Universality in BTW and Manna Sandpile Models

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2012/11/01

Reference

- Asa Ben-Hur and Ofer Biham, 1996. **Universality in sandpile models.** Physical Review E 53, R1317–R1320.

- S. Lübeck, 2000. **Moment analysis of the probability distribution of different sandpile models.** Physical Review E 61, R204-R209.

Outline

- Introduction
- Sandpile Model
  - BTW model
  - Manna model
- Parameter
- Method and Discussion
  - Ben-Hur et al.
  - S. Lübeck
- Conclusion

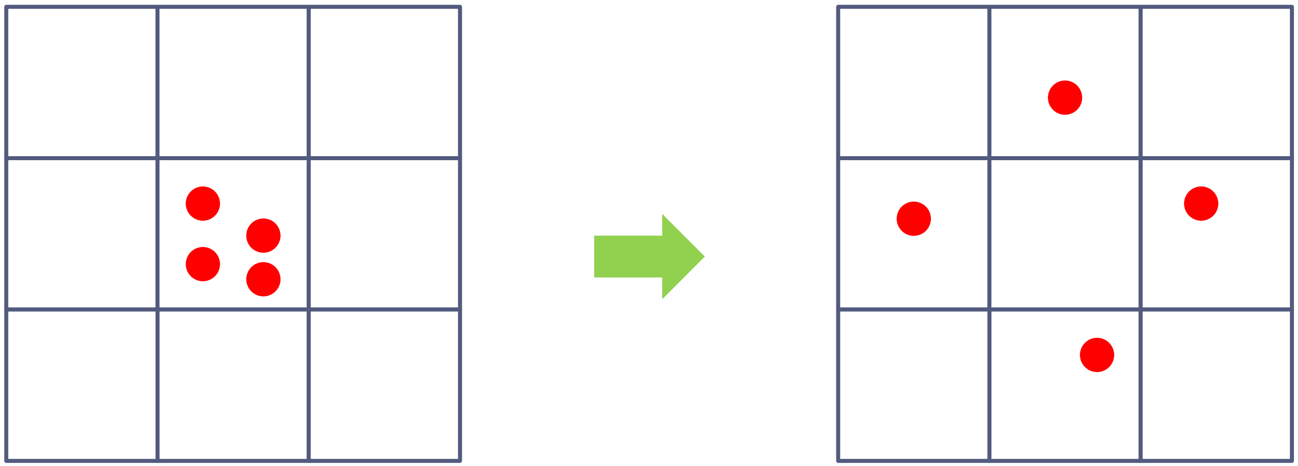
Introduction

- **Model** : BTW model and Manna model
- **Purpose** : Whether the BTW and the Manna model belong to different universality classes
- **Universality classes** : Under the proper conditions, different systems can exhibit the same behavior, as measured by quantitative indices, if they meet the same qualitative criteria.

Sandpile Model

- BTW model

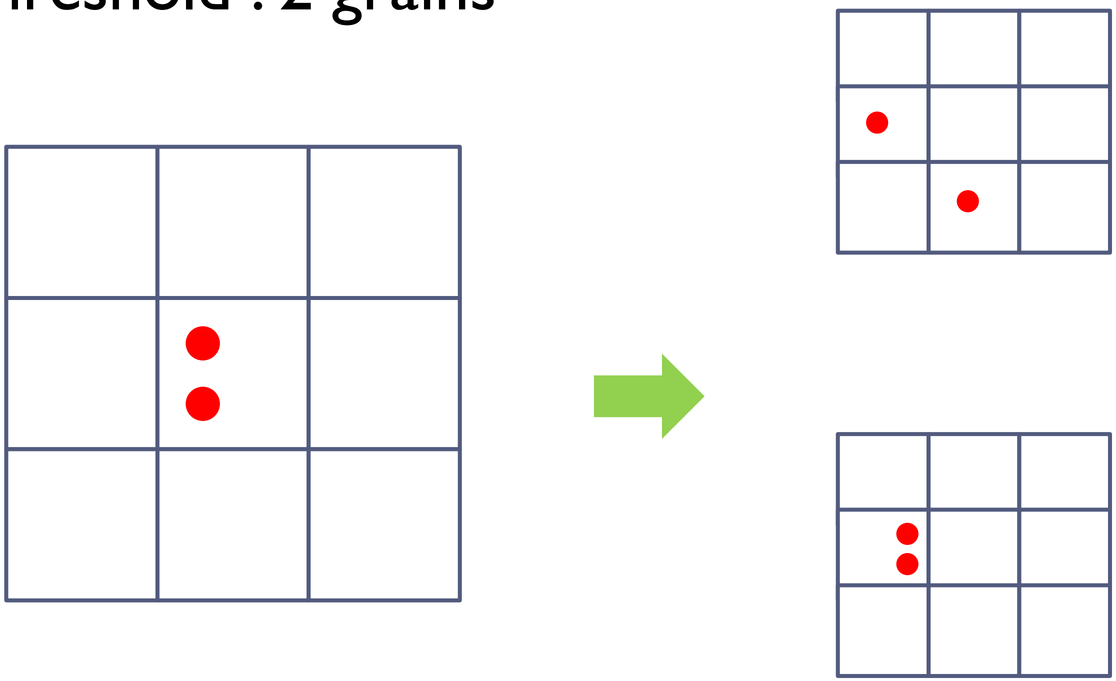
  threshold : 4 grains



Sandpile Model

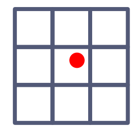
- Manna model

  threshold : 2 grains



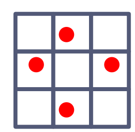
- One perturbes the system by adding particles at a **randomly chosen site r** according to

$$E_{\mathbf{r}} \mapsto E_{\mathbf{r}} + 1, \quad \text{with random } \mathbf{r}.$$



- An unstable site relaxes, its value is decreased by E_c and the two-dimensional nearest neighboring sites are increased by one unit,

$$E_{nn,\mathbf{r}} \rightarrow E_{nn,\mathbf{r}} + 1.$$



$$E_{\mathbf{r}} \rightarrow E_{\mathbf{r}} - E_{\mathbf{c}},$$

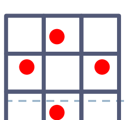
BTW Model

Throw a grain

$$E_{\mathbf{r}} \mapsto E_{\mathbf{r}} + 1,$$

$$E_{\mathbf{r}} \rightarrow E_{\mathbf{r}} - E_{c}, \quad \mathbf{E_c = 4}$$

$$E_{nn,\mathbf{r}} \rightarrow E_{nn,\mathbf{r}} + 1.$$



Manna Model

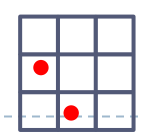
Throw a grain

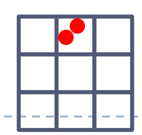
$$E_{\mathbf{r}} \mapsto E_{\mathbf{r}} + 1,$$

When $E_r \geqq E_c$ ➔ avalanche

$$E_{\mathbf{r}} \rightarrow E_{\mathbf{r}} - E_{c}, \quad \mathbf{E_c = 2}$$

$E_{nn,r} \rightarrow$ random





Parameter

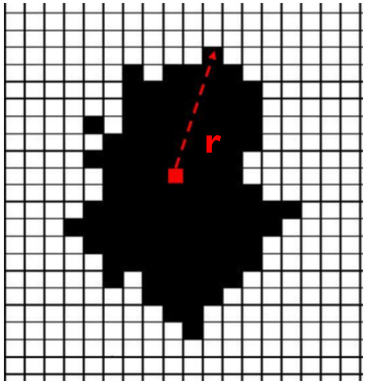
- The avalanches are characterized by several physical properties :

  the **size s** (number of relaxation events)

  the **area a** (number of distinct toppled sites)

  the **radius r** (radius of gyration)

  the **perimeter p** (number of boundary sites)



(Lee,2011)

  the **time t** (number of parallel updates until the configuration is stable), etc.

  the **maximum distance d** (between the origin of the avalanche to sites of the avalanche cluster)

  the **sandpile system size L**

Method - Ben-Hur et al.

- The avalanche variables have probability functions which are assumed to fall off with a **power law** defined by

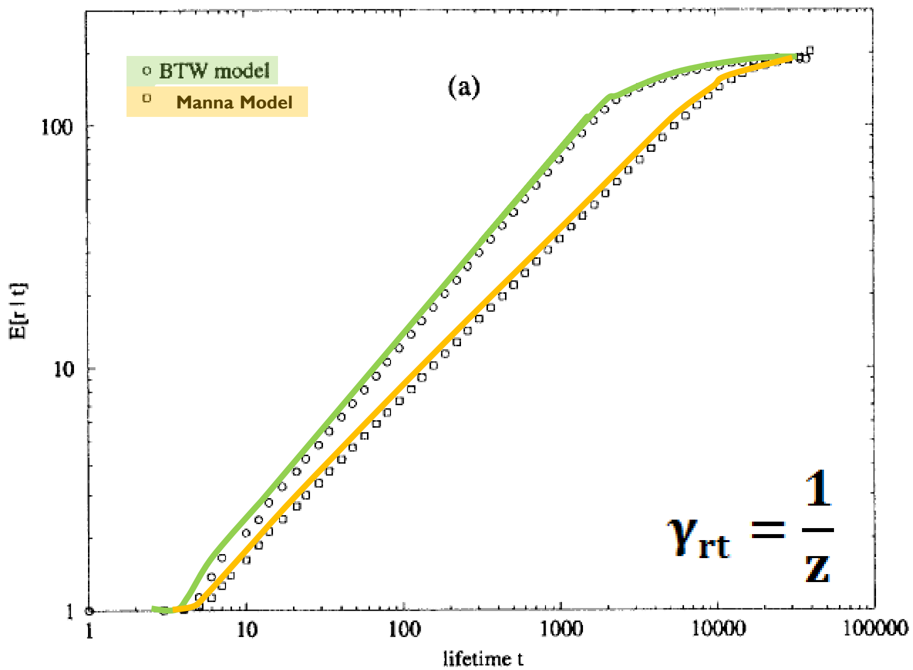
$$\mathbf{P(x)} \sim \mathbf{x}^{1-\gamma_{\mathbf{x}}} \quad , \quad \mathbf{x} \in \{\mathbf{s}, \mathbf{a}, \mathbf{r}, \mathbf{d}, \mathbf{t}, \mathbf{p}\}$$

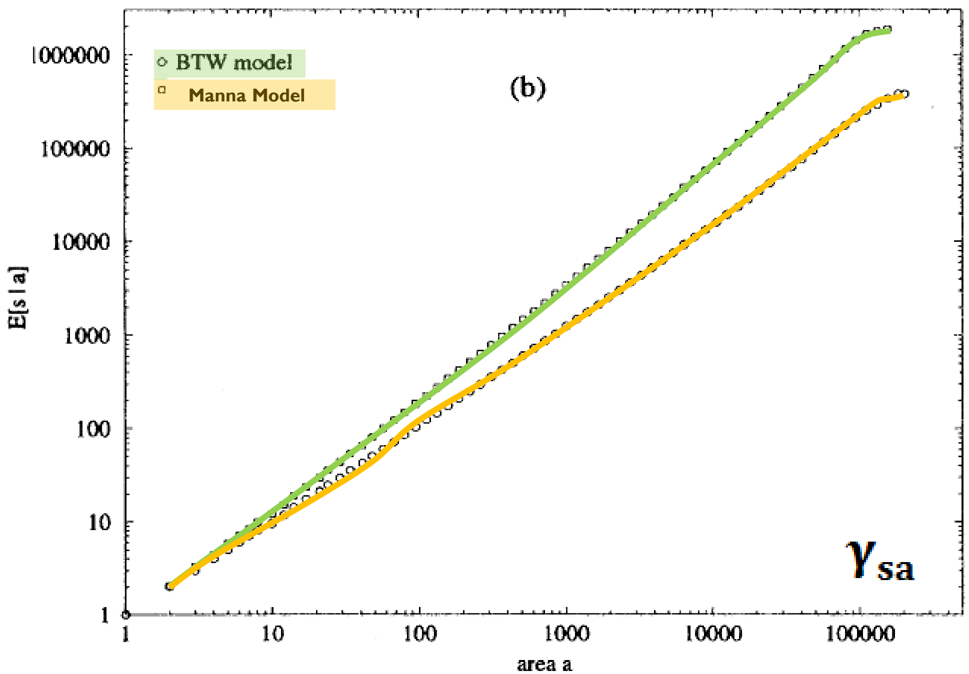
- These variables also scale against each other in the form

$$\mathbf{y} \sim \mathbf{x}^{\gamma_{yx}} \quad \mathbf{for\ x, y} \in \{\mathbf{s}, \mathbf{a}, \mathbf{r}, \mathbf{d}, \mathbf{t}, \mathbf{p}\}$$

- The exact definition of the γ 's is in terms of conditional expectations values:

$$E[y|x] \sim x^{\gamma_{yx}}$$





Exponent	Model BTW	Manna
$1/z$[a]	0.76	0.67
γ_{st}	1.62	1.70
γ_{at}	1.53	1.35
γ_{sa}	1.06	1.23
$D_f \rightarrow \gamma_{pr}$	1.26	1.42

- On the basis of the difference in the γ 's for the BTW model and Manna models we conclude that the two models are not in the same universality class.

Method - S. Lübeck

Power low behavior

$$P_x(x) \sim x^{-\tau_x}$$

Parameters relation

$$x \sim x'^{\gamma_{xx'}} , \quad \gamma_{xx'} = \frac{\tau_{x'} - 1}{\tau_x - 1}$$

Moment Analysis

$$\langle x^q \rangle = \int dx x^q P_x(x)$$

$$x_{max} \sim L^{\gamma_{xr}}$$

$$\langle x^q \rangle_L \sim L^{\sigma_x(q)}$$

$$\sigma_x(q) = \gamma_{xr}(q + 1 - \tau_x)$$

$$\gamma_{xr} = \frac{\tau_r - 1}{\tau_x - 1}$$

slope

$$\sigma_x(q) = \gamma_{xr} q + \Sigma , \quad \Sigma = 1 - \tau_r$$

characteristic quantity of the model

σ(q) value

$$\sigma_x(q) = \frac{\partial \ln \langle x^q \rangle_L}{\partial \ln L}$$

Ben-Hur et al.

$$\mathbf{y \sim x^{\gamma_{yx}}}$$

$$E[y|x] \sim x^{\gamma_{yx}}$$

S. Lübeck

Parameters relation

$$x \sim x'^{\,\gamma_{xx'}} \qquad \gamma_{xx'} = \frac{\tau_{x'} - 1}{\tau_x - 1}$$

Moment Analysis

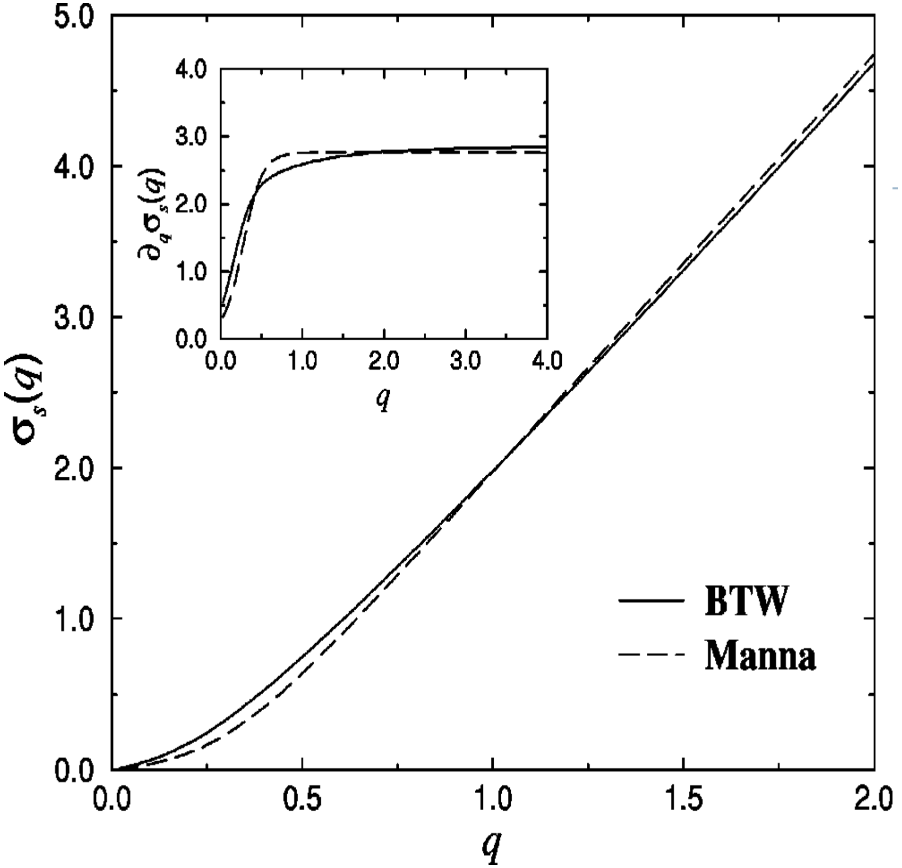
$$\langle x^q \rangle = \int dx\, x^q P_x(x)$$

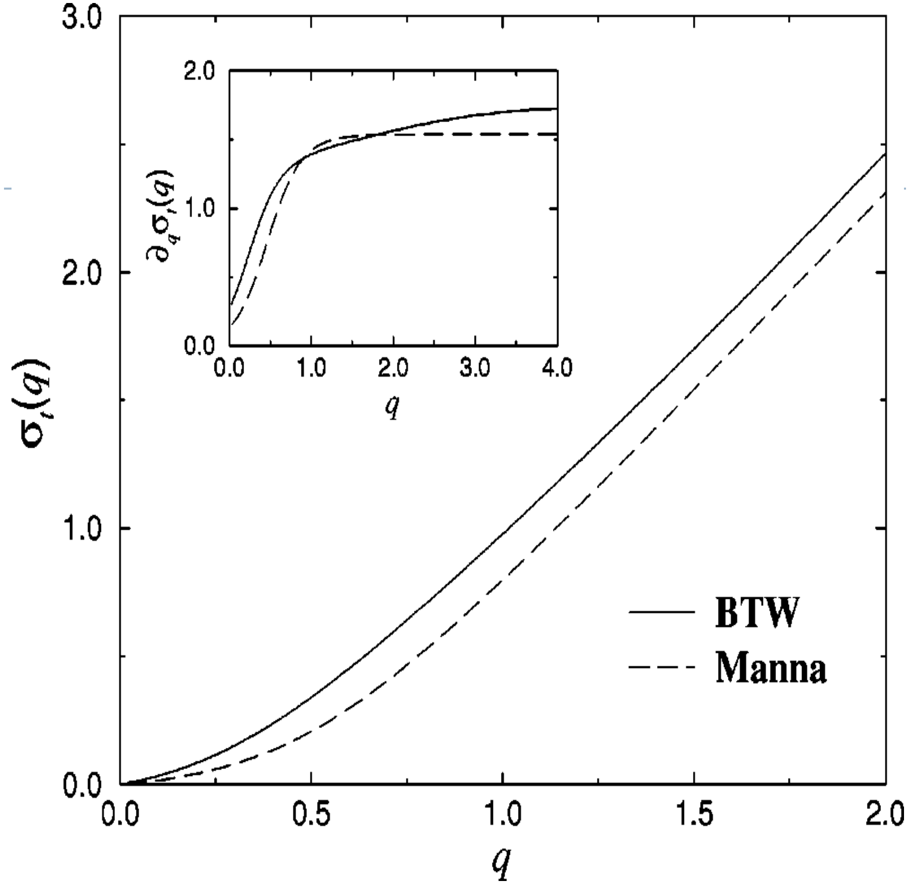
$$\langle x^q \rangle_L \sim L^{\sigma_x(q)}$$

$$\sigma_x(q) = \gamma_{xr} q + \Sigma$$

$$\sigma_x(q) = \gamma_{xr} q + \Sigma, \qquad \Sigma = 1 - \tau_r$$

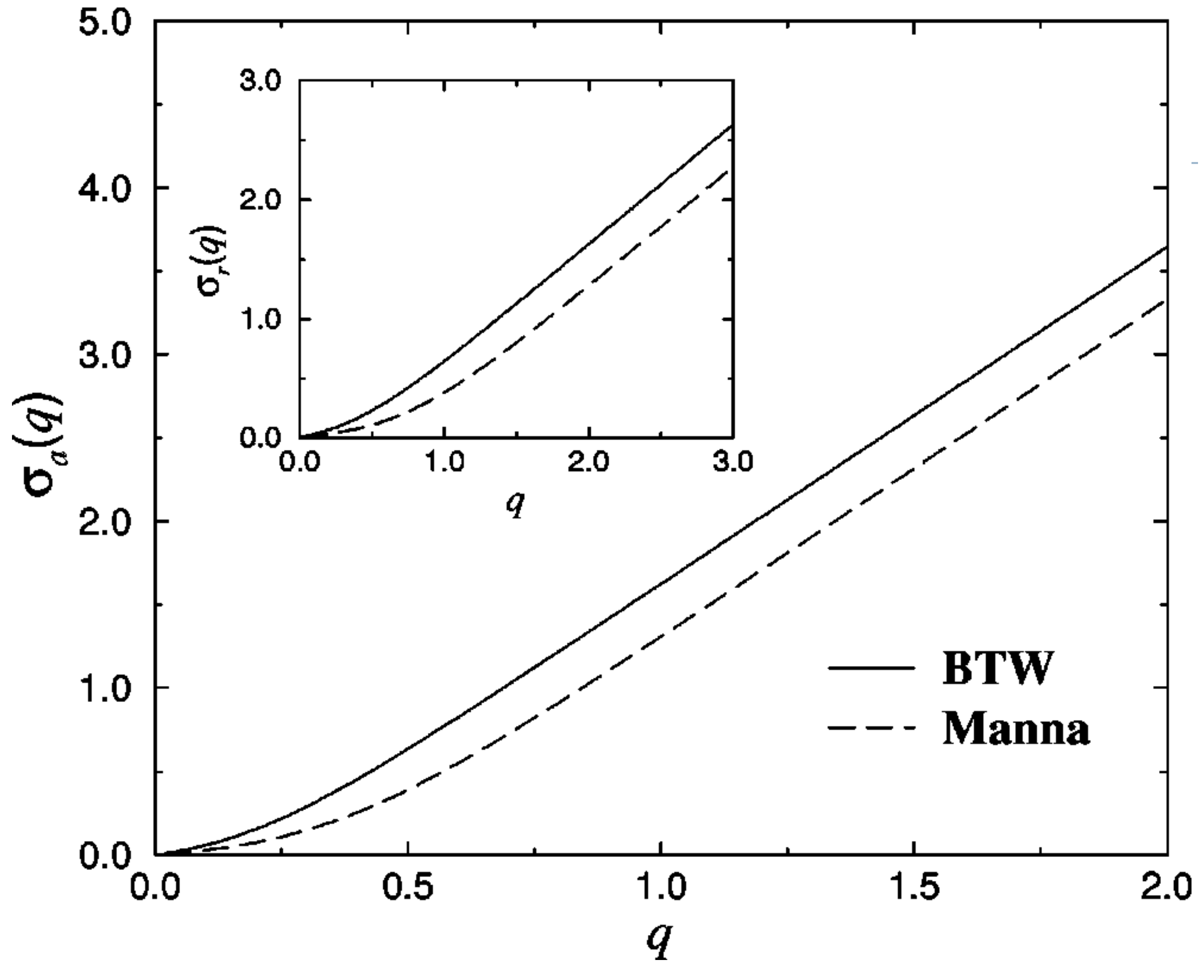
- We get that the intercept **Σ** of the linear q-dependence of the moment exponent $\sigma_x(q)$ is the same for all distributions (size, area, duration, etc.) and is therefore a characteristic quantity of the model.
- Considering two models we get that different values of **Σ** implies different universality classes.
  But the same value of the intercept **Σ** does not imply that both models belong to the same universality class.





	BTW, 2D	Manna, 2D
γ_{sr}		2.764
γ_{ar}	2.021	2.025
γ_{tr}		1.540
γ_{pr}	1.266	1.42 [8]
γ_{rr}	1.008	1.006

$$\sigma_x(q) = \gamma_{xr} q + \Sigma, \quad \Sigma = 1 - \tau_r$$

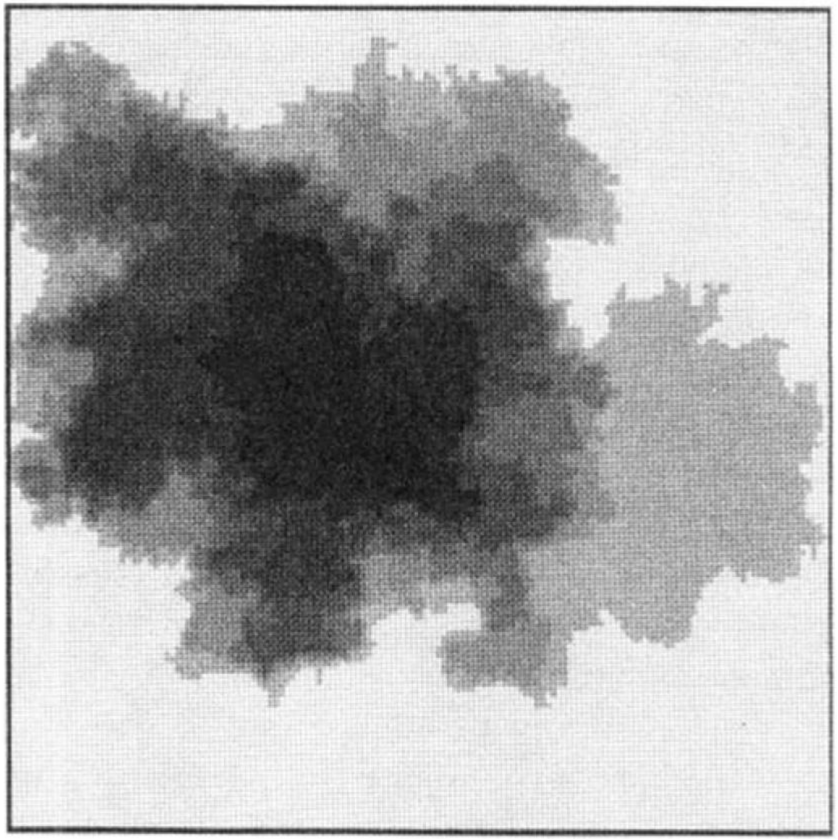


γ_{ar} and γ_{rr} values are the same

	BTW, 2D	Manna, 2D
γ_{sr}		2.764
γ_{ar}	2.021	2.025
γ_{tr}		1.540
γ_{pr}	1.266	1.42 [8]
γ_{rr}	1.008	1.006

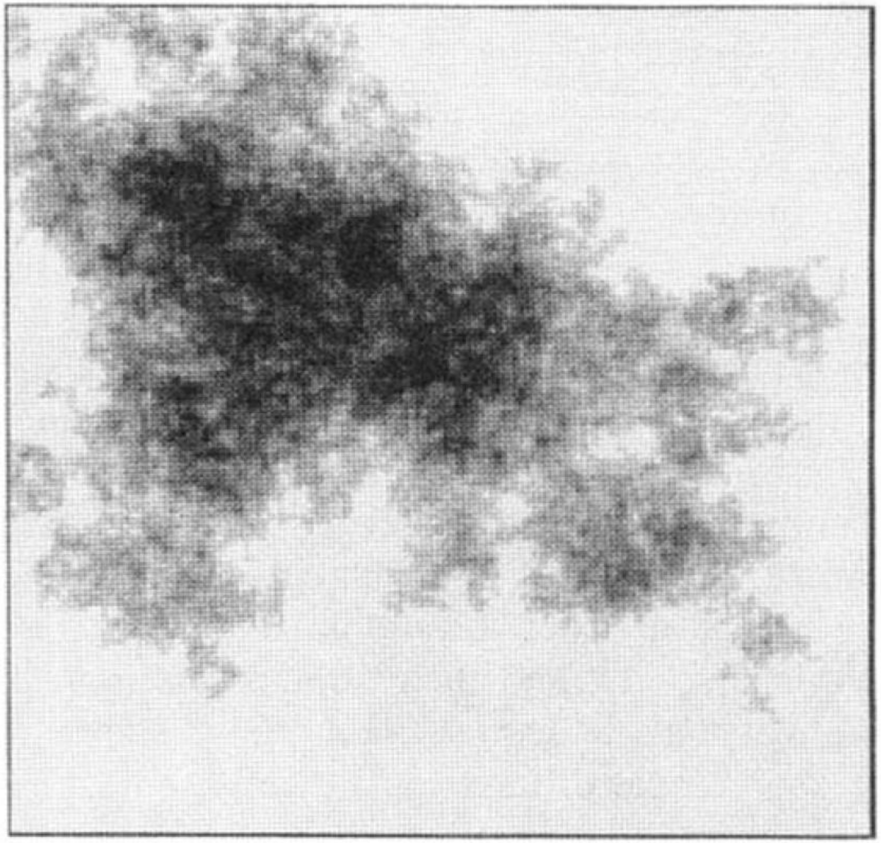
$$\sigma_x(q) = \gamma_{xr} q + \Sigma, \quad \Sigma = 1 - \tau_r$$

BTW Model



Shell structure

Manna Model



Irregular structure

Conclusion

- On the basis of the difference in the γ 's for the BTW and Manna models we conclude that the two models are not in the same universality class.

~Thanks for your attention~