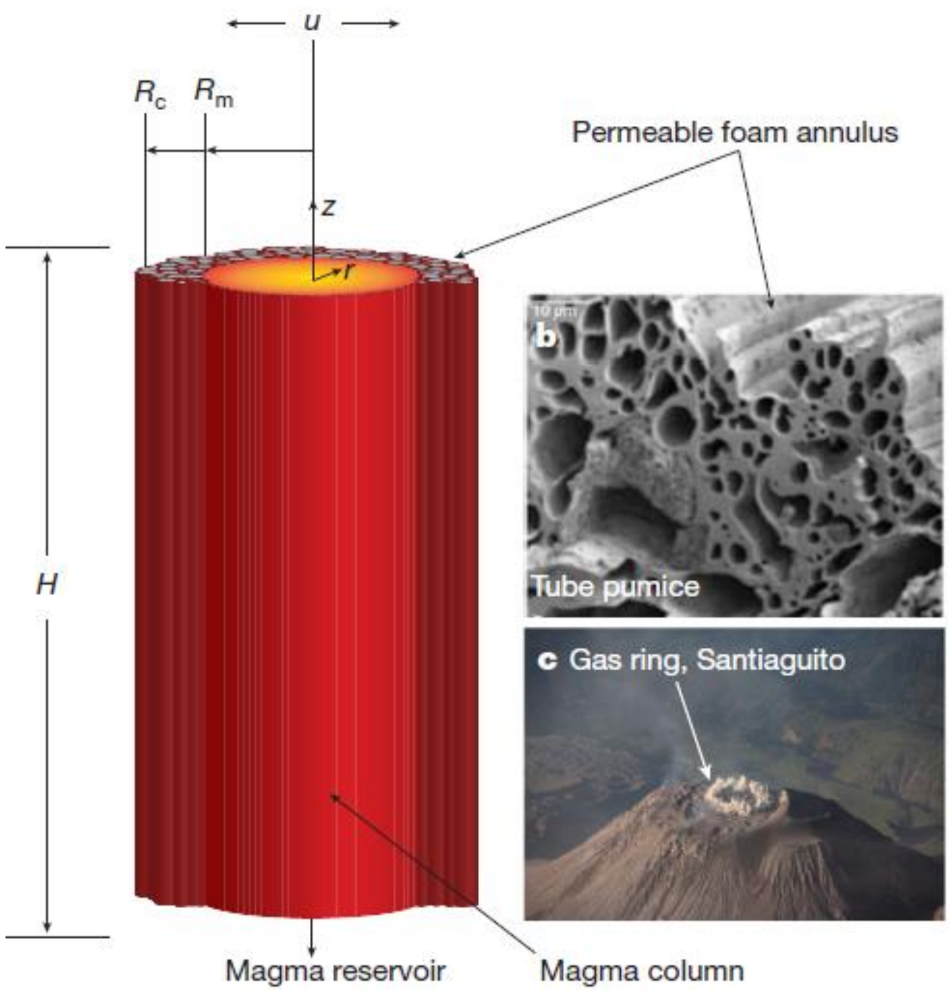
Oscillation or Wagging Magma Column Model (Jellineck and Bercovici, 2011)



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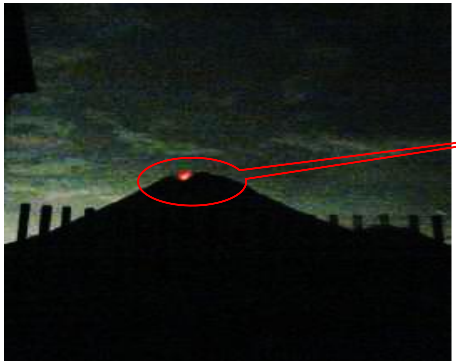
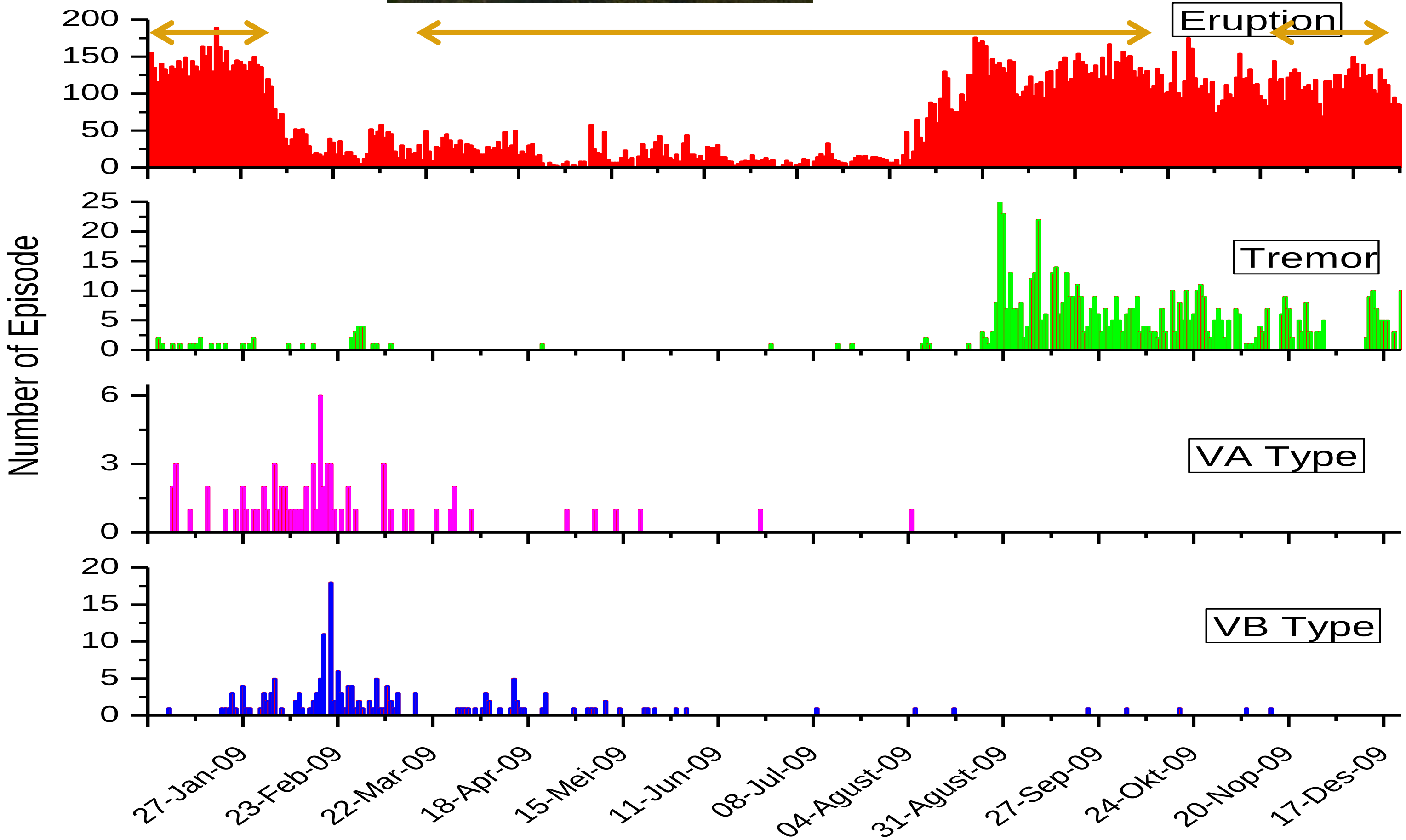



Eruption
Tremor
VA Type
VB Type
Number of Episode
200
150
100
50
0
25
20
15
10
5
0
6
3
0
20
15
10
5
0
27-Jan-09
23-Feb-09
22-Mar-09
18-Apr-09
15-Mei-09
11-Jun-09
08-Jul-09
04-Agust-09
31-Agust-09
27-Sep-09
24-Okt-09
20-Nop-09
17-Des-09

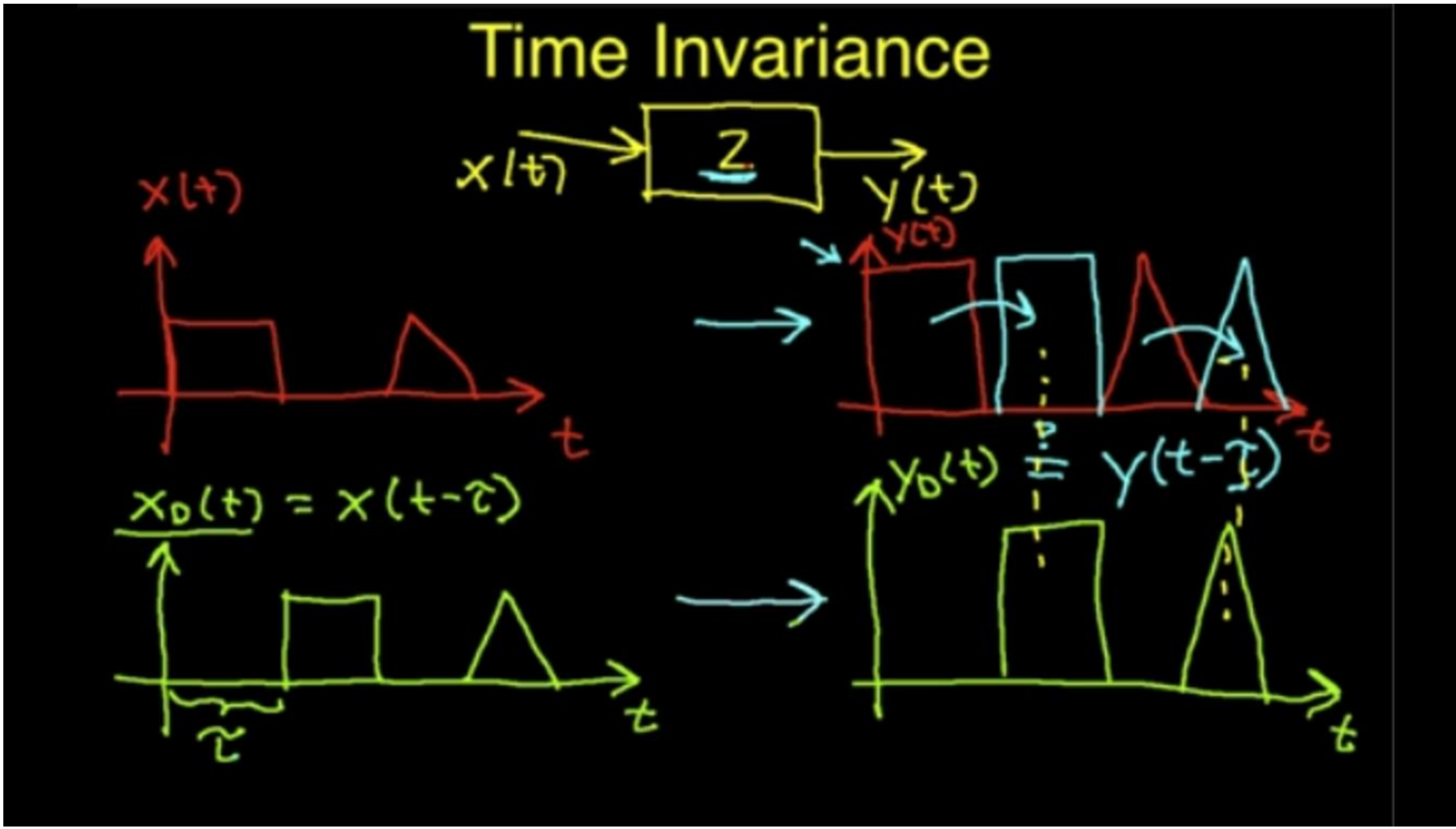
CONCLUSION

- Largest Lyapunov exponent in tremor episodes indicate a nonlinear process in appearance of tremor harmonic in Semeru volcano
- Harmonic tremor may be modeled as a pressure cooker mechanism with a lava dome in the crater acting as a plug

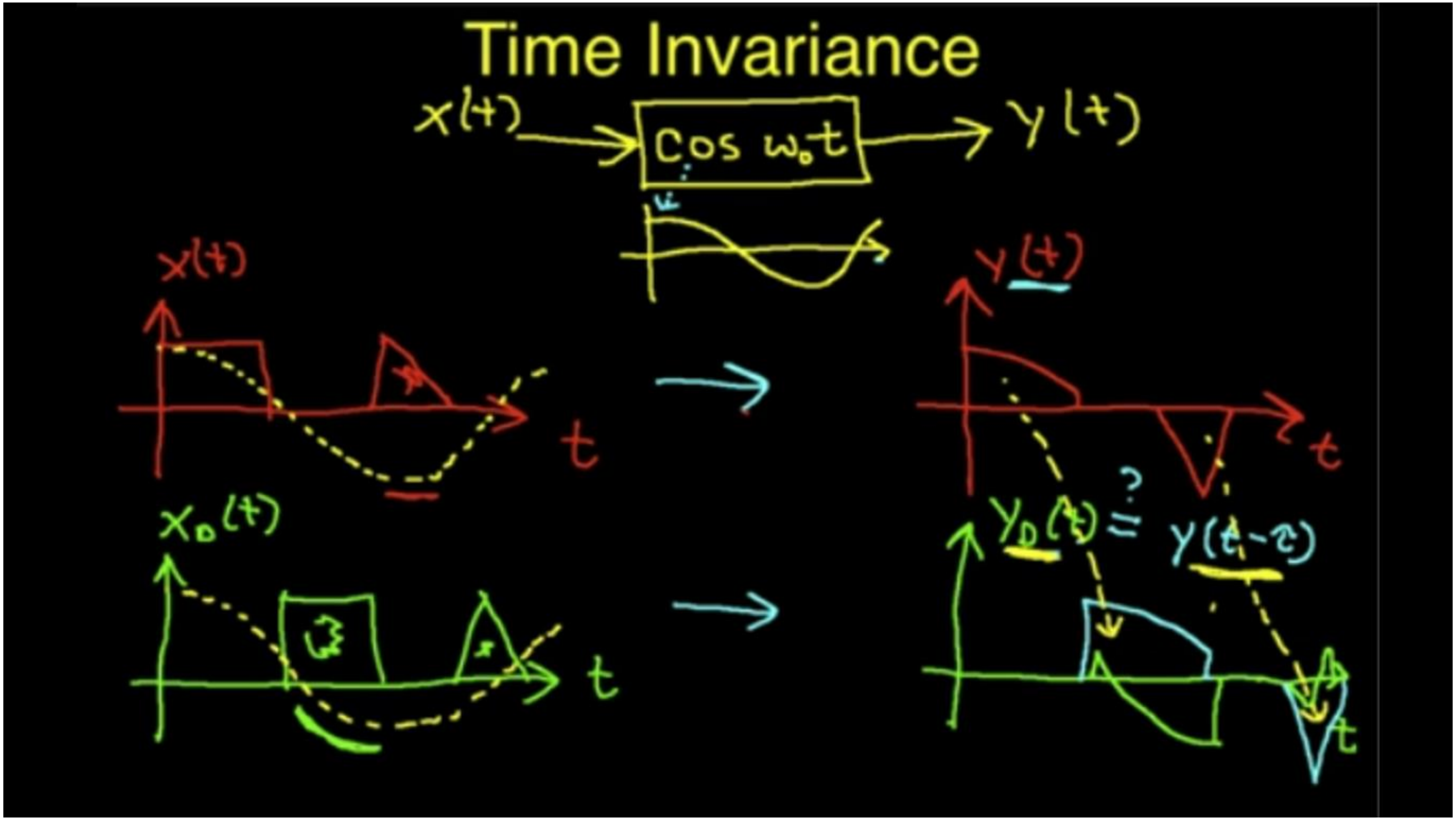
THANK YOU FOR ATTENTION

Discussion and Question are freely open ^_^

TIME INVARIANT (constant gain)



TIME INVARIANT (varying time gain)



Atrractor reconstruction

$$x(t), x(t-\tau), x(t-2\tau), \ldots, x(t-(n-1)\tau)$$

- τ = lag/delay ; n = embedding dimension
- $x(t)$ = (4, 2,7, 4,9,6,10,3)
- Assume τ=2 n = 3. This generates the following lagged coordinate vectors: