

Connectivity Theory – A New Approach to Modeling Non-Archie Rocks¹

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ABSTRACT

So-called Archie rocks are characterized by a resistivity index (RI) versus water saturation that plots as a straight line on log-log scale, the line's slope being equal to $-n$ where n is the saturation exponent. All rocks for which the RI has curvature qualify as non-Archie rocks. This paper presents a relationship (equation 6) that links the true formation resistivity R_t to water saturation S_w and formation porosity ϕ . This differs from Archie's equation in its simplest form by the introduction of a small term called "water connectivity correction index" (WCI) χ_w , to correct for water connectivity effects. This term helps to stabilize

the single exponent in the model called the conductivity exponent μ – equivalent to Archie's equation when $n = m$. A method to derive analytical expressions for χ_w for various formations is introduced. Effective medium theory is used with a modified CRIM mixing law to model the WCI as a function of conductive structures and fluid parameters in the rock. The model agrees with predictions of Waxman-Smits and Dual-Water models for shaly sands (concave down curves) as well as experimental data obtained on strongly oil-wet rocks (concave up curves). Connectivity theory also provides insight as to why so many rocks satisfy Archie's model.

INTRODUCTION

When G. E. Archie (1942) introduced his equation a new era for the oil and gas industry began, where for the first time it became possible to make quantitative hydrocarbon reserve estimates using resistivity and porosity logs. In its simplest form

$$R_t = \frac{R_w}{(S_w \phi)^2}, \quad (1)$$

where R_t is the resistivity of the uninvasion, virgin reservoir rock, R_w is the water (or brine) resistivity, and ϕ is the total porosity of the rock. Archie's equation works remarkably well in "clean" water-wet sandstone formations.

In more general form

$$R_t = \frac{R_w}{S_w^n \phi^m}. \quad (2)$$

Archie's equation requires the determination of two exponents: the so-called cementation exponent m and saturation exponent n . The value of water saturation S_w is quite sensitive to these exponents. For example for a relatively small-sized oil reservoir using $m = n = 1.8$, instead of 2 as in equation 1, easily returns several additional billions of dollars in reserves. These two exponents are known to take different values for various reservoir rocks. This is especially true for carbonates, for which rock typing and pore geometry characterization are essential for petrophysical modeling. Indeed, in carbonates n can range from less than 1.5 to greater than 3, and m can exceed 4 in some vuggy rocks. To make things worse, various rock types tend to be distributed in some carbonate formations with a high level of heterogeneity. Determining a suitable average value to use for n and m in such heterogeneous formations, in order to obtain accurate estimates for hydrocarbons in place, is quite a challenge.

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Besides understanding the stability of Archie’s exponents, or the lack thereof, it is also important to understand when Archie’s equation “works” for a given rock, and when it doesn’t work so well. One obvious feature of equation (2) is that the resistivity index plotted versus water saturation in log-log scale must be a straight line with slope $-n$. Lines with slopes -1 , -2 and -3 are shown in Figure 1 in black. Therefore, as soon as a slight curvature is observed on the resistivity index function, Archie’s equation no longer applies. Such rocks are called “non-Archie” rocks.

The blue curve is characteristic of rocks that tend to remain more conductive than “normal” when the water saturation is lowered. That is the case, for example, with shaly sands. The red curve is observed on strongly oil-wet rocks. As illustrated in Figure 1, it is not possible to define a unique saturation exponent on such curves. In this example the slope picked locally at different points along the curve takes values ranging from -2 to -10 ; i.e., from $n = 2$ to $n = 10$, with n calculated using

$$n = -\frac{d \ln(RI)}{d \ln(S_w)} = -\frac{S_w}{RI} \frac{d(RI)}{d(S_w)}$$

The objective of this article is to propose a theory to explain why Archie’s equation seems to work in many cases, and why it doesn’t for some rocks. The objective is also to propose a model that is applicable to non-Archie rocks, and that reduces to Archie’s equation for the most probable values of its parameters.

The characterization of n and m is usually done on cores by performing special core analysis (SCAL). Cores brought back to the lab from downhole are submitted to various treatments in order to “restore” them as much

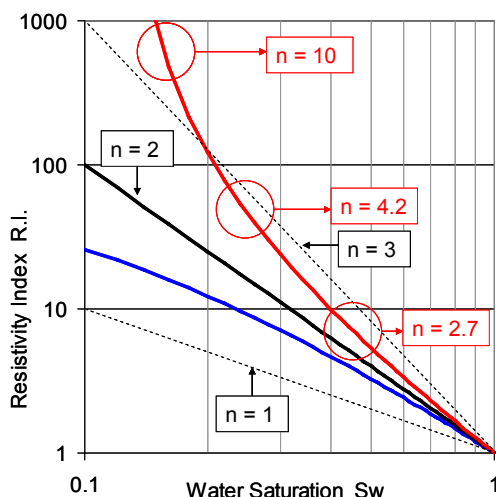


Fig. 1 Resistivity index vs. S_w in log-log scale. Archie’s equation corresponds to straight lines in black (slopes -1 , -2 , -3) and does not apply to curves (blue and red).

as possible; i.e., to put them back to virgin reservoir conditions. The curve obtained when measuring resistivity versus water saturation while slowly injecting crude oil into a water saturated core is called the “drainage curve”. The curve obtained while injecting water into an oil saturated core is the “imbibition curve”. These two curves can be quite different for some rocks, for example oil-wet rocks. The model developed in this paper is intended to apply to virgin reservoir rocks and to drainage curves obtained from SCAL when such curves are considered representative of the reservoir.

PERCOLATION THEORY AND EFFECTIVE MEDIUM THEORY (EMT)

A tremendous amount of work was accomplished by physicists in the 1960s, 1970s and 1980s to understand universal scaling laws for random percolation. See, for example, the text books Stauffer and Aharony (1991) and Lagües and Lesne (2003). There is an interesting parallel between percolation on random resistor networks and transport properties of natural porous rocks. This link was highlighted in a large number of publications in the last 30 years. Figure 2a illustrates these similarities.

A porous rock can be modeled using a simple three-dimensional (3D) cubic grid where a proportion p of cells (blue) are selected at random to be filled with brine of conductivity σ_w . The other cells (white) represent the non-conductive solid. The conductivity of the cube shown in Figure 2a, measured between two opposite metalized sides, is a function of the conductive paths linking the two sides. The cube conducts electricity only if the

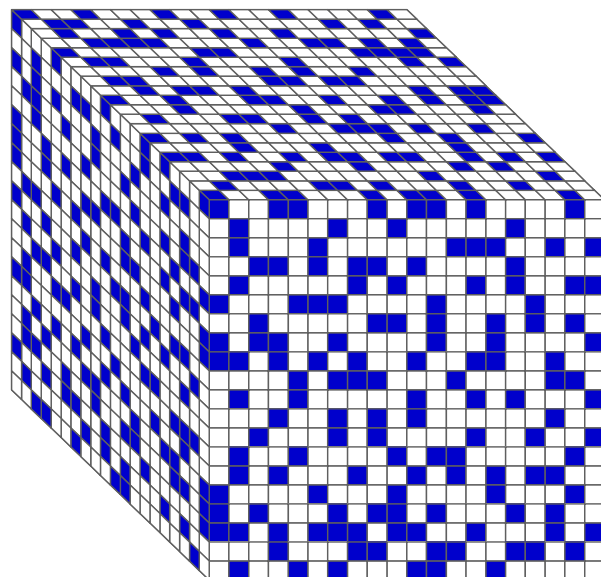


Fig. 2a Numerical experiment for random site percolation on a simple cubic grid $20 \times 20 \times 20$. Only blue cells are conductive and are here in proportion $p = 0.32$.

proportion p exceeds a percolation threshold p_c which in the case of a simple cubic grid is equal to 0.3116. Two conductive cubes are connected only if they share a square side. The maximum coordination number of this cubic network is equal to the number of sides of the cube; i.e., 6. If cubes are allowed to connect through corners, edges and sides the coordination number can be increased up to 26 and the percolation threshold becomes slightly less than 0.1.

Of course the numerical modeling of rocks can be made much more realistic than the cube model in Figure 2a. We generated the rock model shown in Figure 2b using a random process with specific constraints. In this case the rock is made water-wet: the cubic cells that are in contact with the surface of pores contain water.

Grains are grown randomly starting from a 3D distribution of solid nuclei. The growth process stops when the desired porosity value is reached, here 30 percent. The Laplace partial differential equation can be solved on this model using a computer. Unfortunately, this approach does not meet the objective of deriving analytical formulas for the conductivity of porous rocks.

Returning to the simple cube model of Figure 2a, near the percolation threshold the conductivity is known to follow a power law with a critical conductivity exponent, usually denoted t , very close to 2; i.e.

$$\sigma = \sigma_w \left(\frac{p - p_c}{1 - p_c} \right)^t \tag{3}$$

According to numerical simulations performed recently

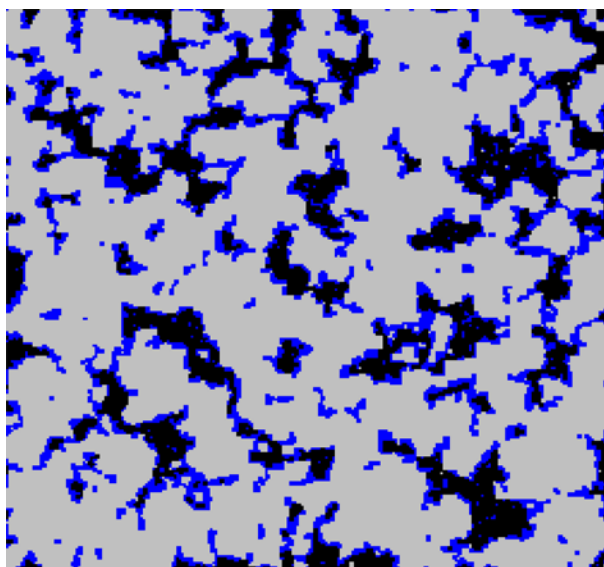


Fig. 2b A more elaborate cube model with 250×250×250 cubic cells. This plane section shows water in blue, oil in black and rock in gray.

by Boris Kozlov and Michel Lagües at ESPCI in Paris (2007, pers. comm.) $t = 2.002 \pm 0.005$. They have also confirmed that the critical region in which the universal behavior is valid is, however, very small.

In his fundamental paper “Percolation and Conduction” Scott Kirkpatrick (1973) used effective medium theory (EMT) applied to conductivity of mixtures to find expressions of the conductivity of a random resistor network on a simple 3D cubic grid for bond and site percolation. Reformulated for the site percolation example given in Figure 2a the Kirkpatrick EMT conductivity model takes the form

$$\sigma \approx \sigma_w p \frac{p - a}{1 - a}, \tag{4}$$

where a is equal to 0.342.

This model is valid for p close to 1, but it appears from Monte Carlo computer simulations that it is an excellent approximation for $p > 0.5$. It is interesting to note that close agreement can be obtained between equation (4) and the power law

$$\sigma = \sigma_w \left(\frac{p - b}{1 - b} \right)^\mu, \tag{5}$$

by adjusting the exponent μ and the threshold b .

This agreement is shown in Figure 3. The red curve labeled ‘Connectivity’ plots equation (5) with $\mu = 1.824$ and $b = 0.271$. The agreement is better than 0.1 percent of

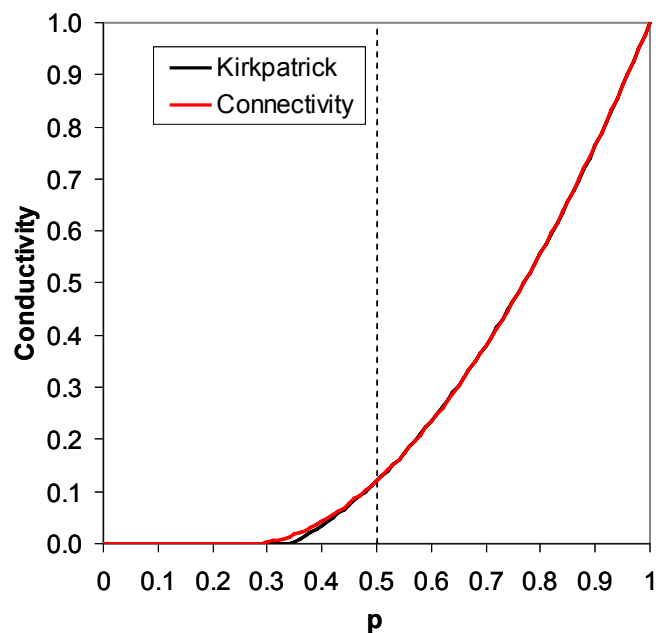


Fig. 3 An excellent match is observed between Kirkpatrick EMT conductivity model for site percolation on a cubic grid and a power law with an exponent equal to 1.824.

full scale for all values of p above 0.5. The fit is even better in the interval [0.75; 1] with $\mu = 1.888$ and $b = 0.250$ with an error less than 0.001 percent of full scale!

This example of the random cube model is interesting because fundamental physics have established the conductivity relationship for both ends of the p range; i.e., at the percolation threshold p_c and close to 1. The two equations are different, but it happens that equation (5) can be fitted to both cases. Of course there is a big difference between the random cube model and real porous rocks. Despite that, we will show here that equation (5) can be used as one key element of a consistent approach to derive analytical models for the conductivity of rocks.

Equations similar to equation (5) have been proposed by several authors to model the conductivity of natural rocks: for example Zhou et al. (1997), Montaron (2005), Hunt (2005), and Kennedy (2007). Berg (2007) gives an excellent survey of various EMTs applied to reservoir rocks.

THE CONNECTIVITY EQUATION

In equation (5) the proportion p of conductive sites is equal to the probability of finding water – the only conductive phase in the medium – at a random point in the cube volume. Therefore, in natural porous reservoir rocks p is equal to the bulk volume fraction of water ϕS_w where ϕ is the total porosity of the rock, and S_w is the water saturation; i.e., the fraction of the porosity occupied by water.

For reasons that will become clear in the rest of the paper the parameter b is renamed χ_w and called the “water connectivity correction index” or “water connectivity index” (WCI). We call equation (5) written in a form similar to equation (2) the connectivity equation; i.e.,

$$R_t = \frac{R_w'}{(S_w \phi - \chi_w)^\mu}, \quad \text{where } R_w' = R_w(1 - \chi_w)^\mu, \quad (6)$$

$$\sigma = \sigma_w'(S_w \phi - \chi_w)^\mu, \quad \text{where } \sigma_w' = \sigma_w / (1 - \chi_w)^\mu. \quad (7a)$$

The exponent μ is called the conductivity exponent. When modeling reservoir rocks with the connectivity equation the typical range observed for the exponent is 1.6 to 2.0; i.e., a range much reduced compared to the range observed for Archie exponents n and m . This stability has positive practical consequences. The parameter χ_w (WCI) takes small values typically ranging from $-0.02 \leq \chi_w \leq +0.02$. Indeed the WCI is a corrective term that adjusts the water volume fraction in equations (6) and (7) to account for positive or negative water connectivity effects in the rock. Several examples of these effects are presented in the remainder of this article.

With such values for the conductivity exponent and the

WCI the resistivity R_w' is almost always within five percent of the true water resistivity R_w . The uncertainty on R_w' is usually larger than this figure and it is acceptable for some calculations to simplify equation (6) by replacing R_w' by R_w .

Equation (7a) is the same as equation (6) but with conductivities instead of resistivities. In some of the developments made later in this paper we will use the approximation

$$\sigma \approx \sigma_w (S_w \phi - \chi_w)^\mu. \quad (7b)$$

As shown in Figure 4, the WCI controls the curvature of the resistivity index (RI) curve. The RI curve is concave down for negative values of the WCI and concave up otherwise. By definition, the resistivity index is the ratio of R_t to the resistivity R_o of the rock fully water saturated. This can be expressed by

$$RI = \left(\frac{1 - S_c}{S_w - S_c} \right)^\mu, \quad (8)$$

by introducing the critical saturation S_c defined by

$$S_c = \chi_w / \phi. \quad (9)$$

When S_c is positive (concave up curve) the resistivity takes very high values when the water saturation is reduced toward S_c . For example in Figure 4 the critical saturation $S_c = 0.12$ corresponds to the solid red curve with $\chi_w = +0.03$. Such an extreme behavior can be observed in strongly oil-wet rocks with very high oil saturations (at least 85 percent).

When the WCI approaches 0 the connectivity equation reduces to Archie’s equation with $n = m = \mu$. This is

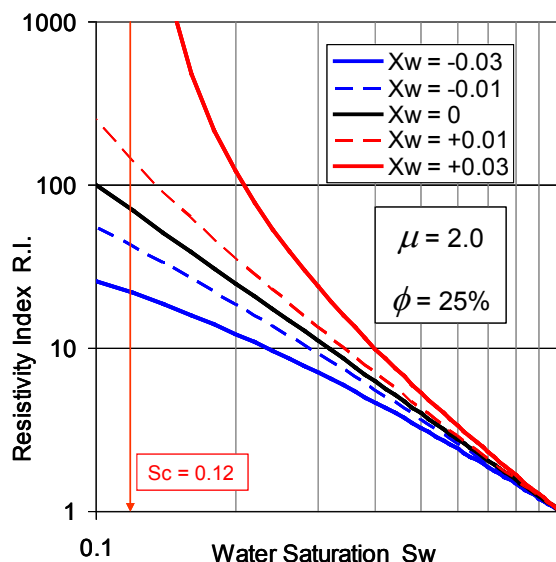


Fig. 4 Resistivity Index curves obtained from the connectivity equation with exponent $\mu = 2$. The parameter χ_w controls the curvature.

an important feature of this model. The large amount of experimental evidence showing the applicability of Archie's equation for many natural rocks makes it a requirement for any good petrophysical conductivity model to contain Archie's equation as the normal case.

The Case of "Clean" Water-Wet Sandstone Rocks

In the site percolation model (Figure 2a) water is distributed at random in space. The distribution of water in naturally porous rocks is actually much more constrained e.g. as shown in Figure 2b.

First, water is in the pore space and the porosity has a specific geometrical structure with spatial correlations. For example the porosity of sandstone results primarily of the stacking of sand grains having a certain size distribution resulting from erosion due to transport. The pore space has subsequently been submitted to the effect of the overburden pressure and cementation. A huge difference with the site percolation model is the fact that the porosity in sandstone rocks is almost entirely connected.

Second, wettability is a self-organizing mechanism that has a dramatic effect on the connectivity of fluid phases inside the pore space. For example "clean" sandstone rocks tend to remain strongly water-wet even when exposed to crude oil at reservoir temperature and pressure conditions. The surface of pores is always covered by a continuous film of water however small the water saturation may be. Reducing the water saturation results in a lower average thickness of this film but the connectivity of the water phase is essentially unchanged. Anderson (1986) published a very detailed, two-part, survey of the literature on wettability.

For a given rock sample these two properties – porosity entirely connected and water-wet pore surface – imply that the conductivity remains positive for any non-zero value of the water volume fraction however small S_w may be. Therefore, in that case χ_w must be negative or equal to 0 in equation (6). The value $\chi_w = 0$ leads to Archie's equation that applies well to most "clean" sandstone reservoir rocks. Note that the term "clean" here refers to clay-free rocks. The effect of clay will be discussed later in the article.

So even though there is a link between the connectivity equation and percolation physics, the WCI should not be interpreted in general as a percolation threshold. χ_w or S_c can be interpreted as such only for very specific cases such as – for example – strongly oil-wet rocks provided there is no other mechanism in the rock enhancing the connectivity of the water phase. Other authors have made similar comments. See for example Zhou et al. (1997), and Kennedy (2007) who calls the WCI a "pseudo"-percolation threshold (PPT).

THE MODIFIED CRIM MIXING LAW

Mixing laws have been introduced in effective medium theories to calculate a physical parameter of a mixture as a function of the volume fractions of the components of the mixture and the values of the parameter for each component.

The Complex Refractive Index Measurement (CRIM) mixing law was introduced by Birchak et al. (1974) to model the complex conductivity of mixtures at high frequency. See also Berryman (1995). Applied to the complex conductivity of a rock made of a mixture of k components with respective volume fractions x_1, \dots, x_k CRIM takes the form

$$\sigma^{1/2} = x_1\sigma_1^{1/2} + x_2\sigma_2^{1/2} + \dots + x_k\sigma_k^{1/2}, \quad (10)$$

with $x_1 + x_2 + \dots + x_k = 1$.

This mixing law was shown to give excellent predictions when compared to dielectric measurements on rocks – Seleznev et al. (2004). However, its application range goes all the way to zero frequency (DC). For example at DC, consider a porous rock made of a mixture of (1) solid grains, (2) water, and (3) oil.

$$\begin{aligned} \text{Then} \quad \sigma_1 &= 0, & x_1 &= 1 - \phi, \\ \sigma_2 &= \sigma_w, & x_2 &= S_w\phi, \\ \sigma_3 &= 0, & x_3 &= (1 - S_w)\phi \end{aligned}$$

Applying equation (10) we get $\sigma = \sigma_w (S_w\phi)^2$ which is Archie's equation in its simplest form (equation (1)). The treatment above is a naïve application of the CRIM mixing law which includes the assumption that water has the same connectivity throughout the rock and at all scales. The requirement for a good model to include Archie's law as a limiting case leads naturally to consider a modified version of the CRIM mixing law where the exponent 1/2 is replaced by $1/\mu$ for a certain value of μ :

$$\sigma^{1/\mu} = x_1\sigma_1^{1/\mu} + \dots + x_k\sigma_k^{1/\mu}. \quad (11)$$

A much more general form that could be considered is

$$f(\sigma) = x_1f(\sigma_1) + \dots + x_kf(\sigma_k), \quad (12)$$

for some continuous function, $f(\sigma)$. This equation is entirely symmetrical; i.e., all components are treated on an equal footing – a property that is highly desirable when modeling random mixtures. To make physical sense it should be possible to change units for conductivity and equation (12) should still be valid. In other words for any constant c one must have

$$f(c\sigma) = x_1f(c\sigma_1) + \dots + x_kf(c\sigma_k). \quad (13)$$

Functions satisfying equation (13) for all c are called ‘homogeneous’ or ‘scale independent’. We have proved – Montaron (2008) – that all mathematical solutions of this functional equation lead to equation (11); i.e., to satisfy equation (13) for all c the function f must be of the form $f(\sigma) = \sigma^{1/\mu}$ for some real number μ . This result gives equation (11) a very special status in the world of mixing laws because it is the only mixing law that is both symmetrical in its variables and homogeneous. Note that this includes the limiting case $\mu \rightarrow \infty$, which corresponds to the geometrical mean:

$$\lim_{\mu \rightarrow \infty} (x_1 \sigma_1^{1/\mu} + \dots + x_k \sigma_k^{1/\mu})^\mu = \sigma_1^{x_1} \dots \sigma_k^{x_k}.$$

The mixing law based on the geometrical mean is obtained with the function $f(\sigma) = \ln(\sigma)$ and is also called the Lichtenecker mixing law. The mixing law in equation (11) was first introduced by Lichtenecker and Rother (1931) to calculate the complex permittivity of mixtures.

APPLICATION TO SHALY SANDS

Now apply equation (11) for the modeling of shaly sands. I use the experimental data published by Clavier et al. (1984) in their paper on the Dual Water model. Clavier’s paper refers to the original work of Waxman and Smits (1968) and Waxman and Thomas (1974). Shown in Figure 5 is a thin section of a typical example of shaly sand. A schematic representation of the shaly

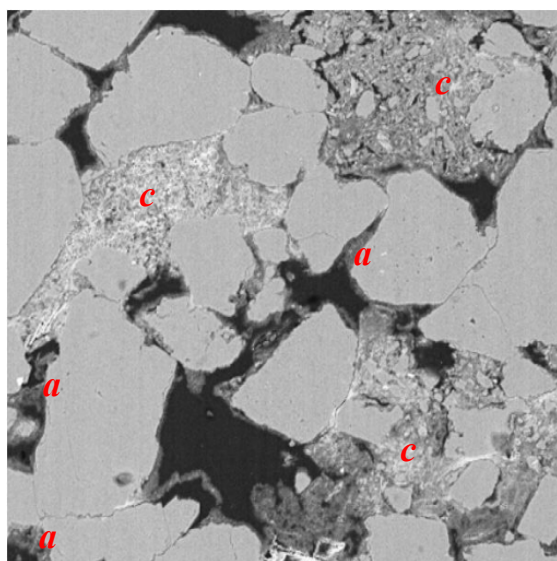


Fig. 5 SEM picture of a cretaceous sandstone in Texas. Abundant pore lining authigenic chlorite (a) and patchy pore filling chlorite (c) clay are visible. The dark color between quartz grains is epoxy.

sand reservoir rock model used here is shown in Figure 6. The clay-bound-water has a much higher conductivity compared to the ‘free’ brine contained in the pores.

Therefore, clay must be considered as a separate phase in the model. To simplify only four phases are assumed to be present in the medium. (In principle, different types of clays, with different conductivities associated to each type, might be distinguished.) Using Clavier et al. (1984) notation the bulk volume fraction of clay water is $S_{cw}\phi$ where ϕ is the total porosity, the conductivity of clay water is σ_{cw} , the bulk volume fraction of ‘free’ water is $(S_w - S_{cw})\phi$ where S_w is the total water saturation, and σ_w is the conductivity of ‘free’ water (brine).

The other two phases – solid quartz grains and oil – have zero conductivity. Equation (11) thus takes the form

$$\sigma^{1/\mu} = S_{cw}\phi\sigma_{cw}^{1/\mu} + (S_w - S_{cw})\phi\sigma_w^{1/\mu}, \quad (14)$$

$$\sigma = \sigma_w (S_w\phi - \chi_w)^\mu, \quad (15)$$

with

$$\chi_w = -S_{cw}\phi \left(\left(\frac{\sigma_{cw}}{\sigma_w} \right)^{1/\mu} - 1 \right). \quad (16)$$

Equation (15) is an approximation of equation (7b) with the WCI provided by equation (16). Since the conductivity of clay water is higher than the conductivity of brine, χ_w takes negative values; therefore, the RI curve is concave down. The corresponding Dual Water model is:

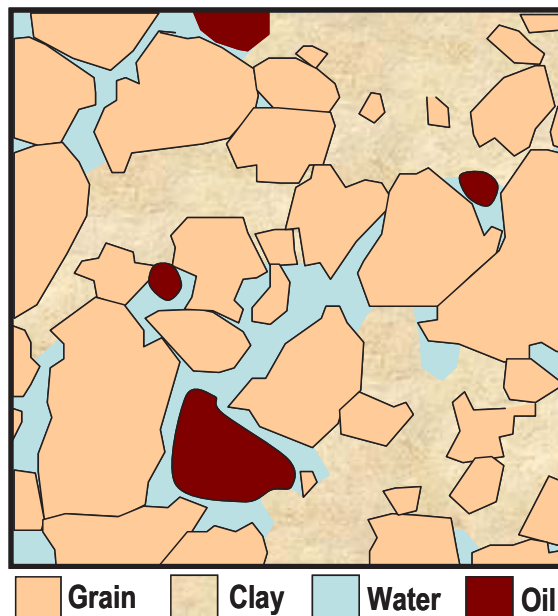


Fig. 6 This shaly sand model assumes that the medium is a mixture of 4 phases: grains, clay, water and oil. The conductivity of the clay bound water is different from the conductivity of free water.

$$\sigma = \sigma_w S_w^n \phi^m \left(1 + \frac{S_{cw}}{S_w} \left(\frac{\sigma_{cw}}{\sigma_w} - 1 \right) \right). \quad (17)$$

As per Clavier et al. (1984), the clay water saturation is equal to the product of the counterion concentration Q_v and not by v_o defined as the clay water volume associated with 1 unit of clay counterions (meq). Clavier et al. (1984) point out that in order to obtain correct values for the conductivity the parameter Q_v used in their model is required to be 25 percent higher than the true “chemical” counterion concentration measured in the lab. This “electrical” Q_v – as they call it – also takes slightly different values for the Dual Water and the Waxman-Smits models. Regardless of which case is considered, it is a fitting parameter.

Using the data in Clavier et al. $n = 1.77$, $m = 1.85$, $\phi = 20$ percent, $\sigma_w = 10$ mho/m, $\sigma_{cw} = 30$ mho/m at 100°C , the fit shown in Figure 7 is obtained by adjusting the conductivity exponent to $\mu = 1.81$ and the counterion concentration by +5.5 percent compared to the Dual Water model. With these values the water connectivity index is equal to $\chi_w = -0.0176$ and the critical saturation S'_c is -0.088 . The maximum relative error between the two curves is ± 3.4 percent.

It is shown here how the modified CRIM mixing law (11) can be used to combine four different phases to model the conductivity of a shaly sand reservoir rock. The application of the mixing law leads directly to the connectivity equation and provides an analytical expression (16) of the WCI as a function of formation properties. The agreement between this model and Waxman-Smits and Dual-Water models is remarkable.

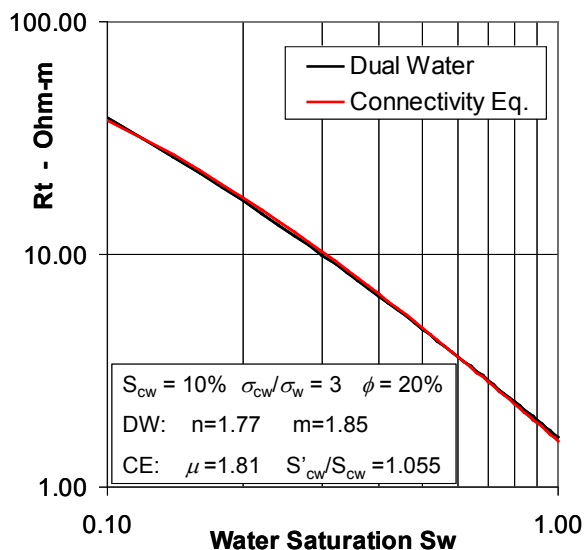


Fig. 7 Comparison between the Dual Water model for shaly sands and the connectivity equation. The fit is done by adjusting counterion concentration (+5.5 percent).

However, all three models use values of the counterion concentration that are not equal to measured chemical counterion concentrations. This is an indication that these models are incomplete. It is however, possible to develop a more sophisticated connectivity model that allows use of the true counterion concentration.

OIL-WET CARBONATE ROCKS

A detailed approach for modeling oil-wet rocks is described in Montaron (2007). Start by considering the model shown in Figure 8. The space can be divided up in ‘oil-wet cells’ and ‘water-wet cells’. Oil-wet cells have 100 percent of the solid surface covered by oil. Water-wet cells have 100 percent of the solid surface covered by water. Presented here is a different way to partition the pore space, as compared to Montaron (2007).

The oil-wet cells are shown in Figure 8 with red boundaries. They are entirely contained in the porosity. The cells are polyedra constructed with respect to the center of gravity of surrounding grains. The water-wet cells are the rest of the porosity with black boundaries.

The pore volume fraction of oil-wet cells is x_o and the pore volume fraction of water-wet cells is $1 - x_o$. According to their definition, water-wet cells and oil-wet cells have porosity 1 but they may have different average fluid saturations. Such a “clean” medium made only of water-wet cells – shown in Figure 9 – follows the connectivity equation with a WCI equal to zero; i.e., it follows Archie’s equation.

A medium made only of oil-wet cells (Figure 9) cannot conduct electricity unless the amount of water in the system is sufficient for isolated water blobs to become connected. This medium has a clear percolation behavior with a percolation threshold for the water volume fraction

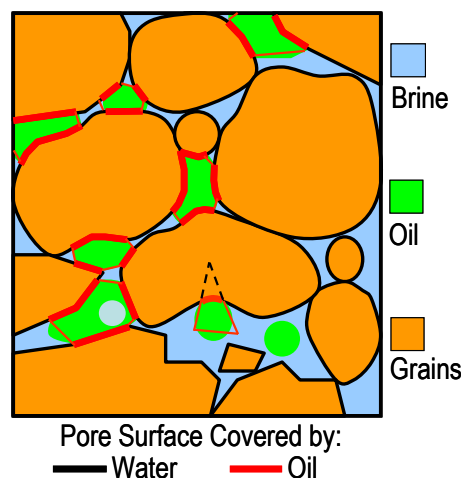


Fig. 8 A simple model for oil-wet porous rocks. The volume is considered a mixture of “water-wet cells” (black boundaries) and “oil-wet cells”(red boundaries).

equal to

$$\chi_w = \chi_{w0} = S_{c0}\phi. \tag{18}$$

χ_{w0} is the maximum possible value for the WCI and S_{c0} is the maximum possible value for the critical saturation. S_w must be higher than S_{c0} for the medium to be conductive. From initial results presented in Montaron (2007) it seems that S_{c0} is of the order of 60 percent to 70 percent.

Oil-wet natural rocks, even strongly oil-wet rocks, are far from having 100 percent of their pore surface covered by oil. In most cases the surface fraction covered by oil is actually very small – typically a few percent. The reason is that micro pores are generally fully water saturated and water-wet. Only macro pores and a fraction of the meso pores can be oil-wet. However, the vast majority of the pore surface area is in micro pores, while the majority of the pore volume is in macro and meso pores. This is a direct consequence of the pore size distribution. The medium described above is a mixture of three phases (solid grains, oil-wet cells (ow), water-wet cells (ww)) with the following parameters:

$$\begin{aligned} \sigma_{sol} &= 0, & x_{sol} &= 1 - \phi, \\ \sigma_{ow} &\approx \sigma_w (S_w^{ow} - S_{c0})^\mu, & x_{ow} &= x_o \phi, \\ \sigma_{ww} &= \sigma_w (S_w^{ww})^\mu, & x_{ww} &= (1 - x_o) \phi. \end{aligned}$$

The total water saturation is equal to

$$S_w = x_o S_w^{ow} + (1 - x_o) S_w^{ww}.$$

Applying equation (11) to the conductivities and volume fractions of the three phases leads to the connectivity equation with a WCI equal to

$$\chi_w = x_o S_{c0} \phi. \tag{19}$$

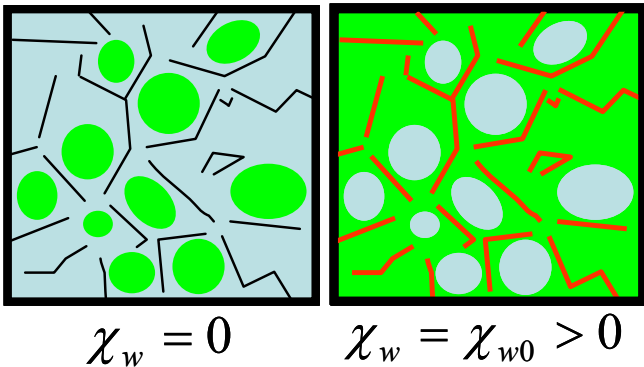


Fig. 9 The water-wet medium follows Archie’s law; i.e., $\chi_w = 0$. A 100 percent oil-wet medium exhibits a large positive percolation threshold χ_{w0} . In these media porosity is 1, only pore boundaries are present.

This result is consistent with Montaron (2007) with a different space partitioning method. It can be shown that x_o remains identical in both methods.

According to equation (19) a medium with 20 percent porosity and an oil-wet bulk volume fraction of 60 percent as defined in Figure 8 would have a WCI of the order of +0.07 and a critical saturation S_c equal to 36 percent. Such a large positive value of the WCI would create a pronounced concave up curvature for the RI curve. That type of extreme behavior can be created artificially on carbonate cores.

A famous data set from Sweeney and Jennings (1960) is shown in Figure 10 in comparison to the connectivity equation. The RI curves shown are based on equation (8) with various values of the critical saturation S_c . The black straight line corresponds to Archie’s equation ($S_c = 0$) and fits well the water-wet data points. The dotted and solid black curves correspond to positive values of S_c up to 0.5. Sweeney and Jennings used rocks from the same carbonate formation for these experiments. The black dots were obtained on a rock rendered completely water-wet using a chemical treatment. The blue and red dots correspond to the same rock type made strongly oil-wet. The blue and red dots match nicely the connectivity

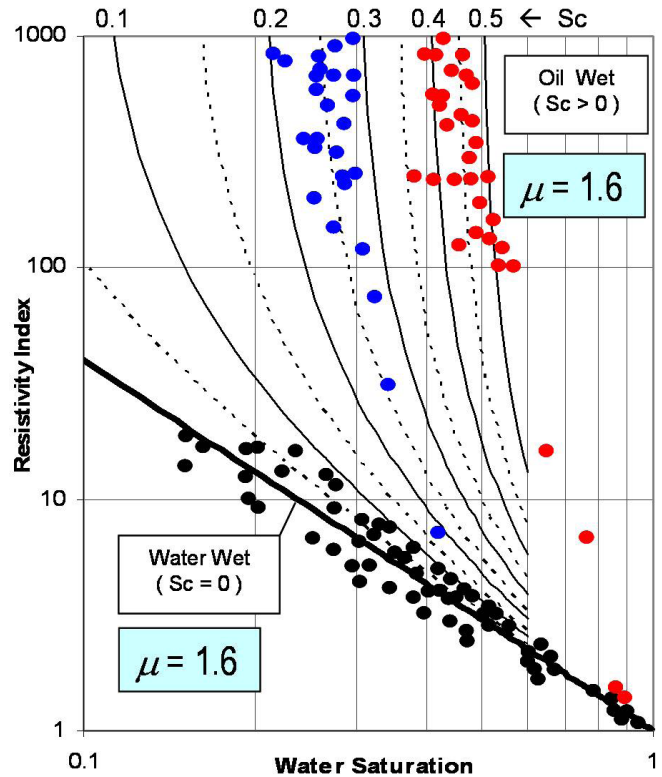


Fig. 10 Data from Sweeney and Jennings (1960) compared to the connectivity equation. The same exponent $\mu = 1.6$ can be used to model a rock made either water-wet or strongly oil-wet.

equation with S_c close to 0.25 (blue dots) and 0.5 (red dots). It is remarkable here that in all cases the same conductivity exponent $\mu = 1.6$ is used. Modeling the black dots with Archie's equation requires $n = 1.6$ for the black dots, $n = 8.6$ for the red dots, and $n = 4.8$ for the blue dots with a poor match in this last case due to the curvature that cannot be represented by Archie's equation.

Archie Exponents Revisited

One way the saturation exponent n can be related to the parameters of the connectivity equation is by equating equation (8) to the RI derived from Archie's (equation (2)):

$$n = \mu \frac{\ln(S_w - S_c) - \ln(1 - S_c)}{\ln S_w}. \quad (20a)$$

Using the example shown in Figure 10 for $S_c = 0.25$ equation (20a) gives $n = 2.13$ at $S_w = 1$ and $n = 5$ at $S_w = 0.26$ just above the critical saturation. Another method is to use the logarithm derivative

$$n = -\frac{d \ln(RI)}{d \ln S_w} = \mu \frac{S_w}{S_w - S_c}. \quad (20b)$$

Equation (20b) gives the local slope n at the point (RI, S_w) . Equation (20a) gives the slope of the line joining the point $(1,1)$ to the point (RI, S_w) . Note that for both expressions the limit value of n at $S_w = 1$ is $\mu / (1 - S_c)$. For small values of $|S_c|$ equation (20b) with $S_w = 0.5$ gives a good approximation of the best matching n for Archie's law. Similarly one can find the expression for m (Montaron, 2007):

$$m = \mu \frac{\ln(1 - S_c) - \ln(1/\phi - S_c)}{\ln \phi}. \quad (21)$$

For example assuming 30 percent porosity, $\mu = 1.6$ and $S_c = 0.25$, then $m = 1.88$. The high stability of the conductivity exponent and the fact that it does not depend on S_w makes the connectivity equation model simpler to use than Archie's equation.

MIXED-WET MICRITIC CARBONATES

The simple model presented above for oil-wet rocks leads systematically to a concave up curvature for the RI curve. The connectivity of water is always reduced by the presence of oil-wet surfaces that create "cuts" in the continuity of the water film.

However, some mixed-wet carbonate rocks do not have RI plots with concave up curvature. In fact some mixed-wet carbonate rocks nicely follow Archie's law (no curvature) and some can even present a concave down curvature, similar to shaly sandstones! The explanation of this apparent paradox requires a detailed analysis

of the pore structure of carbonate rocks and of the connectivity of water in these systems. Swanson (1985) was one of the first to underline the influence of microporosity on the conductivity of rocks. Petricola et al. (2002) have proposed a mixing model for micritic carbonate rocks based on a partition of the pore size distribution in 3 groups: micro, meso and macro pores.

This type of partitioning has been widely adopted by the industry for carbonate petrophysical modeling. Considered here is a model for micritic carbonates based on this partition. Sen et al. (1981) have shown that the conductivity exponent for sphere packings is equal to 1.5. This is also the value known for micrite crystal packings found in micritic carbonate rocks as shown in Figure 11. The size of these crystals is limited to a few microns. The sub-micron pores are so small that very high capillary pressure is required for oil to penetrate them, so high in fact that micritic grains generally remain fully water saturated and perfectly water-wet, at least in formation layers located not too far above the oil-water contact. The low value of the conductivity exponent for micritic grains is due to the high connectivity of the water films in these micro-pores.

A reasonable model for micro-pores is: x_m = bulk volume fraction of micritic grains; ϕ_m = porosity inside micritic

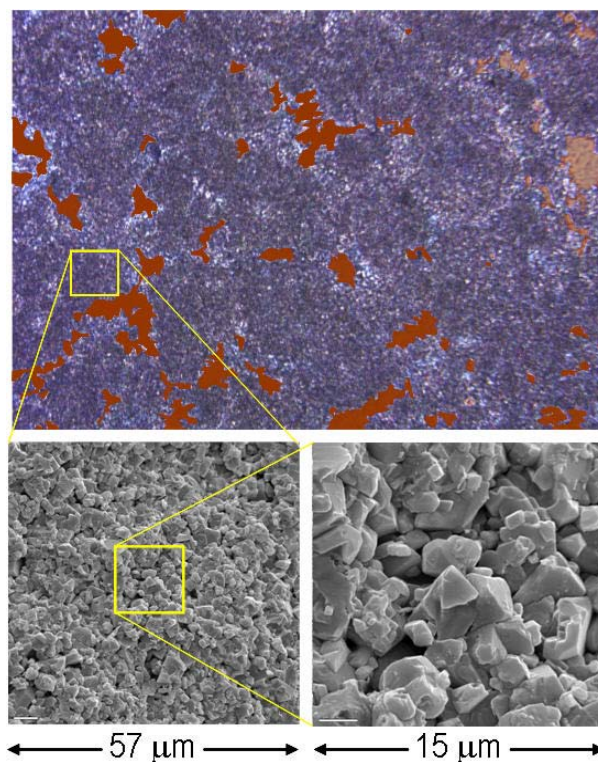


Fig. 11 Thin section of a micritic limestone from the Thamama formation (Abu-Dhabi). The oil patches were added to the picture. Grains are made of tight packings of fully water saturated micrite crystals.

grains; $\mu_m = 1.5$ (conductivity exponent of micritic grains); $S_{wm} = 1$ (fully water saturated); and $\chi_{wm} = 0$ (Archie's equation). The conductivity of micritic grains is $\sigma_m = \sigma_w \varphi_m^{1.5}$.

The oil is contained in meso and macro pores; i.e., pores other than pores located in micritic grains. With this simplified model the meso-macro porosity φ_M must be equal to or lower than $1 - x_m$. The remaining volume fraction $1 - x_m - \varphi_M$ corresponds to solid grains (e.g., dolomite crystals or large calcite crystals) that may be present. The total porosity is $\varphi = x_m \varphi_m + \varphi_M$.

Meso-macro pores have water-wet and oil-wet surfaces. The same definitions as in Figure 8 and Figure 9 are used to define the meso-macro pore volume fraction x_o of oil-wet cells. Finally, this model is a mixture of four phases (1) micritic grains, (2) oil-wet cells, (3) water-wet cells, and (4) solid grains, i.e., grains with no micro-porosity. Its parameters are

$$\begin{aligned} x_1 &= x_m, & \sigma_1 &= \sigma_w \varphi_m^{\mu_m}, \\ x_2 &= x_o \varphi_M, & \sigma_2 &\approx \sigma_w (S_w^{ow} - S_{c0})^\mu, \\ x_3 &= (1 - x_o) \varphi_M, & \sigma_3 &= \sigma_w (S_w^{ww})^\mu, \\ x_4 &= 1 - x_m - \varphi_M, & \sigma_4 &= 0, \end{aligned}$$

where the approximation given by equation (7b) is used for the conductivity, σ_2 , of oil-wet cells. Also assume that the exponent μ in the mixing law given by equation (11) is the exponent of the meso-macro pore network. Water saturations in oil-wet and water-wet cells are S_w^{ow} and S_w^{ww} . The total water volume fraction is equal to

$$S_w \varphi = x_o \varphi_M S_w^{ow} + (1 - x_o) \varphi_M S_w^{ww} + x_m \varphi_m.$$

Using the expression for $S_w \varphi$ above in equation (11) leads to the connectivity (equation (7b)) with

$$\chi_w \approx x_m (\varphi_m - \varphi_m^{um/m}) + x_o S_{c0} \varphi_M. \quad (22)$$

Equation (22) can be used in equation (7a) to get the final expression for the conductivity of the medium.

Numerical Example

With $\varphi_m = 0.15$, and the bulk volume fraction of micritic grains $x_m = 0.70$, the total micro-porosity is $x_m \varphi_m = 0.105$. Assuming a total porosity of the rock sample $\varphi = 0.25$, the meso-macro porosity is $\varphi_M = 0.145$ and the volume fraction for solid grains is 0.155. Using $\mu = 1.905$, $\mu_m = 1.5$, $S_{c0} = 0.6$, and $x_o = 0.60$, then $\chi_w = 0.0$; i.e., despite the fact that 60 percent of the meso-macro pore volume is oil-wet, the presence of a large amount of micrite compensates exactly the effect of oil-wetness and Archie's equation

applies beautifully. Of course the value $\mu = 1.905$ was chosen here on purpose to get $\chi_w = 0$ in order to make the point.

Using $R_w = 0.05$ ohm-m and the numerical data above, the resistivity of this formation is $R_t = 1.98$ ohm-m. The water contained in the micro-pores is non-movable, and the oil saturation in the meso-macro pore volume is 70 percent and even higher in macro pores. This is a typical example of low resistivity pay (LRP) carbonate formation such as described by Petricola (2002).

Discussion: Archie or Not Archie?

The model presented above for mixed-wet micritic carbonates is a simple model. For example, it does not account for vugs that may be present in sufficient quantities to change significantly the petrophysical model. Vugs could have been accounted for by adding another term in the WCI resulting from the addition of a "vug component" in the mixing model.

However, equation (22) is a quantitative model that greatly helps understand water connectivity effects in these rocks. The presence of oil-wet surfaces in meso-macro pores reduces the connectivity of water and the corresponding WCI term is equal to +0.052 in equation (22). Such a large WCI should generate a very pronounced concave up curvature if grains had no micro-porosity. But the presence of micro-porosity in micritic grains and the fact that the conductivity exponent of micrite crystal packings is only $\mu_m = 1.5$ generates a WCI term that is negative. In the numerical example chosen above the two terms compensate exactly and Archie's equation 'works' with $n = m = 1.905$! With different numbers all possible curves shown in Figure 4 can be obtained: Concave up or down and straight lines.

The conductivity of micritic carbonates can behave like shaly sandstones. They can also behave like strongly oil-wet carbonates if, for example, the micrite volume fraction is not sufficient to compensate the effect of oil-wetness. And they can also follow Archie's equation in all situations where χ_w is close to 0; e.g., in the range $-0.005 \leq \chi_w \leq +0.005$.

The fact that Archie exponents n and m , as well as the conductivity exponent μ , are generally close to 2 is correlated with the CRIM mixing law exponent (1/2). This also has to do with the critical conductivity exponent for 3D site percolation – equal to 2 – and the Kirkpatrick EMT model – a polynomial of degree 2!

Application to a Middle East Carbonate Reservoir

When SCAL data is unavailable, or unconvincing, a good start is to apply equation (6) or equation (7) with $\mu = 1.9$ to 2.0, and to use log data to determine the WCI. This

procedure was used on a data set from a Middle East carbonate reservoir.

The water saturation, the porosity and the resistivity of the formation must be known in order to calculate χ_w from equation (6). Unfortunately, the methods available to estimate water saturation independently of deep resistivity rely on instrument responses influenced by drilling-induced changes in formation properties. The deepest response is from the neutron capture sigma tool with about a 6-inch depth of investigation. At “wireline time” the near-wellbore formation logged has probably been completely invaded with mud filtrate. Jia, Buckley and Morrow (1994) concluded that surface-active components used in water based muds adsorb on clays and are removed from the filtrate by formation of the mud cake. Changes in wettability due to mud filtrate invasion are therefore, expected to be minor in carbonate rocks drilled with water based muds. On the other hand, wettability alteration would be severe for oil-based muds due to oil-wetting surfactants used in these fluids. Fortunately, most carbonate rocks, at least in the Middle East, are drilled with water based muds.

The water saturation was provided by an EPT tool. This high frequency tool measures the dielectric constant of the formation that is a direct function of the water saturation. This very shallow measurement was combined with the resistivity provided by an R_{xo} tool.

The WCI was calculated using the equation

$$\chi_w \approx S_{xo} \phi - (R_{mf} / R_{xo})^{1/\mu}, \quad (23)$$

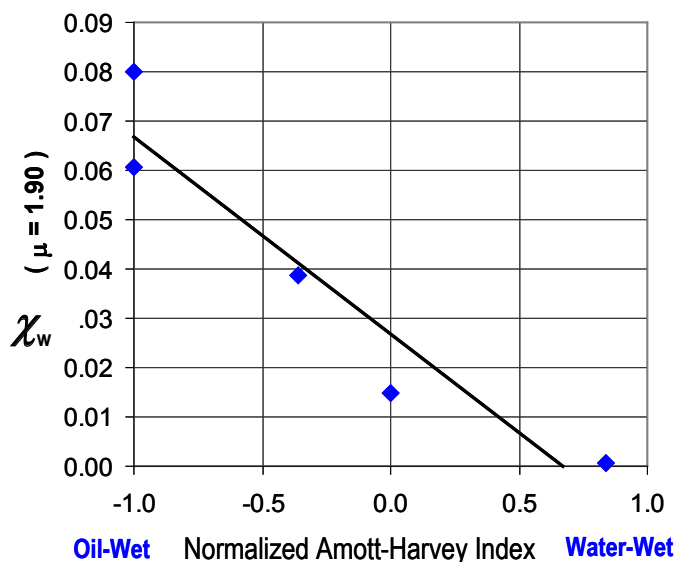


Fig. 12 Normalized Amott-Harvey wettability index measurement on 5 cores compared to the WCI derived from wireline logs in a Middle East carbonate reservoir. A good correlation is observed.

where R_{mf} is the resistivity of the mud filtrate, known accurately, and where $\mu = 2$.

The χ_w and S_c logs are shown in Figure 13. In order to highlight oil-wet zones the intervals with negative WCI or S_c are truncated. Only positive spikes are shown. The depth is increasing from left to right. The light blue curves are the 100ft sliding averages for each parameter, and show a normal trend, with high values towards the top of the reservoir and low values at the bottom of the reservoir showing a water-wet zone close to the oil-water contact.

The logs show a layered reservoir structure with strongly positive WCI zones alternating with mixed-wet zones and some layers that appear to be water-wet with χ_w close to zero at the top of the zone, in the middle and at the bottom. A complete suite of logs was run in this well and an ELAN interpretation made.

Figure 14 shows the agreement between the ELAN and the S_c logs. Tar zones were identified in ELAN by assessing non-movable hydrocarbons using NMR measurements independent of the ones used for the determination of χ_w and S_c .

Tar zones are known to be strongly oil-wet. The tar mat at x100 ft identified with ELAN correlates perfectly with the WCI positive spike shown by the S_c log. The tight limestone bed at x800 ft contains only water and correlates with a water-wet marker on the S_c and χ_w logs.

All the apparently water-wet zones at the bottom of the reservoir are indeed confirmed to contain mostly water by ELAN. The apparently water-wet layers at the top of the reservoir actually have negative values of χ_w ; approximately -0.01 . This requires an explanation.

A comparison of the S_c log with an ECS log (nuclear spectroscopy tool for mineralogy identification) is shown in Figure 14. All the layers with slightly negative χ_w agree perfectly with anhydrite layers. This is easy to understand from equation (23). The porosity of anhydrite is zero. R_{mf} was approximately 0.025 ohm-m. Although the resistivity of anhydrite is expected to be extremely high – i.e. mega-ohm-m – R_{xo} being a very shallow measurement measured only 250 ohm-m due to the influence of the salty mud in the borehole. We should have found $\chi_w = 0$ in front of each anhydrite layer, but instead we found the value provided by equation (23); i.e., $-(0.025/250)^{1/2} = -0.01$. This shows that a zero value for χ_w or S_c can also correspond to a very tight zone with zero porosity. In practice this could be taken care of by applying a cut-off to ignore anhydrite layers.

Figure 14 shows all the other tar layers identified with ELAN. They all match positive WCI and S_c spikes. Five cores from this well were tested for wettability in the lab using the Amott-Harvey method (see Anderson, 1986, his second citation). A good linear correlation was obtained between χ_w and the normalized Amott-Harvey index

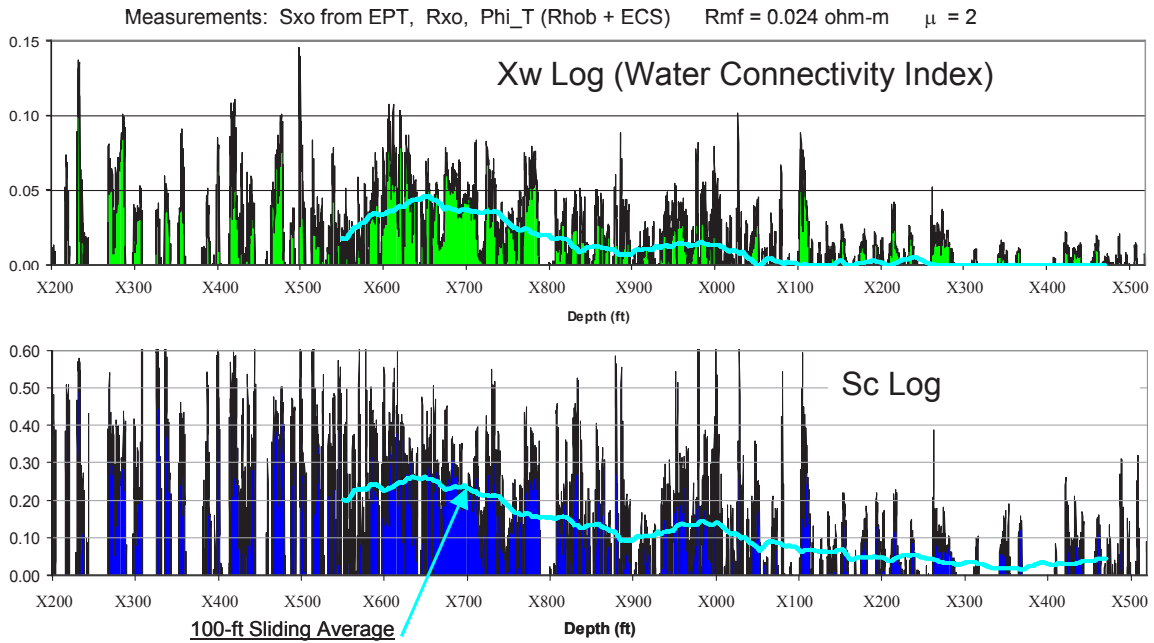


Fig. 13 First χ_w and S_c logs. Light blue curves are 100-ft sliding averages of χ_w and S_c . The trend is normal: Oil-wet at the top of the reservoir and water-wet at the bottom. The highest S_c spikes top at about 0.60. Only the positive values of χ_w and S_c are shown to highlight oil-wet zones with simple water-based muds.

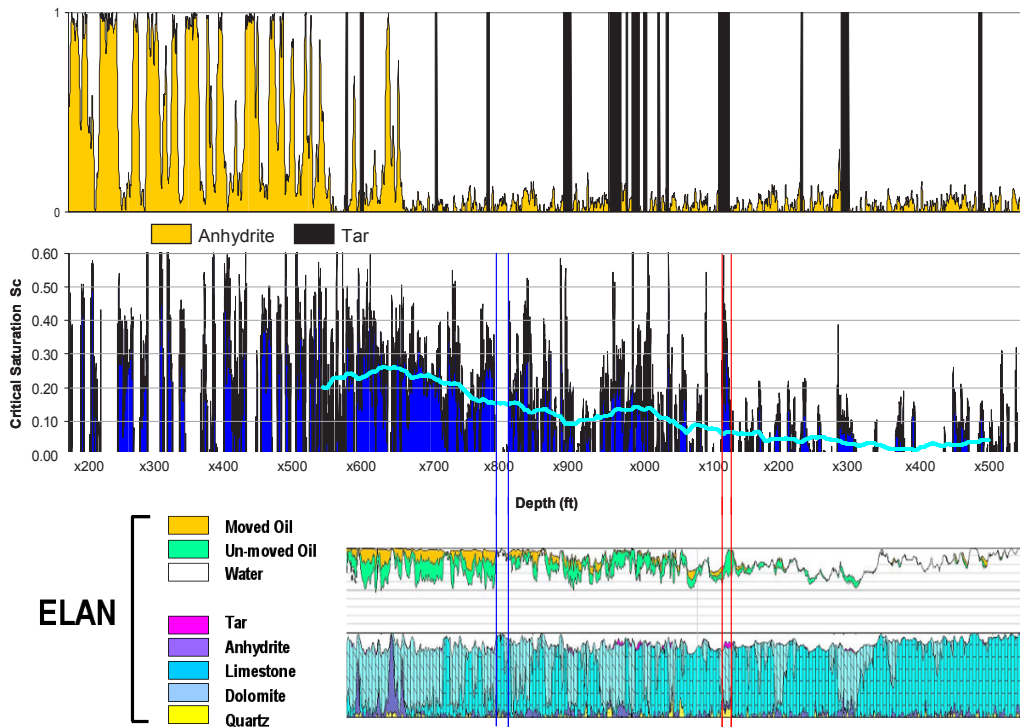


Fig. 14 The tar mat at x100 ft identified with ELAN correlates perfectly with the WCI spike shown by the S_c log. The tight limestone bed at x800 ft contains only water and correlates with a water-wet marker on the χ_w and S_c logs. All the tar beds identified by ELAN correlate with oil-wet spikes on the χ_w and S_c logs. Anhydrite beds detected by the ECS tool correspond to slightly negative values of χ_w (and S_c) due to borehole effects on R_{xo} measurements.

using $\mu = 1.9$ (Figure 12). This correlation is well in line with the model (19), or the more elaborate model (22), expressing the WCI as a linear function of the oil-wet volume fraction in the rock.

CONCLUSION

The connectivity equation provides a nice and practical alternative to Archie's equation. It is a simple equation: Like Archie's equation it has only two petrophysical parameters – the conductivity exponent and the water connectivity index. It reduces to Archie's equation in the limit $\chi_m \rightarrow 0$, and it allows the modeling of non-Archie rocks such as strongly oil-wet rocks and shaly sands. I have shown that the connectivity equation is consistent with the modified CRIM mixing law.

The connectivity equation and the modified CRIM mixing law are the two main analytical tools used in the theory of connectivity. A porous formation is described as a mixture of various solid and fluid phases, each phase being characterized by its volume fraction in the medium and its electrical conductivity. This approach accounts for variations of water connectivity among the different phases. The method was successfully applied to develop analytical models for the conductivity equation of rock types as different as shaly sands, oil-wet rocks, and mixed-wet micritic carbonates. All these rocks were shown to follow the connectivity equation. The conductivity exponent in this model seems much more stable than Archie exponents n and m . The variability in the connectivity theory is transferred to the water connectivity index (WCI). However, this parameter can be expressed analytically as a function of rock properties such as wettability, clay content or the pore structure of rocks.

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