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THE INITIATION AND DEVELOPMENT OF ASYMMETRICAL BUCKLE FOLDS IN NON-METAMORPHOSED COMPETENT SEDIMENTS

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SUMMARY

This paper presents a relatively simple analysis, based on engineering principles, to explain the initiation and development of the type of fold structures which are most commonly observed in the non-metamorphosed zones of the crust; namely, asymmetrical chevron folds or asymmetrical flexures with rounded hinges but long straight limbs. The development of anticlinoria and synclinoria is also considered.

INTRODUCTION

The mode of development of buckle folds has interested geologists for many years. Theoretical analyses of the problem have been carried out by a number of people, more especially, in recent years by Ramberg (1959, 1960, 1961, 1963) and Biot (1965). In these analyses the buckling of a single competent unit, or series of such units, set in a less competent environment is treated as an exercise in either elastic or viscous theory. Compression of the competent unit is assumed to take place along its length; and the resulting folds are invariably upright and symmetrical (by which terms the author means the axial planes are vertical and the lengths of the limbs which comprise any one fold are equal). Also, the shapes of the crests and troughs of the folds are well rounded. Indeed, the final form of the fold profile is often sinusoidal, so that the limbs exhibit no straight portions.

The basic assumption that rock behaves as a viscous material probably holds as a reasonable approximation in the environments of high temperature and confining pressures obtaining during high grade metamorphism. Confirmation of the accuracy of the assumption and of the mode of analysis is obtained from the fact that the shapes of the folds which result from these analyses are often in good agreement with the fold forms observed in metamorphic rocks.

However, outside the zones where metamorphism is taking place, where temperatures and confining pressures are lower, the assumption that competent rock behaves as a completely viscous material is not tenable. The assumption that such rocks will behave as completely elastic materials, even when the strains are not infinitesimal, is also obviously invalid. It is not surprising, therefore, that the shapes of the folds predicted by theory commonly show considerable divergence from those exhibited by the natural folds which develop in competent units in the upper, non-metamorphic zones of the crust.

In profile, the folds in such zones are almost invariably asymmetrical. (Here, the author takes the ratio of the length of the long limb, L_l, to the short limb, L_s, to represent the degree of asymmetry; the higher the ratio, the higher is the degree of asymmetry). Also, it is usually found that some portion of the fold flanks are straight. Indeed, in chevron folds, the whole length of the limb is straight.

Published data which give the ratio of the length of limbs from field observations are scarce; however, by referring to Fig.1. and comparing these shapes with various sections taken from field observations, it is suggested that competent buckle folds which develop in non-metamorphosed rocks rarely exhibit a limb-length ratio of less than 1.5 : 1. The majority of folds exhibit limb-length ratios of between 2 : 1 and 4 : 1, and folds which exhibit ratios of greater than 5 to 6 : 1 are also infrequently observed. Moreover, when the ratio of limb lengths is small, the axial plane of the structure is almost invariably inclined.

However, it must be pointed out that throughout a section, taken in a plane normal to the fold axes, the degree of asymmetry of successive folds is rarely completely constant; neither are the dips of the corresponding long or short limbs of successive folds necessarily similar, nor are the acute angles separating adjacent limbs constant. Indeed, it is frequently observed that the only feature of a fold belt which exhibits any degree of regularity is that the axial planes of the various folds are usually "sub-parallel".

Again, the extent to which axial planes in one group of folds diverge from being parallel is rarely specified in the literature. However, from general field observations it is suggested that the divergence is commonly less than 15°.

It is often assumed in geological literature that the axis of greatest

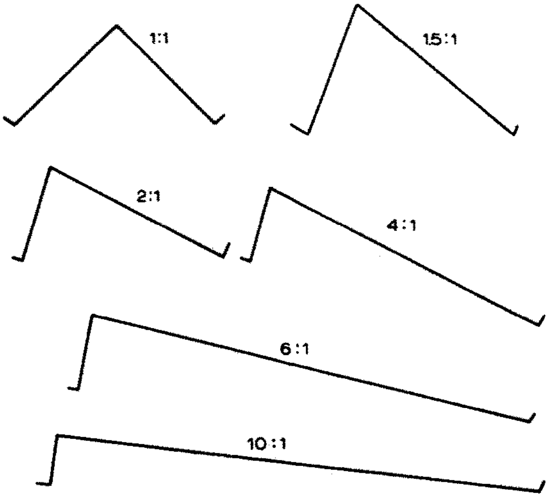

Fig.1. Diagrammatic representation of chevron folds showing various ratios of limb lengths from 1 : 1 to 10 : 1.

principal stress, which caused the folding, acted approximately at right angles to the axial plane. In analyses presented hitherto, this relationship presents no difficulty, for it is considered in these various analyses that the principal compressive stress acted parallel to the length of a horizontal competent unit; and because the folds which result from such compression are symmetrical upright structures, the axial plane of the fold will be exactly at right angles to the initiating stress. The questions which present themselves when folds are asymmetrical are: (1) does the axis of greatest principal stress[1] act at right angles to the axial plane and (2) if this relationship exists, how does it come about?

In the sections which follow, the author briefly considers the behaviour pattern of rock in the non-metamorphic zones of the crust and then conducts an analysis to show how the initiation of asymmetrical buckle folds with straight limbs is brought about.

This is followed by a discussion of the manner in which, during its subsequent development, the axial plane of an asymmetrical fold may become approximately normal to the axis of greatest principal stress.

RHEOLOGICAL BEHAVIOUR OF ROCK IN THE UPPER CRUST

The application of viscous theory to the folding problem is based on the assumption that rock behaves as a viscous Newtonian liquid. There is, however, much evidence which indicates that, in the upper layers of the crust, competent rocks have the properties not of a liquid but of a solid. The evidence and arguments leading to this conclusion have been given elsewhere in detail (see Price, 1966) and need not be repeated here. The rheological model which has been derived and which applies to competent rock for the relatively short time it takes a fold to develop is represented in Fig.2A, while the time–strain and stress–strain relationships which such a model exhibits are shown in Fig.2B and C.

It can be inferred from these figures that the important feature of a solid is that it possesses a "yield strength". If such a material is subjected to stresses which are lower than the yield value, deformation is wholly elastic; and this behaviour is independent of the length of time during which the load is applied. When the level of stress reaches that of the yield strength, the behaviour pattern changes, for, depending upon the environmental conditions, the material will fail as a result of brittle, or semi-brittle rupture, or it will deform in a plastico-viscous, ductile manner.

Quantitative data for the long-term yield strength of competent rock in the environmental conditions which obtain in the upper layers of the crust have not yet been obtained, and almost certainly will not be available for many years to come. However, enough experimental data are at hand to enable one to suggest that the yield strength of a competent compacted or cemented quartzose rock, at a temperature of 150–200°C and confining

[1]The orientation of the principal stresses in and adjacent to the competent unit will, of course, be profoundly influenced by the disposition of the unit itself. Hence, when it is stated that the principal compressive stress acted normal to the axial plane of a fold it is tacitly assumed that the stress field is being considered as a statistical average, rather than in the precise sense that throughout any fold the axes of maximum principal stress are everywhere in the fold exactly normal to the axial plane.

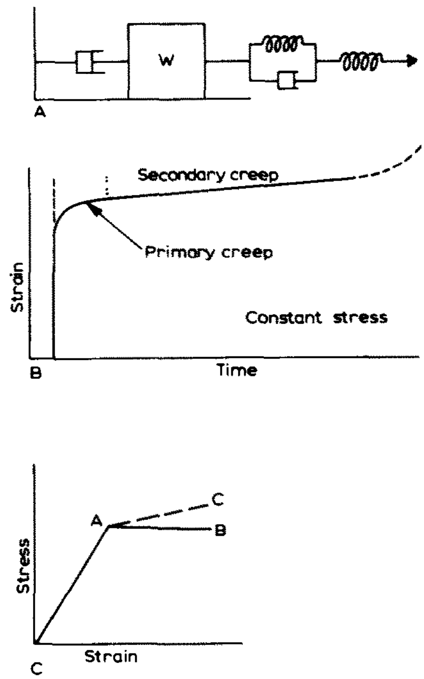

Fig.2A. Diagrammatic representation of a Bingham–Voigt model. B. Time–strain relationship, at constant stress, exhibited by a Bingham–Voigt model. C. Long-duration stress–strain relationship for a Bingham–Voigt model obtained at constant stress (*AB*) and when stress increases with time beyond some initial value (*AC*).

pressure of approximately 20,000 lb./sq.inch. is almost certainly greater than 50,000 lb./sq.inch. and may easily be in excess of 100,000 lb/sq.inch. These conditions are those which may obtain at a depth of approximately 20,000 ft.

The behaviour of rock when submitted to a tensile stress is similar in many ways to its behaviour in compression; however, the tensile strength of competent rock is very much smaller than the strength of the same rock type in compression. Thus, the uniaxial long-term yield strength in extension is of the order of 1,000–2,000 lb./sq.inch. and appears to be relatively little influenced by moderate values of confining pressure.

The competence of any particular unit is controlled mainly by its mineralogical composition and its diagenetic history. Although much work needs to be done on the rheological properties of individual minerals, it seems probable that the minerals most resistant to deformation are in the felspar group. These are followed in order of increasing ease of deformation by quartz, dolomite, calcite and some members of the clay-mineral group. Hence, arkoses and quartzites are among the most competent sediments.

The competence of a unit, for any specific composition will be largely determined by the degree of cohesion of the component minerals. An openstructured, porous unit will be less competent than a unit of low porosity in which component grains are in contact and cohere along the whole of their boundaries. The intergranular cohesion is affected by one of two processes, namely cementation and compaction.

Units which are cemented may attain their full competence soon after deposition, at a time when surrounding incompetent material may exhibit little or no signs of compaction. In such instances, the differences in the elastic or strength properties of the competent and incompetent materials may be truly enormous. When cohesion is brought about by compaction, however, the process is a progressive one. The cohesion will reach a maximum when the main factors causing compaction, namely temperature and pressures, due both to depth of burial and tectonic processes, have their maximum effect. Thus, in general, a competent unit only reaches its maximum compaction and hence competence when tectonic deformation ceases.

Because the process of progressive compaction applies to neighbouring incompetent units as well as to the competent units, the differences in competence of the two types of unit is likely to be very much smaller than the differences which obtained when the competent unit was cemented. Moreover, the continually variable physical "constants" which will be associated with such progressive compaction mean that a rigorous mathematical analysis of the buckling problem is rendered extremely difficult.

Where the competent unit has been cemented one may assume that the values of the various physical constants were in fact reasonably constant throughout the whole of the unit's deformational history. However, because the neighbouring incompetent material is often uncemented it will undergo progressive compaction during the deformation; so that one must bear in mind that the physical "constants" of these latter units will vary progressively.

The problem of buckling will be considered in the following sections in the light of the conclusions expressed above regarding the physical behaviour of competent and incompetent sediments in the upper layers of the crust.

THE INITIATION AND DEVELOPMENT OF SYMMETRICAL FOLDS

Before dealing with the development of asymmetrical folds it is first necessary to establish certain principles which are most readily illustrated by considering the mechanism of formation of perfectly symmetrical buckles.

From the remarks made in the preceding section regarding the rheological behaviour of competent rock, it is clear that one may consider the initial deformation of a single cemented competent unit as a problem in elasticity. In this analysis a competent unit of semi-infinite extent is assumed to be buckled into a periodic wave form. Attention in the subsequent analysis is centred upon a single wavelength within the wave-train. Moreover, if the adjacent incompetent material is uncompacted and extremely weak one may assume, as a first approximation, that it has negligible effect on the buckling of the competent layer. Thus, although relatively high pressures may act normal to the surfaces of the competent unit, one can estimate the critical force (p_{crit}) needed to initiate buckling from the Euler equation:

$$p_{crit} = \frac{\pi^2 E \cdot I}{L^2} \tag{1}$$

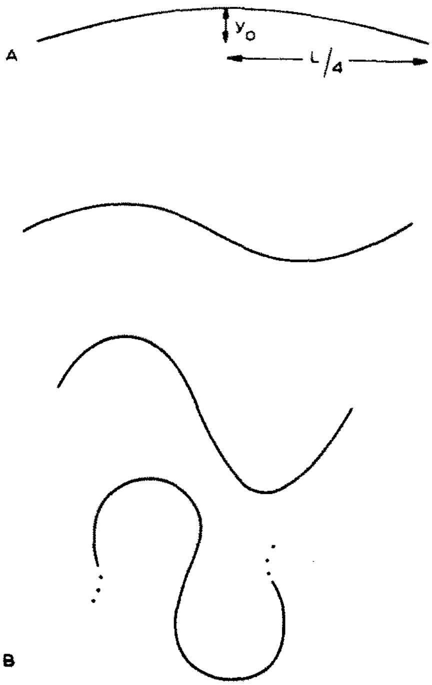

Fig.3. A. Half-wavelength of an elastic buckle. B. Stages in the development of the "elastica".

where E is Young's modulus, L is the wavelength of the elastic flexure (see Fig.3A) and I is the moment of inertia.

When deformation is elastic it may reasonably be assumed that the wavelength and the arc length are equal. Later, when the fold develops beyond the elastic limit, L represents the arc length, or for chevron folds it equals the sum of the lengths of the long and short limbs.

It must be pointed out that Euler's equation is an approximation: there are more accurate buckling criteria (see Salmon, 1952). However, the Euler form is probably the most widely known of the buckling criteria and it is certainly sufficiently accurate for the purposes of this paper.

If the buckled strut possesses a rectangular section the moment of inertia is given by $I = a^3 \cdot b/12$, where a is the thickness of the strut, or unit, and b is its width.

In this paper, the width of the strut, or unit, will always be taken as equal to unity, so that one is almost considering a simple two dimensional case. Consequently, the Euler equation may be written as:

$$\sigma_{crit} = \frac{p_{crit}}{a} = \frac{\pi^2 \cdot E}{\left[\frac{\sqrt{12}\,L}{a}\right]^2} \tag{2}$$

where σ_{crit} is the stress needed to initiate buckling and L/a represents the "slenderness ratio" of the competent unit. This equation was developed for engineering problems in which buckling occurred when the confining pressure was one atmosphere. Hence, in a geological environment where the least principal stress (σ_3) acts vertically and the horizontal, or greatest principal stress (σ_1) initiates buckling in the competent unit, the principal stresses are related to the critical buckling stress by the expression:

$$\sigma_{crit} = \sigma_1 - \sigma_3$$

It follows from eq.2 that if a long unit is subjected to a certain critical stress acting along its length, it will buckle. The actual initial form taken by this elastically deformed unit is a cosine curve which, if carried to extreme deflection gives rise to a form known as the "elastica", the development of which is represented in Fig.3B (see Wilson, 1952). Such a form may develop in rubber-like materials, spring steel, etc., but will not occur in rock. However, before discussing this further it is necessary to consider a certain point deriving from eq.2.

It may be inferred from this equation that σ_{crit} is not a constant but is dependent upon the slenderness ratio (L/a) of the unit. Thus, for high ratios the resistance to buckling is small. In fact, the equation indicates that as L/a approaches infinity, σ_{crit} approaches zero. This is a sophism which results from neglecting body forces in the derivation of the Euler equation. As we shall see later, limits are set to the slenderness ratio when the influence of the incompetent material is not assumed to be negligible. However, neglecting for the time being the influence of the incompetent material, the relationship represented by eq.2 can be taken as a reasonable approximation for moderate values of L/a (i.e., 10 to 20 : 1).

To enable specific values to be attached to σ_{crit} it is necessary to assume values for Young's modulus (E). Upper and lower limits to the value of this constant for cemented competent rock types may be taken as 10^7 and 10^6 lb./sq.inch, respectively. Using these values the relationship between σ_{crit} and L/a is as shown in Fig.4. It will be seen that the stresses necessary to cause buckling when the ratio is less than 20 : 1 increase rapidly, reaching extremely high values for slenderness ratios of 5 to 6 : 1, even when the value of E is as low as 10^6 lb./sq.inch.

There are obvious criticisms which arise when one uses specific values, determined in the laboratory, of elastic moduli, strength etc. and apply them to rocks postulated to be undergoing tectonic deformation. Nevertheless, it is instructive to use such data which, even if they only approximate to the correct values, provide an insight into the mechanics of deformation.

It has been shown by Biot (1965) and Ramberg (1961) that when the influence of adjacent incompetent material is taken into consideration the slenderness ratio of the competent buckle is given by the relationship:

$$\frac{L}{a} = 2\pi\sqrt[3]{\frac{G_c}{6G_i}} \tag{3}$$

where G_c and G_i represent the shear moduli of the competent and incompetent material respectively.

The shear modulus is not readily measured. However, if Young's modulus (E) and Poisson's number (m), quantities which can be measured, are known; then values for the shear moduli can be obtained from the relationship:

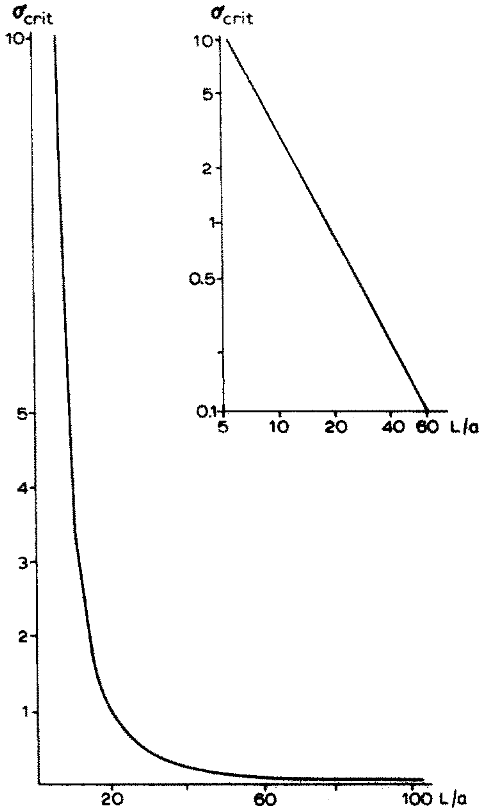

Fig.4. Relationship between the critical buckling stress (σ_{crit}) and the slenderness ratio (L/a). The units of stress are dependent upon the value of Young's modulus (E). When $E = 10^6$ lb./sq. inch and 10^7 lb. sq.inch the buckling stress is in units of 10^4 lb./sq.inch and 10^5 lb./sq.inch, respectively. The inset diagram shows the same relationship using a log-log graph.

$$G = \frac{m \cdot E}{2(m+1)} \tag{4}$$

The author has found that the ratio of the shear modulus of an extremely competent cemented sandstone (uniaxial strength of 70,000 lb./sq.inch) to that of a very weak, but compacted, mudstone (uniaxial strength of less than 1,000 lb./sq.inch) can be as high as 1,000 : 1. Putting such a value of the ratio in eq.3, the resulting value of the slenderness ratio is approximately 35 : 1. The values of the elastic moduli for the rocks quoted were determined after compaction had taken place. Thus, for this example, the initial ratio of the moduli, when the adjacent material is uncompacted, could easily have been in excess of 1,000 : 1, with a corresponding increase of the slenderness ratio of the resultant fold. Indeed, if the surrounding incompetent material is extremely weak it approximates to a Newtonian liquid of relatively low viscosity.

Thus, the analysis becomes one in both elastic and viscous behaviour and it follows that the resistance to elastic buckling of the competent layer by the surrounding "liquid" will be negligible when the rate of deflection (dy/dt) of the competent unit is low.

However, more commonly, the ratio of shear moduli for competent and incompetent material, after compaction and deformation, is usually from 5 : 1 to 50 : 1, so that the corresponding values of L/a are from 6 : 1 to 12 : 1. When both competent and incompetent material are undergoing progressive compaction, the author thinks it likely that the ratios of the shear moduli will not vary by more than an order of magnitude throughout the process of compaction, so that the ratios of L/a of 6 to 12 : 1 are likely to obtain whenever the buckles are initiated.

Examples of the slenderness ratios for single competent units folded in incompetent material are shown in Fig.5. The ratios of L/a (where, in this context, L is the arc length) which are most commonly observed in the field lie between 7 and 10 : 1; buckles with a slenderness ratio of 5 : 1 or less, or greater than about 15 : 1, are rarely observed. It is emphasised that these ratios refer to a single competent unit, such as may form a minor "drag".

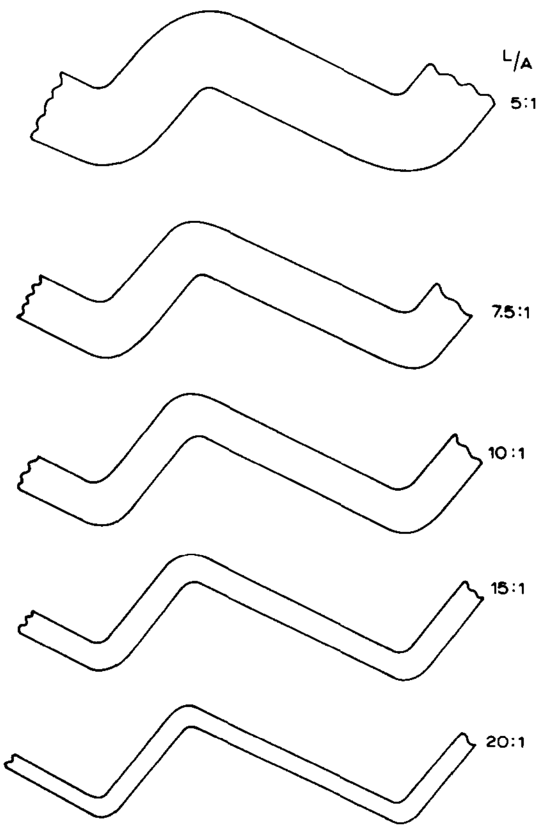

Fig.5. Examples of slenderness ratios of single competent units.

or parasitic fold, or a single major unit. The slenderness ratio of one of a series of thin competent units which have been folded together as one composite unit may, of course, attain values of many thousands to one. Such examples are not being considered here.

It will be clear from these remarks that the ratios predicted by theory are not in complete agreement with field observations, for whereas theory indicates the possibility of buckles existing with L/a ratios of 30 : 1 or more, field data indicate that the upper limit to the slenderness ratio is approximately 15 : 1. This discrepancy will be discussed later. In the meantime, it will be assumed that folds with large L/a ratios may in fact be initiated.

Local stresses within the competent unit set other limits on elastic buckling. When a beam or strut is deflected, stresses are set up within the unit purely related to its curvature. The stress at any point distance d from the neutral surface is: $\mp \sigma_d = E \cdot d/R$, where R is the radius of curvature of the neutral surface. At the upper and lower surface of the unit the stresses due to bending are a maximum and a minimum and are:

$$\mp \sigma = E \cdot a/2R \tag{5}$$

where, as before, a is the thickness of the unit. The stresses are tensile at the upper limit when curvature is anticline and, conversely, tensile at the lower limit of the unit when the curvature is synclinal. When the unit is undergoing elastic buckling, these local stresses are superimposed upon the general compressive stress causing the buckling. Hence, at the crest of an "anticline", the total stresses at the upper and lower surfaces of the competent unit are given by:

$$\sigma_{up} = \sigma_1 - E \cdot a/2R \tag{6a}$$

and:

$$\sigma_{low} = \sigma_1 + E \cdot a/2R \tag{6b}$$

In the trough of the elastic "syncline", the signs of eq.6 are reversed.

It is evident from eq.6a that σ_{up} becomes tensile when:

$$E \cdot a/2R \gg \sigma_1 \tag{7}$$

Consider now, specific values of E, etc., to see how readily the stresses become tensile. Take for example a fold with a L/a ratio of 15 : 1 and let E for the competent material be 10^6 lb./sq.inch. From Fig.4 it will be seen that the buckling stress (σ_{crit}) is approximately 16,000 lb./sq.inch. Hence, if it is assumed that σ_3 = 20,000 lb./sq.inch it follows that σ_1 = 36,000 lb./sq.inch. Substituting these values in eq.7 one obtains the relationship that $R = 14a$.

It is now of interest to estimate the maximum value of the elastic deflection which occurs when the unit is on the point of failing in tension. To do this, it is convenient to assume that the half wavelength of the elastic buckle represented in Fig.3A is an arc of a circle. Then, from elementary geometry it can be shown that:

$$y_0(2R - y_0) = (L/4)^2 \tag{8}$$

Substituting the value $R = 14a$ in eq.8 and remembering that for this example L/a = 15 : 1, then solving for y_0 one obtains the deflection in terms of the thickness of the unit. In this instance $y_0 = 0.5a$. Similarly when L/a is 20 : 1 and 10 : 1, the deflection when the stresses become tensile are approximately

0.75a and 0.375a respectively. Thus, in general, $y_0 = k \cdot a$, where k will vary directly as L/a. Because the deflection of the unit actually follows a cosine curve and not an arc of a circle, these values for the deflection are a little too large.

If one takes the yield strength of a competent unit, folded into an anticline and under a confining pressure of 20,000 lb./sq.inch, to be 100,000 lb./sq.inch, E being 10^6 lb./sq.inch,then when the ratio of L/a equals 10 : 1 and the deflection is 0.375a, the unit will be about to fail in compression at its lower surface, as well as being on the point of tensile failure at the upper surface. For the values of E and σ_3 quoted above, when L/a is less than 10 : 1, failure always occurs in compression. This relationship helps explain why, in a fold belt, although some of the larger folds with relatively high L/a ratios may be chevron structures, smaller folds and those with low L/a ratios possess rounded crests and troughs.

The tensile and compressive stresses given by eq.6 do not develop uniformly along the upper and lower limits of the competent units. The form of the elastic buckle is not a series of circular arcs but is a cosine curve. Now the buckling moment is given by the critical buckling force (p_{crit}) multiplied by the deflection at the various points along the strut. Therefore, the buckling moment curve will also have the form of a cosine curve. It can be shown that a moment (M) is related to the rate of change of slope of the elastic curve by the expression:

$$\frac{1}{R} = \frac{d^2y}{dx^2} = \frac{M}{E \cdot I}$$

Clearly, the rate of change of slope of the elastic buckle reaches a maximum when M is a maximum. This occurs at the crest and trough of the elastic fold and it is here that failure occurs, whether it be due to tensile or compressive stresses.

If failure occurs by the development of a tensile fracture the crack will tend to propagate downward at the crest of an anticline, or upward at the trough of a syncline, until the fractures may cut completely through the unit. Further development of the fold merely results in the rotation of two straight limbs (the elastic curvature may be neglected) to form an upright symmetrical chevron fold. Pressure solution often removes material from the competent unit at the crests and troughs, thus resulting in the close contact between adjacent limbs usually exhibited by chevron folds (see Fig.6A).

If failure occurs by plastic flow in compression, deformation is initiated at the lower surface of the unit in the crest of an anticline and at the upper surface in the trough of a syncline. The small zones of plastically deformed material reduce the resistance to buckling of the unit (see Fig.6B). If the stresses causing buckling remain constant, this reduction in effective resistance will result in an increase of the rate of deformation in the competent material in the vicinity of the crest and trough. As one may infer from Fig.2C, this will have an effect on the stress–strain relationship comparable with "strain-hardening". The stress necessary to deform the rock unit in the zone of plastico-viscous deformation will increase from yield stress σ_y to $\sigma_{y'}$. Meanwhile, when the stress level reaches σ_y in areas adjacent to the initial zone of plastico-viscous deformation, this new zone will yield. In this way the plastico-viscous deformation spreads progressively further into the competent unit and laterally away from the trough and crest. This process, which

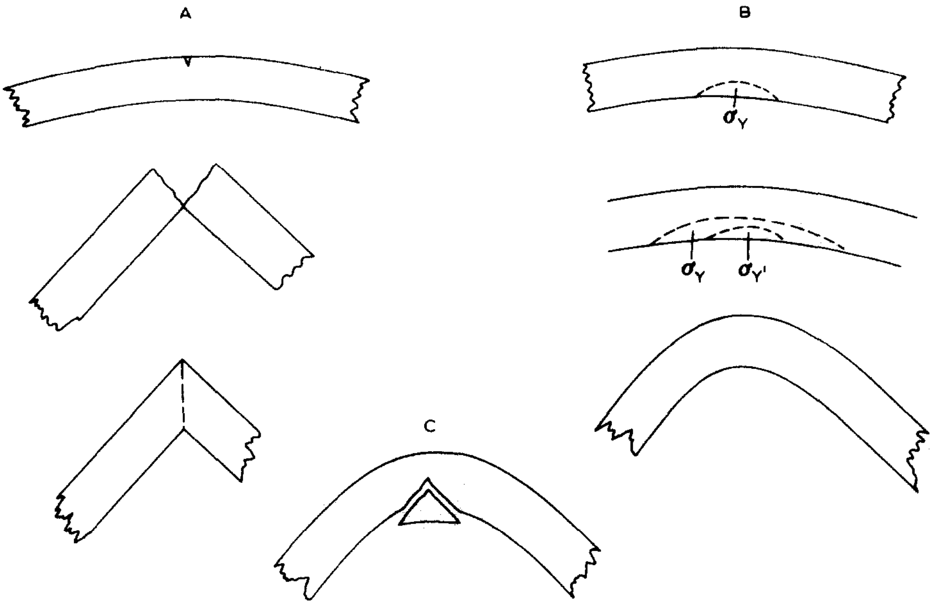

Fig.6. A. Stages in the development of a chevron fold. B. Stages in the development of a fold with a rounded crest. C. Fold with shear failure in the zone of local compression below the neutral surface.

has been noted by Wilson (1952), will result in a fold with straight limbs but rounded troughs and crest. This is the "plastica" type of curve cited by Wilson.

A third possibility, of course, is that given by Seidl (1934) when failure in compression occurs by shear failure as indicated in Fig.6C.

It should be emphasised that lateral migration of the crests and troughs of folds, of the type described by Ramberg (1963) in his paper on "drag folds", is only possible when deformation is wholly elastic or wholly viscous. Once the competent unit yields by either tensile fracture or by rupture or plastic flow in compression the position of the crest and troughs are fixed and the lengths of the limbs are determined. In this section the fold form under discussion is symmetrical, so that the lengths of the limbs are the same. However, this mechanism also holds when the fold form is asymmetrical.

THE INITIATION OF ASYMMETRICAL BUCKLES

Where, as described above, conditions of orthorhombic symmetry hold, one may consider the maximum principal stress as acting normal to the axial plane of the fold. Similarly, because the axial planes of asymmetrical folds are usually inclined, it follows that at the end of a period of folding, the axes of maximum principal stress are also inclined. It is reasonable to assume, therefore, that when the asymmetrical folds were initiated, the axes of principal stress were also probably inclined. This obvious conclusion has been reached by Goguel (1948) and Bredin and Furtak (1963), but neither of these authors outlined the mechanism by which the competent layers develop into asymmetrical buckles.

The stress conditions which are assumed to have obtained when such asymmetrical folds were initiated are, therefore, as indicated in Fig.7A. The principal stresses can be resolved parallel and perpendicular to the competent unit (which is assumed to be horizontal before deformation). These vertical and horizontal stresses are comparable in orientation with those which bring about symmetrical folding; and when the intensities of the stresses are suitable such symmetrical folds tend to develop. However, it will be noted that because the axes of principal stresses are inclined, a distributed shear stress (τ) acts along the upper and lower surface of the competent unit. Therefore, because the shear stresses are the only new factor, it is the action of τ which must be instrumental in determining that the resultant buckles are asymmetrical rather than symmetrical. To study the influence of these shear stresses it is first necessary to consider the "moments of force" involved in buckling and bending.

The quantity which is responsible for the degree of deflection of a unit during buckling is the "buckling moment" (M_{buc}) which at any point of distance x from a nodal point along the line of action of the buckling force is given by:

$$M_{buc} = P_{crit} \cdot y$$

where y is the elastic deflection at point x. For a symmetrical fold, it has been noted that the maximum moment occurs at the crest and trough of the elastic fold form and immediately prior to elastic failure has a value given by:

$$\begin{aligned} M_{buc_{max}} &= P_{crit} \cdot y_{max} \\ &= a \cdot \sigma_{crit} \cdot k \cdot a \\ &= a^2 \cdot k \cdot \sigma_{crit} \end{aligned} \qquad (10)$$

where k is the constant which as previously noted is dependent upon the

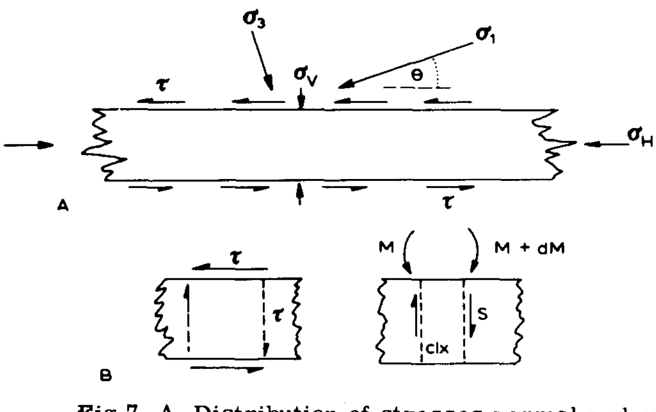

Fig.7. A. Distribution of stresses normal and parallel to a horizontal competent unit when the axis of greatest principal stress is inclined at an angle θ. B. Relationship between shear stresses (τ), shear force (S) and bending moment (M).

slenderness ratio. For example, when $L/a = 20:1$, $k = 0.75$.

Consider now the action of the shear stress (τ); for elastic equilibrium, the shear stress must also act perpendicular to the surface of the unit, as indicated in Fig.7B. These vertical shear stresses give rise to a corresponding shear force (S) acting normal to the surface of the unit where:

$$S = a \cdot \tau \tag{11}$$

It can be shown (see Salmon, 1952, p.58) that there is a relationship between a shearing force in a unit and a bending moment (M_{ben}) such that:

$$S = \mathrm{d}M_{\text{ben}}/\mathrm{d}x$$

or:

$$M_{\text{ben}} = S\int_0^x \mathrm{d}x$$

Thus, when, as here, S is a constant, M_{ben} changes in a linear fashion along the length of the unit. If M_{ben} is arbitrarily put equal to zero at point O, the distribution of the bending moment along the length of the unit due to the shearing stress (τ) is as indicated in Fig.8, so that for the fold length L this becomes:

$$M_{\text{ben}} = \tau \cdot a \cdot L \tag{12}$$

The deflection of the competent unit will be influenced by both the buckling moment and the bending moment. Because the problem, at this stage,

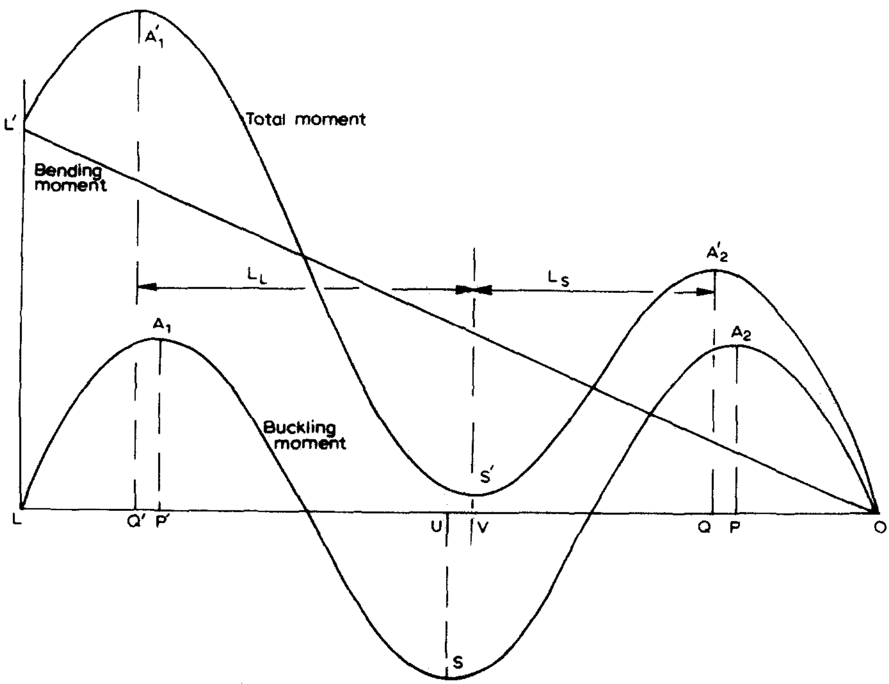

Fig.8. Showing how the bending and buckling moment curves may be combined to give an asymmetrical total moment curve.

is one in elasticity, one may superimpose these moments (M_{buc} and M_{ben}) represented by curves OA_2SA_1L and OL' respectively in Fig.8. The total moment (M_{tot}) is therefore represented by curve $OA_2^1SA_1^1L'$.

It will be noted that the points of maxima of the M_{tot} curve have moved to the left relative to the corresponding points on the M_{buc} curve. The lateral shifts (represented by PQ and $P'Q'$) are the same for both sets of maxima so that the wavelengths of the curves are the same. However, it will be noted that the minimum value on the M_{tot} curve is to the right of the corresponding point on the M_{buc} curve by an amount represented by UV.

It has been pointed out that elastic failure occurs where d^2y/dx^2 are maxima and minima and these conditions are met when the moment curve reaches maximum and minimum values. Using the argument presented in the previous section it is therefore at Q, Q' and V respectively, in Fig.8, that the anticlinal and synclinal axes will be initiated and fixed.

It will be seen from Fig.8 that the distance VQ is less than VQ', so that the syncline which would develop from the moment system represented would be asymmetrical. It is emphasised that the curves represented in this diagram refer to the various moments and not to deflections. Elastic deflections, for moderate slenderness ratios, will be relatively small and as a first approximation may be neglected. Consequently, one may take the distances VQ and VQ' as representing the length of the limbs of the folds. In this instance the ratio of the length of the long limb (L_l) to the short limb (L_s) is 1.4 : 1. It may readily be inferred from Fig.8 that the ratio L_l/L_s will depend upon the relative intensities of M_{buc} and M_{ben}. When the maximum value of M_{ben} is small compared with that of M_{buc} the ratio L_l/L_s will approach unity. However, when the ratio of the moments is large, the ratio of the limb lengths will also be large. Consequently, before pursuing this argument it is of interest to estimate values for the ratio of the moments which may develop in various stress fields which may give rise to folding.

Assuming that the competent unit is initially horizontal and that the axis of maximum principal stress is inclined at an angle θ to the horizontal, then the horizontal critical stress and the shearing stresses are respectively given by:

$$\sigma_H = \sigma_{crit} + \sigma_{vert} = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2} \cdot \cos 2\theta \qquad (13)$$

and:

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \cdot \sin 2\theta \qquad (14)$$

If the ratio of principal stresses is expressed as $\sigma_1/\sigma_3 = K$, then eq.13 and 14 may be rewritten:

$$\sigma_H = \frac{\sigma_3[K+1]}{2} - \frac{\sigma_3[K-1]}{2} \cdot \cos 2\theta \qquad (15)$$

and:

$$\tau = \frac{\sigma_3[K-1]}{2} \cdot \sin 2\theta \qquad (16)$$

Hence, if one also attributes some specific value to the slenderness ratio (n), then:

$$M_{\text{ben}} = \tau \cdot n \cdot a^2 = \sigma_3 \frac{(K-1)}{2} \cdot \sin 2\theta \cdot n \cdot a^2 \qquad (17)$$

However, it has been noted that if a specific value is attributed to the slenderness ratio and also to the elastic modulus (E), this automatically fixes the critical buckling stress (see Fig.4)[1]. Thus, if $L/a = 20:1$ and $E = 10^6$ lb./sq.inch, then it follows that σ_{crit} is 8,500 lb./sq.inch. If it is further assumed that the vertical stress is 20,000 lb./sq.inch, then it follows that the total horizontal stress to cause buckling is 28,500 lb./sq.inch. These conditions for the horizontal and vertical stresses can be satisfied by a whole range of σ_3, K and θ. However, if specific values are assigned to θ*, the corresponding values of σ_3 and K can be calculated by using eq.15 (the data for the vertical stress as well as the horizontal stress must, of course, be used). From the values thus derived one may calculate the corresponding values for the bending moment from eq.17.

Also, by using the value of the maximum elastic deflection ($k \cdot a$) which may occur for any specific slenderness ratio, the corresponding values of the maximum buckling moment can be obtained by using eq.10.

It is possible, therefore, to calculate the ratio of the bending to buckling moments for any specific value of θ, K and σ_3, when the vertical stress and elastic modulus are also assigned specific values. The relationship between $M_{\text{ben}}/M_{\text{buc}}$ and θ (when $\sigma_{\text{vert}} = 20{,}000$ lb./sq.inch and $E = 10^6$ lb./sq.inch) is shown in Fig.9. Spot values for K (i.e., σ_1/σ_3) are indicated on the various curves. The curve for $L/a = 10:1$, when θ is greater than 18°, is represented dotted, because the value of K is greater than 5.0, the rock would fracture or deform plastically rather than buckle.

It should be realised that the data used to calculate the maximum buckling moment are based on two approximations. The first is that the incompetent material does not influence the critical buckling stress. This assumption will result in an underestimate of σ_{crit}, which in turn will result in too small a value for M_{buc}. This error will be large for small values of L/a. The second is that used in estimating the maximum elastic deflection. Here it has been assumed that the form of the elastic deflection approximates to an arc of a circle rather than a cosine curve, which results in an overestimate of the elastic deflection that is possible before failure of the competent unit. This, in turn, results in too large an estimate of the buckling moment. Hence, the two errors incurred by making these assumptions tend to cancel each other out, so that the ratios of the moments represented in Fig.9 may be taken as reasonably correct. It is clear therefore, that the ratio of the moments ($M_{\text{ben}}/M_{\text{buc}}$) may reach high values for quite modest values of K and θ.

The importance of the ratio of the moments and its influence upon the

[1]It is emphasised that data presented in Fig.4 are based on the assumption that the influence of the incompetent material may be neglected. However, in order that the principal stresses may act at an angle to the competent unit and develop a shear stress between the incompetent and the competent materials, the incompetent material must be sufficiently strong to support the shear stress. For low values of K and θ, the shearing stress will be small, consequently, errors incurred by using the data represented in Fig.4 will not be excessive, but will, of course, become increasingly larger and important as a source of error as these values increase.

*θ cannot exceed 45°. For values greater than 45°, stresses normal to the unit must be greater than those parallel to it, so that the unit will not buckle.

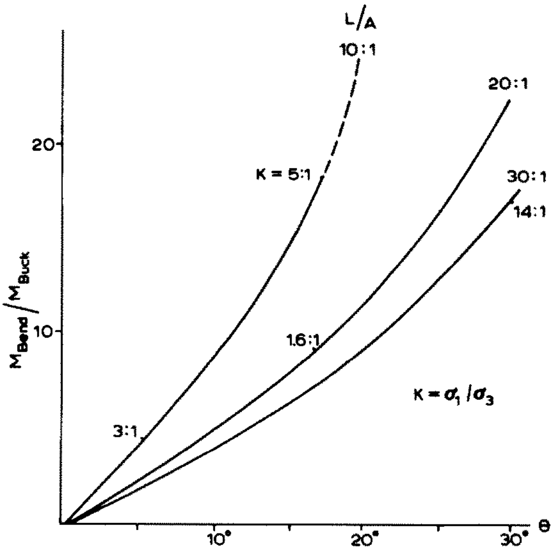

Fig.9. Relationship between the ratio of moments and the angle θ for various slenderness ratios (L/a).

degree of asymmetry of the resulting buckle can now be investigated.

The total moment curve can be represented by an expression of the form:

$$M_{tot} = Q(\cos x + r \cdot x) \tag{18}$$

where Q is a constant with the dimensions of a force and r is the ratio of the moments. The positions of the maxima and minima total moments can be obtained by finding the first differential coefficient of eq.18 and equating it to zero. It is more instructive, however, to determine the position of the maxima and minima, for various ratios of M_{ben}/M_{buc}, by a graphical method. Data for ratios of moments from 1 : 1 to 6 : 1 are represented in Fig.10. It will be seen that the total moment curves show well-defined maxima and minima when compiled from ratios of moments from 1 : 1 to 5 : 1. As before, we take the position of the maxima and minima to fix the position of the sharpest flexure in the deflection curve and these in turn give the points at which the elastic limit is first reached. This exceeding of the elastic limit determines the position of the synclinal and anticlinal hinges and completely controls the subsequent development of the fold.

It will be seen that the dotted lines A_σ–A_σ and S_σ–S_σ which join the points of maxima and minima total moments, converge as the ratio of M_{ben} M_{buc} increases in value. The horizontal distances between points A_1 and S_1, A_2 and S_2, etc., therefore represent the lengths of the short limbs. Because the wavelength of the buckle is not altered by the influence of the bending moment, one can use these data to calculate the lengths of the long limbs and hence indicate the relationship between the ratios of M_{ben}/M_{buc} and the degree of asymmetry as represented by the ratio of the limb lengths (L_l/L_s), see Fig.11.

It will be noted that when the ratio of the moments reaches 6 : 1 the total moment curve no longer possesses a maximum and a minimum but merely shows points of inflexion. It is therefore suggested that when the ratio of the moments reaches 6 : 1, or higher values, the sites of the anticlinal and synclinal axes are not wholly determined by the total moment curve, but are in part determined by minor variations in the physical or dimensional properties of the competent unit. For example, a local 5% reduction in the thickness of the competent unit would result in a corresponding reduction in resistance to buckling of almost 15%. Consequently, although it may be inferred that for a ratio of moments of 6 : 1, the ratio of limb lengths should also be approximately 6 : 1, minor variations in the dimensions of the unit may result in a fold with ratio of limb lengths in excess of 6 : 1. However, since the distribution of points of relative weakness in the unit is likely to be random, there is no way of estimating the probable ratio of limb lengths. This argument

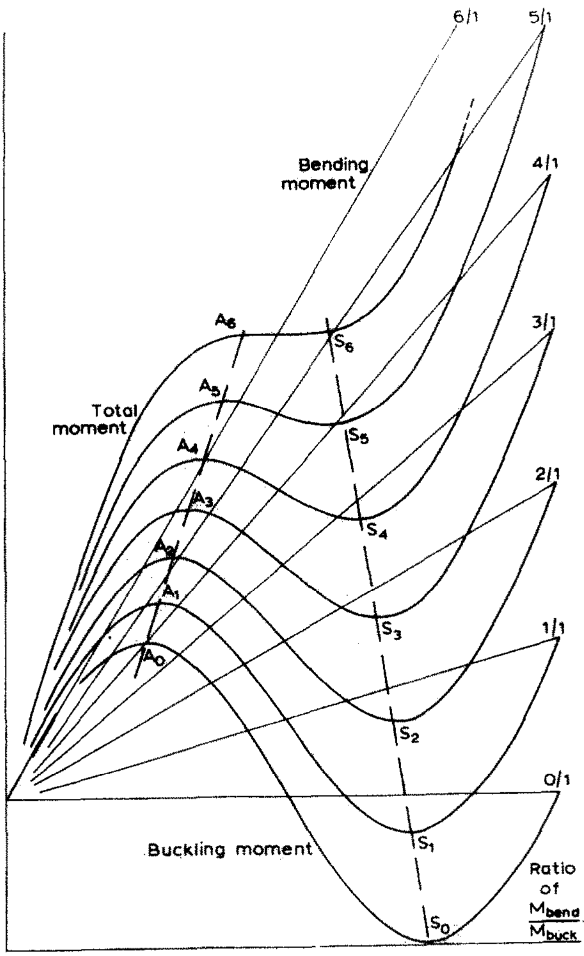

Fig.10. Showing various total moment curves compiled from ratios of bending to buckling moments from 1 : 1 to 6 : 1.

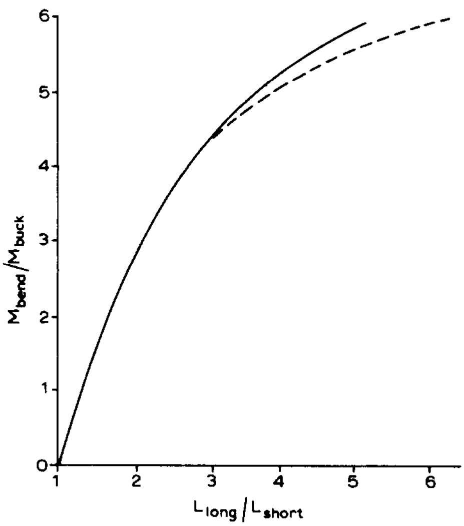

Fig.11. Relationship between the ratio of moments and the resulting ratio of the limb lengths (L_l/L_s).

applies to some extent when the ratio of the moments is less than 6 : 1. However, for small ratios of moments, it will be seen from Fig.10 that the fall-off in the value of the total moment on either side of the maxima and minima points is relatively rapid so that minor variations in thickness will have less influence.

It has been noted that perfectly symmetrical folds are not often observed in the field. The reason for this can be inferred from Fig.9 and 11. The chances of the axis of maximum principal stress acting exactly parallel to the competent unit is relatively small. It will be seen from Fig.9 that for modest values of K of 1.2 : 1 to 1.4 : 1, if the axis of principal stress is inclined to the unit at as little as 2–3°, the ratio of the moments is 1 : 1. From Fig.11 it will be seen that this ratio of moments will give rise to a fold with limb lengths of 1.2 : 1; an "asymmetry ratio" which would be very readily detected in the field.

DEVELOPMENT OF ASYMMETRICAL FOLDS

Once the elastic limits of the material of the competent unit has been reached at the points of maximum curvature of the elastic buckle, the subsequent positions of the hinges of the "plastica" curve are established.

In symmetrical folds, once the elastic limit has been exceeded, development of the fold takes place largely by rotation of the limbs (this is completely true if the fold is of the chevron type) where the rotations are equal in amount but opposite in sense in the adjacent limbs. Clearly, the work done in bringing about this rotation will be equally divided between the two limbs.

The author is of the opinion that one may apply this principle of equal work expended on the rotation of the two limbs to the development of

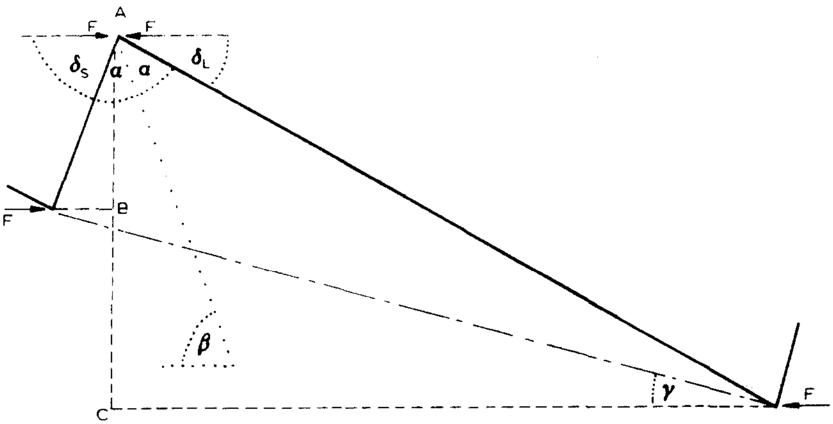

Fig.12. Showing angular relationships discussed in text, when the limbs are rotated in a force-field (F).

asymmetrical folds. If, for the sake of simplicity, it is assumed that the fold type under discussion is a chevron fold, the forces causing rotation can be thought of as two horizontal sets acting upon the ends of the limbs, as indicated in Fig.12. These forces (F) give rise to a turning moment causing rotation of the short limb and the long limb which are $F \cdot AB$ and $F \cdot AC$ respectively.

If it is assumed that the force remains constant in value and direction of action throughout the development of the fold, it can be shown that the work done (W) by the couple in rotating the short limb from the horizontal, through an angle δ_s is given by:

$$W_s = F \cdot L_s \cdot \sin \delta_s \cdot \delta_s \tag{19}$$

Similarly, the work done in rotating the long limb is:

$$W_l = F \cdot L_l \cdot \sin \delta_l \cdot \delta_l \tag{20a}$$

or:

$$W_l = F \cdot n' \cdot L_s \cdot \sin \delta_l \cdot \delta_l \tag{20b}$$

where $L_l/L_s = n'$.

If $W_s = W_l$, the rotation of the two limbs is linked by the expression:

$$\sin \delta_s \cdot \delta_s = n' \cdot \sin \delta_l \cdot \delta_l \tag{21}$$

Thus, the difference in the rotation of the two limbs is related to the ratio of the limb lengths. If the unit was originally horizontal and the long and the short limbs rotated through angles of δ_l and δ_s respectively, the angle of inclination of the axial plane is uniquely defined, for, from Fig.12:

$$\delta_s + \delta_l + 2\alpha = 180° \tag{22}$$

and:

$$\alpha + \delta_l = \beta \tag{23}$$

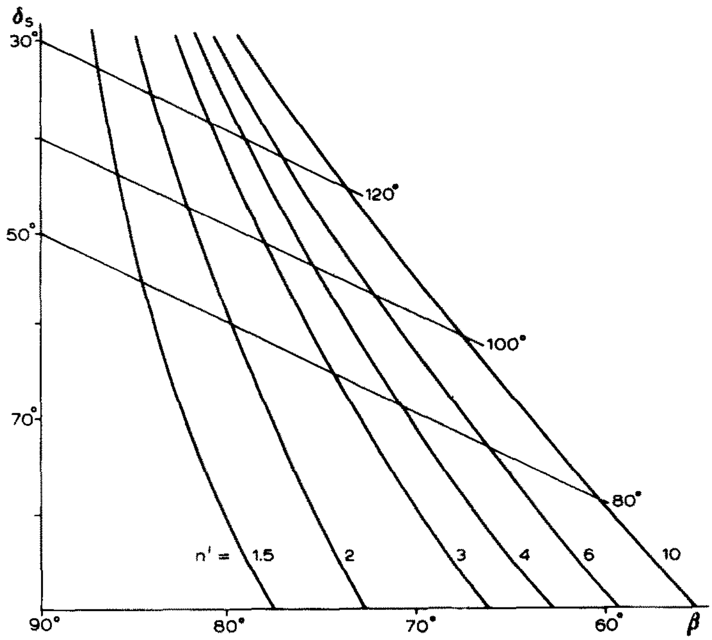

Fig.13. Inter-relationship between the angle of rotation of the short limb (δ), the angle between the two limbs (2α) and the inclination of the axial plane (β), for various ratios of limb lengths (n').

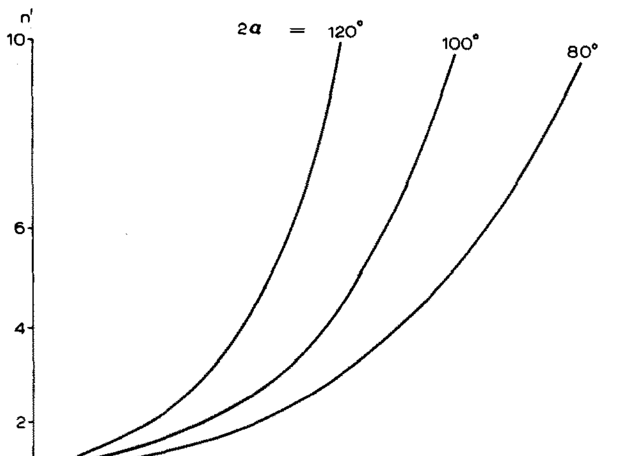

Fig.14. Relationship between the limb length ratio (n') and the inclination of the axial plane (β) for various values of 2α.

The inter-relationship between the angle of dip of the short limb and the inclination (β) of the axial plane, for various values of 2α and n', obtained by using eq.21–23, is shown in Fig.13. Clearly, from eq.21 the ratio of the angles of rotation is determined by the value of n'. Consequently, it follows that the dip of the axial plane is related to the extent of the folding, represented by the value of 2α, and to the ratio of the limb lengths, represented by n'. The smaller the value of 2α, the smaller is the angle of dip of the axial plane (β): similarly, the larger the value of n', the smaller the value of β. These relationships, obtained from the data presented in Fig.13 are most readily seen in Fig.14. It may be inferred that in a fold belt (assumed to have originated in the manner indicated in this and previous sections) in which the ratio of the limb lengths (n') is from 2 : 1 to 4 : 1, and the angle between the limbs (2α) is from 80 to 100°, the inclination of the axial plane will be from 71 to 81°. So that the axial planes in adjacent folds could reasonably be described as sub-parallel.

If the axis of maximum principal stress acted at right angles to the axial plane then, in the instance cited it would be inclined at 9–19° to the horizontal.

It will be recalled that the ratio of limb lengths of a fold is fundamentally determined by the intensity of the shear stress (τ) acting along the boundary of the competent unit. The intensity of this stress is controlled by both the intensities of the principal stresses and the angle the axes of principal stresses make with the competent unit. Consequently, a unique solution of the problem, using this type of analysis, is not possible.

Nevertheless, the data presented in Fig.9, 11 and 14 indicate that the axis of greatest principal stress (which it will be remembered is, in this context, being used in a statistical sense) commonly acts at high angles to the axial plane. Thus, from Fig.11 it will be seen that for folds with ratios of limb lengths of 2 : 1 to 4 : 1, the ratios of moments which initiate such folds are from 3.3 : 1 to 5 : 1. From Fig.9 it will be seen that for values of K from 1.2 to 3.0, such a ratio of moments will be obtained when θ lies between 4 and 12°. This range of angle overlaps the data obtained from Fig.14 and the maximum difference is only 15°. The data presented in Fig.9 and 14 do not, of course, represent all possible variations of stress orientations, tightness of folding, etc. Even so, taking into account the various simplifying assumptions made in this analysis, the data are sufficient to indicate the general validity of the principle that axial planes tend to develop approximately at right angles to the statistical axis of maximum principal stress. It should be noted, however, that this principle has greatest validity when the interlimb angle is small. Thus, when the fold is initiated and $2\alpha \approx 180°$, the axial plane will be normal to the competent unit and, at this stage of the development of the fold, will be independent of the orientation of the principal stress.

If the ratio of the limb lengths (n') and the ratio δ_s/δ_l are fixed, the angle between adjacent synclines (i.e., of the fold envelope) in Fig.12 is determined only by the degree of folding, as represented by 2α. The relationship between γ and n' for various values of 2α are indicated in Fig.15A. It is interesting to note that for values of n' greater than 3.0 the maximum variation of γ for any value of 2α is only about 3°. Thus, this angle is much more constant than the inclination of the axial plane; this relationship is probably worthy of study in the field. The data presented in Fig.15A are based on the assumption that the fold is of the chevron type and that the

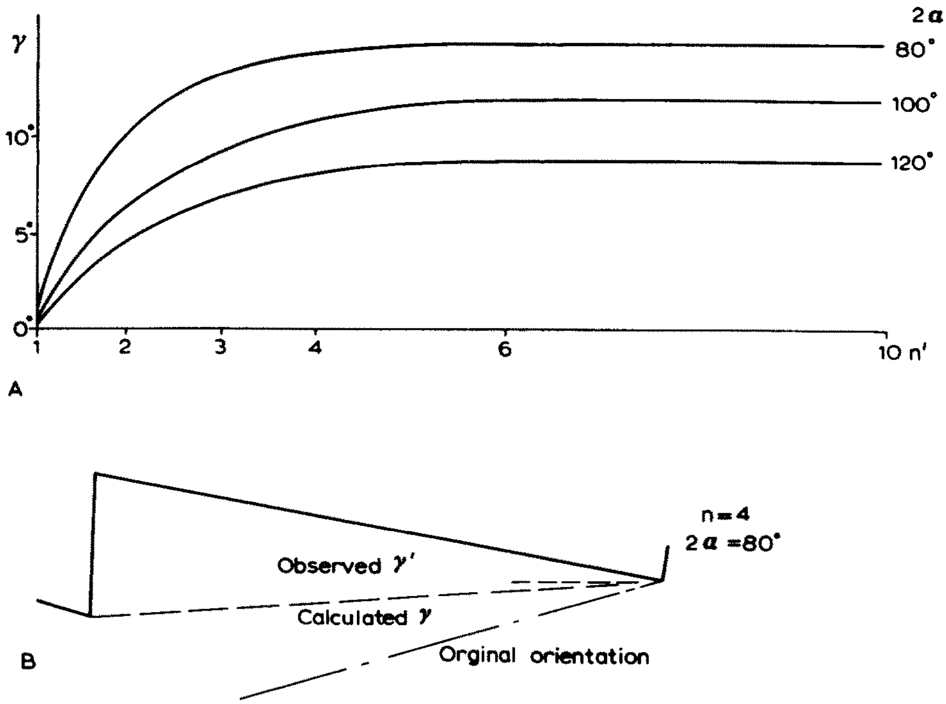

Fig.15. A. Relationship between γ (see Fig.12) and limb length ratio (n') for various values of 2α. B. Indicating possible method of estimating original orientation of competent unit prior to buckling.

competent unit was originally horizontal. Therefore, for chevron folds, or in folds in which troughs and crests are sharply rounded and the limbs are relatively long and straight, one may use the measurement of γ' based on field measurements to indicate the probable inclination of the competent unit prior to the initiation of buckling, see Fig.15B. This relationship may, of course, be modified by subsequent compressive strain or shear.

FOLDS IN MULTI-LAYERED COMPLEXES

For the sake of simplicity in presenting the main argument, it has so far been assumed that we have been dealing with a single competent band set in thick incompetent material. However, competent and incompetent units of comparable thickness are commonly interbedded. Ramberg (1961) and Biot (1965) have shown that the behaviour of such complexes is in many ways comparable with the behaviour of a single competent unit set in incompetent material. The mathematical treatment of multi-layered complexes is, however, somewhat laborious, for the thickness of the individual competent and incompetent layers in addition to their respective elastic or viscous constants must be taken into account; but it is possible to demonstrate the

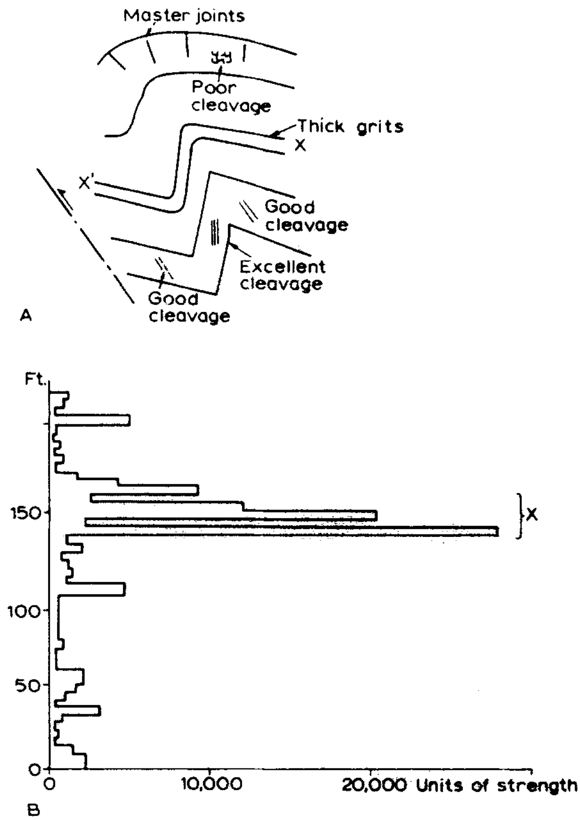

Fig.16. A. Composite profile of structure developed in the Aberystwyth Grits in cliff sections near Llanrhwstyd, Cardigan, Wales. B. Showing resistance to buckling of various rock units exposed in the profile represented above.

importance of the thickness of the various competent units from field evidence.

The structure represented in composite section in Fig.16A occurs in a series of thin interbedded grits and shales of the Aberystwyth Grits. From this section it may be inferred that in the upper portion of the structure the compressive stresses never developed the same intensities as were attained by the stresses in the lower portions of the structure. By analogy to elastic buckling, the existence of a neutral surface may be inferred in the neighbourhood of *XX'* in Fig.16A.

Detailed measurements were made of the thickness of the grits and shale bands which comprise the 220 ft. of sediments represented in the section. The grit bands varied considerably in thickness, but they were similar in composition throughout the succession. The lengths of individual bands in the fold can reasonably be taken as constant. Also, if one assumes that the elastic properties of all the competent bands are alike and that the influence of the incompetent material was negligible, then, from eq.1, the resistance to buckling of any individual competent layer is proportional to the cube of the thickness of the layer.

If it is assumed that a one inch thick grit band has unit resistance to buckling, then the equivalent resistance of any unit can be represented numerically by taking the cube of the unit's thickness, measured in inches. This operation was carried out for every competent unit in the succession illustrated. For ease of presentation of the data, the succession was divided into 55 parts, each being 4 ft. in thickness. The resistance to buckling of each 4 ft. division was then taken as the sum of the resistances of every competent band within the division, and are represented graphically in Fig.16B.

It is obvious that the divisions which comprise group *X*, in Fig.16B, must have profoundly influenced the development of the fold. In fact, the neutral surface *XX'*, in Fig.16A, is situated within the limits of group *X*. The few thick grit bands which are concentrated in group *X* can therefore be thought of as a major control group, comparable in many ways with a single competent unit.

DEVELOPMENT OF ANTICLINORIA AND SYNCLINORIA[1]

The theory outlined above indicates that folds will develop which possess slenderness ratios of 35 : 1 or more. Yet, structures composed of a single competent unit exhibiting such slenderness ratios are not observed in the field. This is an apparent contradiction, upon which comment must be made.

For folds to develop which have large slenderness ratios, the ratio of the shear moduli of the competent and incompetent material must be very large. As we have seen, this means that the competent material must be cemented and the incompetent material must be extremely weak. When such conditions obtain, the author believes that folds with a very large slenderness ratio will in fact be initiated by a critical stress of a few thousands of pounds/sq.inch, acting along the length of the competent unit. If the principal stress acted exactly along the competent unit, it would give rise to a symmetrical fold. Further, assuming that the value of Young's modulus of the competent material is 10^6 lb./sq.inch or more and that the depth of cover is moderate (say 20,000 ft.), then the fold will develop in a chevron form as indicated in Fig.17A. In the light of the data and conclusions of the previous paragraphs, although each section or profile represented in this figure is of a single competent unit, they could equally well represent a multi-layered complex.

When the limbs of the fold attain an inclination of 30°, the amount of horizontal shortening of the competent unit brought about by folding is approx. 15%. In the incompetent material, it is almost certain that this horizontal shortening will mainly be accommodated by volume flow. That is, progressive compaction will occur and cause a corresponding progressive increase in the strength and values of the elastic moduli of the incompetent material. Thus, as folding progresses, the compressive stress needed to close this fold must, of necessity, increase in intensity. The elastic modulus of a cemented unit

[1]The words "anticlinoria" and "synclinoria" are here used to represent compounded anticlines and synclines in which the wavelengths are commonly between a few hundred yards and a few miles.

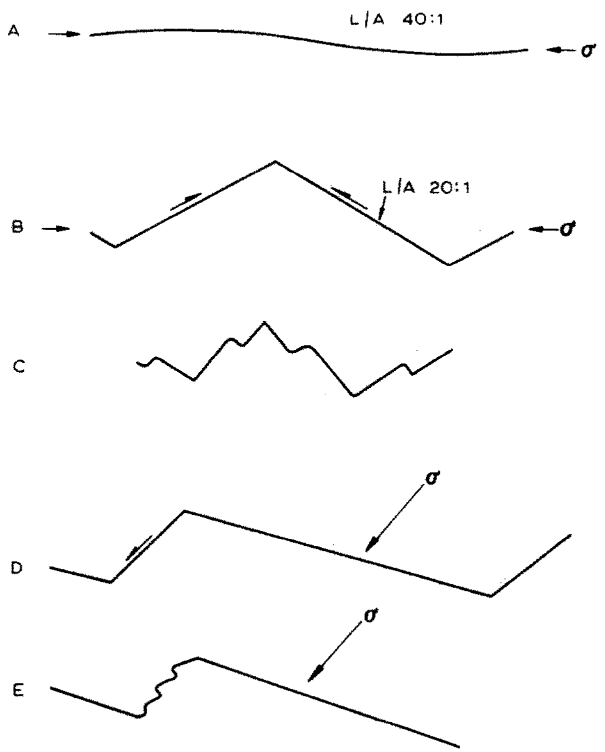

Fig.17. A–C. Stages in the development of symmetrical anticlinoria and synclinoria. D–E. Stages in the development of an asymmetrical anticlinorium when the axis of greatest principal stress initially makes a large angle with the competent unit.

is likely to be little changed by such compaction, whereas the shear modulus of the incompetent, but compacting material will eventually increase a hundred-fold or more. With the ratio of the shear moduli thus reduced, folds with a slenderness ratio of perhaps 10 : 1, or less will form when the buckling stresses reach a sufficient intensity. Such buckles, which may be termed "secondary" folds, will form on the limbs of the original structure, which may be referred to as a "primary" fold. Repetition of folding resulting in anticlinoria and synclinoria may thus develop and their component folds will in general exhibit slenderness ratios of less than 15 : 1. If the slenderness ratio of the primary fold is sufficiently large, this mechanism may even give rise to "tertiary" folds.

The limbs of the primary fold may make an angle of 30° or more with the axis of greatest principal stress when the secondary folds are initiated. Consequently, the shear stress acting along the competent units, at this time, will result in the secondary folds being asymmetrical. This asymmetry is, in this instance, in opposite senses on the opposing limbs of the primary structure. Whereas the primary fold may have been of the chevron type, the secondary folds, because of their smaller slenderness ratio more probably have rounded hinges. If the axis of greatest principal stress was originally at low angle to the length of the unfolded competent unit the primary fold will be slightly asymmetrical so that the final anticlinorium will also be asymmetrical. Such structures are found in the cliff sections in the Culm of North Devon and in the Carboniferous Series of Pembroke. If the axis of greatest principal stress makes a large angle (30–40°) with the unfolded competent unit the primary structure will again be asymmetrical, but will have a larger ratio of limb lengths. But if, at the stage of development shown in Fig.17B, the long limb makes an angle of more than 45° with the axis of greatest principal stress, the stress intensity normal to the surface of the competent unit in the long limb must be greater than the intensity of stress acting along the length of the unit. When such stress conditions prevail, secondary buckling of the long primary limb is not possible. The short limb, however, will be subparallel to the axis of greatest principal stress and is ideally orientated for secondary buckling. If the short limb is in fact parallel to the axis of greatest principal stress, the resulting secondary buckles will be symmetrical; but, of course, their axial planes will be inclined and not vertical. If the axis of greatest principal stress is inclined at an angle greater than the dip of short primary limb, then the shear stress acting along the surface of the competent unit will be "left-handed", as indicated in Fig.17D. The secondary folds which subsequently develop on the short primary limb will, therefore, be asymmetrical and present the same "movement picture" as that of the whole primary structure. This disposition of primary and secondary structures (represented in Fig.17E) results from such a mechanism and have, in fact, developed in the Aberystwyth Grits (Price, 1962).

CONCLUSION

The shape of folds in competent units in the upper, non-metamorphosed zones of the crust cannot be explained by theories based on the assumption that the behaviour of competent rock approximates to that of a Newtonian fluid. Indeed, in these upper zones, competent rock behaves as either a brittle or a plastic solid. Consequently, the initial deformation of such rock is wholly elastic, but when certain limits of stress are reached the rock fails either by rupture or by plastico-viscous deformation.

When the maximum principal stress acts along a competent unit, the resulting elastic buckle is symmetrical. The elastic limit of the rock is first reached at the crests and troughs of the buckled unit, where the rate of change of curvature of the unit is greatest. The sites of these points of initial elastic failure determine the subsequent development of the fold. At relatively shallow depths and when the value of Young's modulus (E) of the rock is high, whether the unit fails in tension and develops into chevron folds or whether it fails in compression and forms "plastica" type folds depends upon the slenderness ratio (L/a) of the buckle. The higher the value of Young's modulus and the greater the depth of burial the greater must be the slenderness ratio before the unit will fail in tension.

These remarks apply equally to asymmetrical structures which, it is shown, are initiated when the axis of maximum principal stress is inclined to the unfolded competent unit or group of units. The quantity which is fundamentally responsible for the asymmetry of the fold is the shear stress. Such shear stresses generate a bending moment which when added to the buckling moment (which results from compression along the unit) results in a total moment curve which is asymmetrical. The asymmetry of this total moment curve controls the asymmetry of the resulting buckle. Such a mechanism, it is shown, can give rise to folds with a marked asymmetry in which the ratio of the limb lengths may be in excess of 6 : 1.

Once the fold is initiated and the position of the fold axes determined by elastic failure, subsequent development of the flexure, whether it be symmetrical or asymmetrical, largely takes place by rotation of the limbs. The work done in rotating each limb is taken to be equal. As a result, for asymmetrical folds, the shorter limb rotates through a larger angle. The orientation of the axial plane of such asymmetrical structures is determined by the ratio of the limb lengths and the degree of folding (represented by the acute angle between the straight limbs). Unfortunately, a unique solution is not possible, but when an analysis is made, the results indicate that the axis of greatest principal stress will act approximately normal to the axial plane.

Although most of the arguments presented in the paper are restricted to the folding of a single competent unit, field evidence is presented to indicate that the principles involved in folding a single unit or a multi-layered complex are similar.

Finally, it is argued that a highly competent unit, or group of units, set in extremely incompetent material may be first folded into a primary structure with a high slenderness ratio. As a result of the shortening that takes place, the incompetent material is compacted and becomes progressively more competent so that, eventually, a second and even a third generation of folds exhibiting progressively smaller slenderness ratios may develop on the limbs of the primary folds, resulting in anticlinoria and synclinoria. The final form of the complex of flexures is determined by the angle which the axis of greatest principal stress makes with the competent unit, or units. When this angle is large, the resulting structure can be completely asymmetrical, in that one of the primary fold limbs (the long one) may remain unfolded throughout subsequent phases when secondary and tertiary folds are developing in the short limb.

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