



## Magnitude estimation using the first three seconds P-wave amplitude in earthquake early warning

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[1]  $P_d$  is the peak amplitude of displacement in the first three seconds after the arrival of the  $P$  wave. We investigated the attenuation of  $P_d$  with the hypocentral distance  $R$  in southern California as a function of magnitude  $M$ , and obtained the following relationship:  $\log(P_d) = -3.463 + 0.729 \times M - 1.374 \times \log(R) \pm 0.305$ . Given an earthquake location determined by the  $P$ -wave arrival times at stations close to the epicenter, this relationship can be used to define a so-called “ $P_d$  magnitude” of earthquakes. Our result shows that for earthquakes in southern California the  $P_d$  magnitudes agree with the catalog magnitudes with a standard deviation of 0.18 for events less than magnitude 6.5. Therefore,  $P_d$  is a robust measurement for estimating the magnitudes of earthquakes and has practical application in earthquake early warning systems. **Citation:** Wu, Y.-M., and L. Zhao (2006), Magnitude estimation using the first three seconds P-wave amplitude in earthquake early warning, *Geophys. Res. Lett.*, 33, L16312, doi:10.1029/2006GL026871.

### 1. Introduction

[2] When large earthquake occurs, earthquake early warning (EEW) provides alerts to urban areas near the epicenter of the forthcoming strong ground shaking. Depending on the specific geometry of the earthquake and the stations used in EEW, the early warning time window can be a few seconds to a few tens of seconds. This short warning can be extremely important since even a few seconds can be sufficient for pre-programmed systems to take emergency measures, such as the deceleration of rapid transit vehicles to prevent possible derailment, protective measures for sensitive facilities such as nuclear reactors to avoid damage and leakage, orderly shutoff of gas pipelines to minimize fire hazards, controlled shutdown of manufacturing operations to reduce potential losses, and the activation of structural control. Structural control, also known as “seismic response control”, is the latest structural technology to assure the structural safety of buildings during extremely large earthquakes by controlling the buildings’ vibration characteristics or providing a control force to suppress their responses using seismic response control devices.

[3] There are two different approaches to EEW: Regional warning and on-site warning. In regional warning, traditional seismological methods are used to determine the locations and magnitudes of earthquakes and to estimate

the ground motion in the region involved. In on-site warning, the beginning of the ground motion (mainly the  $P$  wave) observed at a given site is used to predict the ensuing ground motion (the later arriving  $S$  and surface waves) at the same site, with usually no attempt to locate the event or to estimate its magnitude. The regional approach is more comprehensive and accurate. However, it takes a longer time to operate since it requires information from a number of stations. Therefore, it cannot be used for early warning purposes in areas very close to the epicenter. The EEW systems currently implemented in Japan, Taiwan, and Mexico [e.g., *Odaka et al.*, 2003; *Horiuchi et al.*, 2005; *Wu and Teng*, 2002; *Espinosa-Aranda et al.*, 1995] belong to this category.

[4] As an amplitude parameter, the peak amplitude of the initial  $P$ -wave displacement,  $P_d$ , reflects the attenuation relationship of the ground motion with distance. In our previous studies, we showed that  $P_d$  is well correlated with the peak ground velocity (PGV) in Taiwan [*Wu and Kanamori*, 2005a] and in Southern California. This suggests that we can predict the intensity of the ground motion using the initial  $P$  waves. Therefore, determining the attenuation relationship of  $P_d$  naturally leads to practical applications for EEW. For instance, when large earthquake happens, its location can be quickly obtained from a few  $P$ -wave arrival times at nearby stations.  $P_d$  can then be used to determine the magnitude via the attenuation relationship, which can then be used in EEW for emergency operations.

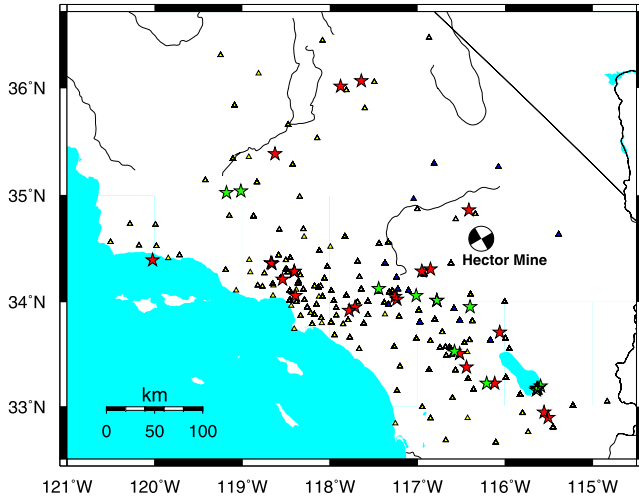
[5] In this paper, we first investigate the attenuation relation of  $P_d$  with hypocentral distance in southern California using seismic records of regional earthquakes from the Southern California Seismic Network (SCSN) operated jointly by the United States Geological Survey and the California Institute of Technology. Then, we use the attenuation relation to obtain the “ $P_d$  magnitude” and compare them with the magnitudes in the catalog, and we show that the  $P_d$  amplitude is robust in estimating earthquake magnitudes and can be used for earthquake early warning purpose in southern California.

### 2. Seismic Data and Analysis

[6] We selected 25 regional earthquakes (see auxiliary material<sup>1</sup> Table S1) from the SCSN catalog with  $M \geq 4.0$ , including one  $M > 7$  earthquake (Hector Mine earthquake), in southern California. To ensure a good station coverage for each event and to avoid the complexity of path effects for  $P$  waves at longer distances, our criteria for event selection is that for each earthquake at least six records

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**Figure 1.** Locations of seismic stations (triangles) of the Southern California Seismic Network (SCSN) and the epicenters of 25 events (red stars) used in this study. Green stars show the locations of 13 earthquake used in the test of near-source  $P_d$  amplitudes in Section 5. The focal mechanism of the Hector Mine earthquake is shown by the beach ball, and  $P_d$  amplitudes for this event were collected from 13 stations (blue triangles).

must be available within 120 km of the epicenter. In the SCSN catalog, the magnitudes are local magnitude  $M_L$  for events with  $M_L < 6$  and moment magnitude  $M_W$  for larger events. Here, we denote both types of magnitudes simply by  $M$ . All the events occurred during the period from 1992 to 2004. Figure 1 shows the distributions of the epicenters and the SCSN stations used in this study.

[7] Most of the SCSN stations have both high-gain broadband velocity and low-gain force-balance acceleration sensors [Hauksson *et al.*, 2001]. Signals are digitized at 100 or 80 samples per second with 24-bit resolution. The low-gain channels record all large earthquakes without clipping. High-gain channels provide high-resolution signals for the initial 3 seconds of the  $P$  waves used in this study.

[8] We used a total of 684 vertical-component records (see auxiliary material Table S1) to determine  $P_d$  in this study. The velocity signals were numerically differentiated to obtain the accelerations and integrated to obtain the displacements. We applied a 0.075 Hz high-pass recursive Butterworth filter to remove the low-frequency drift after the numerical integration. An automatic  $P$ -wave picker described by Allen [1978] is used to detect the  $P$  arrivals from the vertical acceleration records. Following the analysis of Kanamori [2005], Wu and Kanamori [2005a, 2005b], and Wu *et al.* [2006], we used a duration of 3 seconds for the estimation of  $P_d$ .

### 3. Result

[9] Analogues to the definition of surface-wave magnitude, e.g., equation (4.14) given by Fowler [1990], we assumed a simple linear regression model among the logarithmic  $P_d$ , the reported magnitude  $M$ , and the logarithmic hypocentral distance  $R$ :

$$\log(P_d) = A + B \times M + C \times \log(R), \quad (1)$$

where  $A$ ,  $B$ , and  $C$  are constants to be determined from the regression analysis. It is expected that in addition to the attenuation along the path, the  $P$ -wave peak amplitude  $P_d$  also depends on the  $P$ -wave radiation pattern, just like other types of seismic phases used in magnitude estimation. Therefore, it is important that a good azimuthal coverage of  $P_d$  observations from each event be used in the regression analysis of equation (1) so as to reduce the standard deviation. Furthermore, in such a regression one usually also needs to consider a linear  $R$  term. However, regression experiments with this term included show that the linear  $R$  term is not statistically significant. Similar results were also found in our previous study for earthquakes in Taiwan [Wu *et al.*, 2005]. With a total of 684 records as the constraints, the resulting best-fitting attenuation relationship for  $\log(P_d)$  is given by

$$\log(P_d) = -3.088 + 0.630 \times M - 1.322 \times \log(R) \pm 0.313. \quad (2)$$

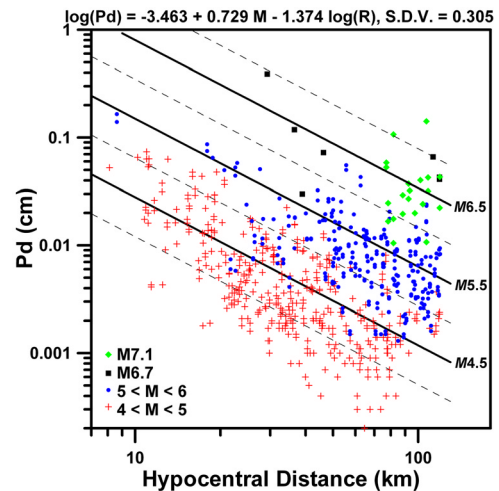
However, we found that the measurements from the  $M = 7.1$  Hector Mine event did not fit well in the regression. Thus, we only used the records from events with  $M < 7$  in the regression analysis. With a total of 662 records as the constraints, the resulting best-fitting attenuation relationship for  $\log(P_d)$  is given by

$$\log(P_d) = -3.463 + 0.729 \times M - 1.374 \times \log(R) \pm 0.305. \quad (3)$$

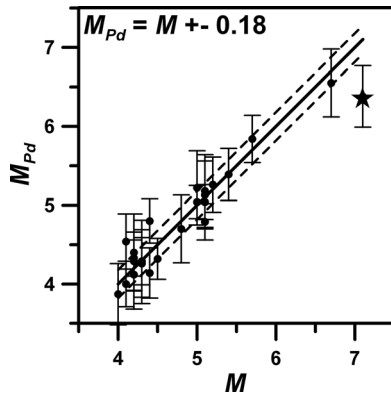
In Figure 2, the observed  $P_d$  values are compared with those predicted by equation (3) separately for magnitudes 4.5, 5.5, and 6.5.

### 4. Application for Earthquake Early Warning

[10] In the regional approach, the earthquake locations can be determined from the  $P$  waves at the same stations involved in the EEW system. With hypocentral distances



**Figure 2.** Distribution of the observed  $P_d$  measurements. The three solid diagonal lines are calculated from the linear  $\log(P_d)$ - $\log(R)$  relationship, solid lines for magnitudes 4.5, 5.5, and 6.5, and dashed lines magnitudes 4, 5, and 7.

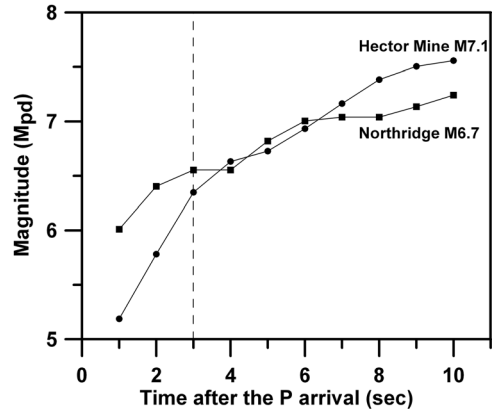


**Figure 3.** Magnitudes determined from  $P_d$  versus the reported magnitude  $M$ . Solid line shows the least squares fit and the two dashed lines show the range of one standard deviation. The star represents the Hector Mine event.

available, we can use the regression result in equation (3) to estimate the magnitudes from the  $P$ -wave peak amplitude  $P_d$ . We call it the “ $P_d$  magnitude”, defined in the standard magnitude form derived from equation (3):

$$M_{Pd} = 4.748 + 1.371 \times \log(P_d) + 1.883 \times \log(R). \quad (4)$$

Figure 3 compares the  $P_d$  magnitudes of the 24 events of magnitude less than 7.0 in auxiliary material Table S1 with their catalog magnitudes. On the  $45^\circ$  line,  $M_{Pd} = M$ , and the dashed lines show the one standard deviation (0.18) locations. The small uncertainty of magnitude determination from  $P_d$  for  $M < 7$  events suggests a consistency with the earthquake initiation model proposed by *Olson and Allen* [2005] which states that the overall magnitude of an earthquake is determined by the earliest stage of the rupture process. Interestingly, for the magnitude 7.1 Hector Mine earthquake the  $P_d$  magnitude of 6.37 is much lower than its catalog magnitude. For this earthquake, we used stations from a variety of azimuths (blue triangles in Figure 1), so this lower  $P_d$  magnitude estimation was not due to the effect of the radiation pattern, but a real lack of  $P$ -wave peak amplitude. The same saturation problem of  $P_d$  amplitude was also observed for large earthquakes in Taiwan [*Wu et al.*, 2006] providing another evidence that the earthquake initiation model of *Olson and Allen* [2005] may not be applicable to large earthquakes. In Figure 3, the  $P_d$  magnitude of the northridge earthquake ( $M = 6.7$ ) is located within the range of the one standard deviation. However, waveform simulations [*Kanamori*, 2005] using the kinematic source model of *Sato and Hirasawa* [1973] and our  $P_d$ -magnitude study in Taiwan [*Wu et al.*, 2006] suggest that the  $P_d$  magnitude  $M_{Pd}$  saturates for  $M > 6.5$ . Figure 4 shows the growth of the  $P_d$  magnitude for Hector Mine and Northridge earthquakes with the lengths of  $P$ -wave records used in determining  $P_d$ . The  $P_d$  growth pattern in Figure 4 implies that  $P_d$  determined from the first 3-second  $P$  waves cannot be used to estimate the magnitude for earthquakes of magnitude  $M = 6.7$  and above. However, for earthquakes of  $M < 6.0$ ,  $M_{Pd}$  values determined from equation (4) are

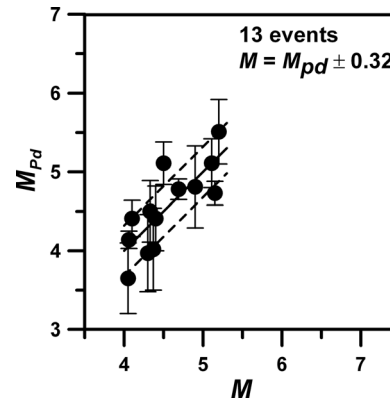


**Figure 4.** Growth of  $P_d$  magnitudes for Hector Mine and Northridge earthquakes versus the lengths of the initial  $P$ -wave records used in estimating  $P_d$ .

reliable estimates for regional earthquake monitoring purpose and EEW operations.

### 5. Test for Near-Source Signals

[11] Near-source signals are commonly used for EEW purpose. Here we use thirteen additional events with shorter epicentral distances for testing. These events (see auxiliary material Table S2) occurred between January and October in 2005 and are all of magnitude  $M > 4.0$ , and for each event at least five records within 40-km epicentral distance are available. Results from these near-source signals are plotted in Figure 5 in which the near-source  $P_d$  determined magnitudes  $M_{Pd}$  are plotted against the catalog magnitudes  $M$ . Similar to the result in Figure 3, the  $M_{Pd}$  values in Figure 5 are still linearly correlated to the  $M$  values. On the  $45^\circ$  line,  $M_{Pd} = M$ , and the dashed lines show the one standard deviation locations (0.31). Although the standard deviation of 0.31 is large than that from the twenty-four events in auxiliary material Table S1, it is sufficient for early warning purpose. The standard deviation increases from 0.18 to



**Figure 5.** Magnitudes determined from near-source  $P_d$  amplitudes versus the reported magnitudes  $M$  for 13 supplementary events (green stars in Figure 1). Solid line shows the least squares fit and the two dashed lines show the range of one standard deviation.

0.31, which is not surprising since in this near-source experiment fewer stations were used.

## 6. Discussion and Conclusion

[12] For regional EEW, rapid and reliable determination of earthquake magnitudes is more difficult than the estimation of other parameters because the shear wave trains may not completely arrive within a few to ten seconds time window. *Wu et al.* [1998] proposed the  $M_{L10}$  method, a magnitude based on the first 10 seconds of the signal that can be linearly related to the local magnitude  $M_L$  for regional EEW purpose. Taiwan Central Weather Bureau adopts a similar method and requires an average of 22 seconds of signals to determine the earthquake parameters with an uncertainty of  $\pm 0.25$ . That approach provides an early warning for areas beyond about 70 km from the epicenter [*Wu and Teng*, 2002]. However, the  $M_{L10}$  method is a purely empirical approach, whereas  $Pd$  is a more physically fundamental and source-dependent parameter. It is more characteristic of the earliest stage of an earthquake's rupture process and also less affected by the scattering of the complex velocity structure than the other types of arrivals used in determining magnitudes. The effectiveness of  $Pd$  amplitude in estimating the magnitude and in EEW practice is mutually supportive of the model proposed by *Olson and Allen* [2005] that the eventual sizes of earthquakes are in fact controlled by the initial stages of the rupture, rather than the entire rupture process. Our result shows that the magnitudes obtained from only a few initial  $P$ -wave peak amplitudes agree with the well-established catalog magnitudes with an uncertainty of  $\pm 0.18$ . In EEW practice,  $Pd$  provides more advanced warning time since it is determined from only 3 seconds of the early signal. In particular, it is shown in the test for near-source signals in Section 5 that acceptable magnitude estimation can be achieved by using records within 40 km from the epicenter. In Southern California, it takes  $\sim 7.5$  sec for the  $P$  wave to propagate 40 km from a source of 5 km depth. Adding a 3.0 sec telemetry delay and another 3.0 sec to record the  $P$  waves needed to determine  $Pd$ , we can expect that this method is capable of generating earthquake warning information within about 15 sec of the occurrence of a large event, thus providing early warnings to places farther than 45 km from the epicenter.

[13] For EEW purpose, the more effective way for magnitude determination may be to use the average period  $\tau_c$  [*Kanamori*, 2005; *Wu and Kanamori*, 2005a, 2005b; *Wu et al.*, 2006] or the predominant period [*Nakamura*, 1988; *Allen and Kanamori*, 2003] from the initial  $P$  waves, because both the local magnitude  $M_L$  and the magnitude determined from  $Pd$  may have the saturation problem [*Wu et al.*, 2006]. However, magnitudes determined from  $Pd$  can be regarded as the lower bounds of the magnitudes of large

earthquakes with small uncertainty and are still very important for practical EEW operations. It will be very effective to avoid the false alarm caused by small and moderate events.

[14] **Acknowledgments.** We wish to thank Hiroo Kanamori for his stimulating ideas and encouragements. We also wish to thank Ta-liang Teng for his constructive comments. This research was supported by the National Science Council of the Republic of China under Grant No. NSC94-2116-M-002-021 and NSC95-2119-M-002-043-MY3. We also thank the Southern California Seismic Network and the Southern California Earthquake Data Center for providing public access to their seismological data, and their contributors at the Seismological Laboratory, Caltech and the U.S. Geological Survey, Pasadena.

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