Moment analysis of the probability distribution of different sandpile models

S. Lübeck*

Theoretische Tieftemperaturphysik, Gerhard-Mercator-Universität Duisburg, Lotharstrasse 1, 47048 Duisburg, Germany

(Received 1 April 1999)

We reconsider the moment analysis of the Bak-Tang-Wiesenfeld and the stochastic sandpile model introduced by Manna [J. Phys. A **24**, L363 (1991)] in two and three dimensions. In contrast to recently performed investigations our analysis reveals that the models are characterized by different scaling behavior, i.e., they belong to different universality classes.

PACS number(s): 64.60.Ht, 05.65.+b, 05.40.−a

I. INTRODUCTION

The Bak-Tang-Wiesenfeld (BTW) model was introduced as a paradigm of the concept of self-organized criticality which describes the emergence of spatiotemporal correlations in slowly driven dissipative systems [1,2]. Despite its analytical tractability [3] the scaling behavior of the two-dimensional BTW model is not well understood. Especially the exponents which determines the avalanche distributions are not known exactly. Several numerical attempts were made but do not provide consisting results [4–10]. Recently De Menech*et al.* performed a moment analysis of the BTW model [11] which was extended by several authors to different sandpile models [12–14]. Especially the moment analysis of the size distribution of the BTW and Manna sandpile model has led Chessa *et al.* to the conclusion that both models are characterized by the same scaling exponents and thus belong to the same universality class [12]. In this work we reconsider the moment analysis and compare the scaling behavior of various avalanche quantities for the BTW and Manna model. Our analysis turns out that in contrast to [12] the moment behavior of both models differs significantly, i.e., the BTW and the Manna model belong to different universality classes.

II. MODELS AND SIMULATIONS

The BTW model is defined on a D-dimensional square lattice of linear size L in which non-negative integer variables $E_{\mathbf{r}}$ represent a physical quantity such as the local energy, stress, height of a sand column, etc. One perturbes the system by adding particles at a randomly chosen site $\mathbf{r}$ according to

$$E_{\mathbf{r}} \mapsto E_{\mathbf{r}}+1, \quad \text{with random } \mathbf{r}. \tag{1}$$

A site is called unstable if the corresponding variable exceeds a critical value E_c, i.e., if $E_{\mathbf{r}} \geqslant E_c$, where the critical value is given by $E_c = 2D$. An unstable site relaxes, its value is decreased by E_c and the two-dimensional (2D) nearest neighboring sites are increased by one unit, i.e.,

$$E_{\mathbf{r}} \to E_{\mathbf{r}} - E_c, \tag{2}$$

$$E_{nn,\mathbf{r}} \to E_{nn,\mathbf{r}}+1. \tag{3}$$

In this way the neighboring sites may be activated and an avalanche of relaxation events may take place. These avalanches are characterized by several physical properties like the size s (number of relaxation events), the area a (number of distinct toppled sites), the time t (number of parallel updates until the configuration is stable), the radius r (radius of gyration), the perimeter p (number of boundary sites), etc. In the critical steady state the corresponding probability distributions should obey power-law behavior [1]

$$P_x(x) \sim x^{-\tau_x} \tag{4}$$

characterized by the avalanche exponents τ_x with $x \in \{s,a,t,r,p\}$. Assuming that the size, area, etc. scale as power of each other,

$$x \sim x'^{\gamma_{xx'}}, \tag{5}$$

one obtains the scaling relations $\gamma_{xx'} = (\tau_{x'}-1)/(\tau_x-1)$. The scaling exponents $\gamma_{xx'}$ describe the static avalanche properties as well as its propagation. For instance, the exponent γ_{sa} indicates if multiple toppling events are relevant ($\gamma_{sa}>1$) or irrelevant ($\gamma_{sa}=1$). The exponent γ_{ar} equals the fractal dimension of the avalanches. A possible fractal behavior of the avalanche boundary corresponds to the inequality $D-1<\gamma_{pr}<D$. Finally, the exponent γ_{tr} is usually identified with the dynamical exponent z.

A stochastic version of the BTW model was introduced by Manna [15]. Here, critical sites relax to zero, i.e., $E_{\mathbf{r}} \to 0$ if $E_{\mathbf{r}} \geqslant E_c$ and the removed energy is randomly distributed to the nearest neighbors in the way that one chooses randomly for each energy unit one neighbor. For $E_c=1$ the behavior of the model corresponds to a simple random walk. Above this value ($E_c \geqslant 2$) is the choice of the critical energy irrelevant to the scaling behavior (see Fig. 1).

Recently Dhar introduced a modified version of the two-dimensional Manna model where the energy of critical sites is not reduced to zero but $E_{i,j} \to E_{i,j}-2$. The energy $\Delta E = 2$ is then equally distributed with probability 1/2 to the sites $(i \pm 1, j)$ or otherwise to the sites $(i, j \pm 1)$ [16]. In this case it is possible to extend an operator algebra, which was successfully applied in studying the BTW model [3], to this modified Manna model.

Compared to the BTW model the dynamics of the Manna model with its stochastic distribution of the energy to the nearest neighbors can be interpreted as a disorder effect. A different kind of disorder effects were investigated in di-

*Electronic address: sven@thp.uni-duisburg.de

