GEOPHYSICAL RESEARCH LETTERS, VOL. 12, NO. 1, PAGES 33-36, JANUARY 1985

SOURCE PULSE ENHANCEMENT BY DECONVOLUTION OF AN EMPIRICAL GREEN'S FUNCTION

Charles S. Mueller*

U.S. Geological Survey, Office of Earthquakes, Volcanoes, and Engineering, Menlo Park, CA

Abstract. Observations of the earthquake source-time function are enhanced if path, recording-site, and instrument complexities can be removed from seismograms. Assuming that a small earthquake has a simple source, its seismogram can be treated as an empirical Green's function and deconvolved from the seismogram of a larger and/or more complex earthquake by spectral division. When the deconvolution is well posed, the quotient spectrum represents the apparent source-time function of the larger event. This study shows that with high-quality locally recorded earthquake data it is feasible to Fourier transform the quotient and obtain a useful result in the time domain. In practice, the deconvolution can be stabilized by one of several simple techniques.

In this paper, the method is implemented and tested on high-quality digital recordings of aftershocks of the Jan. 9, 1982 Miramichi (New Brunswick) earthquake. In particular, seismograms from a Jan. 17 aftershock (017 13:33 GMT, local mag.=3.5) exhibit path or site effects which complicate the determination of source parameters. After deconvolution, the apparent far-field source of this event is a simple pulse in displacement with duration ≈ 0.07 second for both P and S.

Introduction

Small-earthquake seismograms were used as empirical Green's functions by Hartzell [1978] in synthesizing strong-motion records from a complex earthquake. For the purposes of Hartzell's study, several subsequent studies [e.g. Kanamori, 1979; Hadley and Helmberger, 1980; Irikura, 1983; Frankel and Kanamori, 1983] and this paper, a useful empirical Green's function is a seismogram written by a simple earthquake which includes the responses of path, recording site, and instrument:

$$G(t)=S(t) * P(t) * R(t) * I(t), \text{ where } S(t) \approx \delta(t).$$

I assume in what follows that instrumentation is constant or at least that differences are well known. Then, assuming that a small earthquake has a simple source-time function, the question of its usefulness as an empirical Green's function reduces to how well its seismogram captures the essential aspects of path and recording site relevant to the large/complex earthquake under study. Practically speaking, the answer to this question involves earthquake locations and focal mechanisms.

In modeling near-source high-frequency strong ground motion, it may be appropriate to use different Green's functions for different parts of a large fault [e.g. Hartzell, 1978; Irikura, 1983]. Modeling longer periods is less restrictive as it is only necessary that the Green's function correspond to an average path and recording site [e.g. Kanamori, 1979]. The above studies, as well as many forward-modeling studies using theoretical Green's functions [e.g. Heaton, 1982], all share the need to assume a space-time source behavior for the earthquake to be modeled. In contrast, the apparent source-time function is obtained directly using the method outlined in this paper.

It is conceptually straightforward to isolate the source-time spectrum of a large/complex event (considered as a point source) by dividing its spectrum by the spectrum of an empirical Green's function. Bakun et al. [1976] used this technique to examine the spectral amplitudes of small central California earthquakes. As the present study shows, with high-quality data the quotient spectrum can be transformed to the time domain, yielding the apparent far-field source-time function of the large/complex event. Time-domain observations have advantages over amplitude spectra alone in seismic source modeling. In either domain, meaning can be attached to the result by assuming linearity and assuming that the far-field source-time function of the small earthquake is a delta function in displacement. The spectral division serves to deconvolve path, recording-site, and instrument complexities. Then for frequencies less than the corner frequency of the Green's function (where the delta-function assumption holds) the result represents the source-time function of the large/complex earthquake in units of the seismic moment of the Green's function event. Equivalently, the result represents a scaled version of the displacement seismogram which the large/complex earthquake would have written in a homogeneous whole-space.

In earthquake seismology, deconvolution techniques have primarily been used in two areas of teleseismic-body-wave research. In one type of study, upper-mantle or crustal structure was determined using spectral division to deconvolve an independently determined source wavelet from teleseismic waveforms [e.g. Helmberger and Wiggins, 1971; Clayton and Wiggins, 1976]. In another type of study, source-time functions of large earthquakes were isolated using time-domain deconvolution of simple theoretical Green's functions [e.g. Boatwright, 1980; Kikuchi and Kanamori, 1982; Ruff and Kanamori, 1983]. Here I apply the spectral-division technique to relatively high-frequency local earthquake data; the main objective being the determination of the source function. Divisional deconvolution does not require any assumptions about the empirical Green's function as a time series [Clayton and Wiggins, 1976] and also provides a useful intermediate result, the quotient spectrum. Incorporation of the empirical Green's function also makes the method robust seismologically; if the Green's function is well chosen, no assumptions need to be

---

* also at Dept. of Geophysics, Stanford Univ.

Paper number 4L6368.

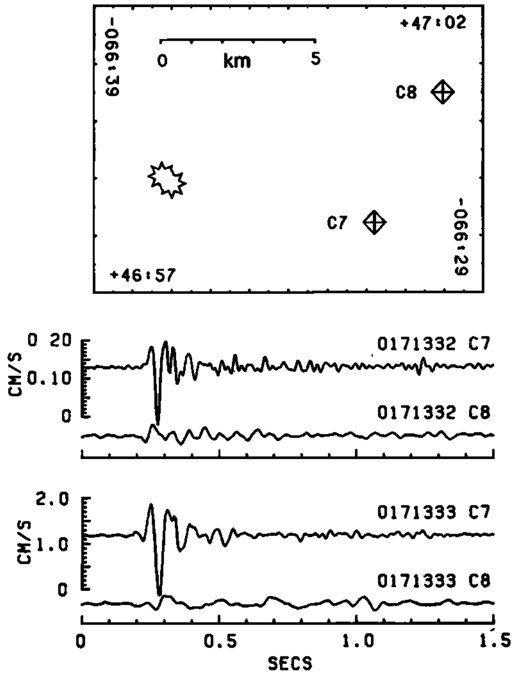

Fig. 1. Map with stations C7 and C8 and epicenters 13:32 (mag.=2.8) and 13:33 (mag.=3.5). Transverse-component velocity traces reveal the striking difference in ground motion at C7 and C8.

made about the details of source, path, recording site, or instrument. In this paper, one possible implementation of this technique is described and tested using high-quality digital recordings of aftershocks of the Jan. 9, 1982 Miramichi (New Brunswick) earthquake.

Data

From Jan. 15 to Jan. 22, 1982 the U.S. Geological Survey recorded aftershocks of the Jan. 9 Miramichi (New Brunswick, Canada) earthquake (m_b=5.7 [Wetmiller et al., 1984]) using portable event-recording seismographs deployed in the mainshock epicentral area [Cranswick et al., 1982]. Output from 2-Hz, three-component geophones was anti-alias filtered (50 Hz) and recorded digitally at 200 samples/second/channel. The largest aftershock recorded by the USGS portable network was an event on Jan. 17 (017 13:33 GMT, mag.=3.5). This event and a foreshock (017 13:32 GMT, mag.=2.8) were recorded at two stations: C7 and C8. (Figure 1). Aftershock locations and magnitudes are from R. Wetmiller (written communication, 1983); details of these determinations are given in Wetmiller et al. [1984]. Transverse-component velocity seismograms from these two events are plotted in Figure 1. Cranswick et al. [1982] noticed the remarkable character of these seismograms; ground motion at C7 was higher amplitude and shorter duration than at C8. In a study of Miramichi-aftershock source parameters, Mueller and Cranswick [1984] found corner frequencies of 15 and 6 Hz at C7 and C8, respectively, for event 13:33. They noted that the resolution of this discrepancy was important in the context of seismic-source scaling in eastern North America.

In the following section, the 13:32 seismograms are treated as empirical Green's functions and deconvolved from 13:33 seismograms in order to determine the apparent source-time function at C7 and C8. Wetmiller's results show that these events had similar hypocenters, suggesting that the deconvolution is well posed in this case. Focal mechanisms were not determined.

Method

Figure 2 shows the deconvolution results for vertical and transverse components at C7 and C8: (a) C7-PZ, (b) C7-SH, (c) C8-PZ, and (d) C8-SH. Numerator and denominator traces are shown at the top of each figure. The method does not require the use of displacement seimograms, but this facilitates comparison with the deconvolution result which is essentially a displacement waveform. Displacement records were obtained from seismograms via instrument correction, integration, and a 1-Hz zero-phase-shift low-cut filter. In each case 1.0 second of seismogram was windowed, tapered, and padded prior to the Fourier transform. Numerator, denominator, and quotient spectral amplitudes are shown at the center of each figure. Dividing the complex spectra accomplishes the deconvolution. This result, shown below the spectra, corresponds to the apparent far-field source-time function of the numerator event in units of the seismic moment of the denominator event.

As is well known from previous studies, the deconvolution is an unstable process; zeroes in the denominator dominate the quotient spectrum. In microearthquake data, this instability is typically observed at high frequencies where the signal-to-noise ratio is small. The spectral division must generally be modified to obtain a useful result. A high-cut filter applied to the numerator spectrum will reduce high-frequency instability but is not suitable if the denominator has zeros within the data band. When it can be used, the high-cut filter serves a dual purpose, both stabilizing the deconvolution and filtering out frequencies meaningless to the result (the Green's-function source approximates a delta function only at frequencies below its corner frequency). Helmberger and Wiggins [1971] suggested another technique for stabilizing the deconvolution in which squared spectral amplitudes of the denominator are not allowed to fall below a fraction of the peak squared spectral amplitude. Clayton and Wiggins [1976] have called the fraction parameter the waterlevel. This technique proved successful with the relatively high-frequency data used in this study. The deconvolutions in Figure 2 have been stabilized with waterlevel=0.001. Consideration of simple spectral shapes reveals that low frequencies may be erroneously modified by a waterlevel applied indiscriminately to acceleration or velocity spectra. This problem could be avoided by applying a spectral equalization and/or an instrument correction. The high-cut filter discussed above avoids this problem.

Apparent source functions for the 13:33 event in Figure 2 are one-sided pulses with similar amplitudes and duration ≈ 0.07 second. The simplicity of these pulses is remarkable considering the complexity of the input seismograms and spectra. This complexity must be due to path or site (note the coincidence of spectral peaks and troughs). After deconvolution, corner frequencies in the C8

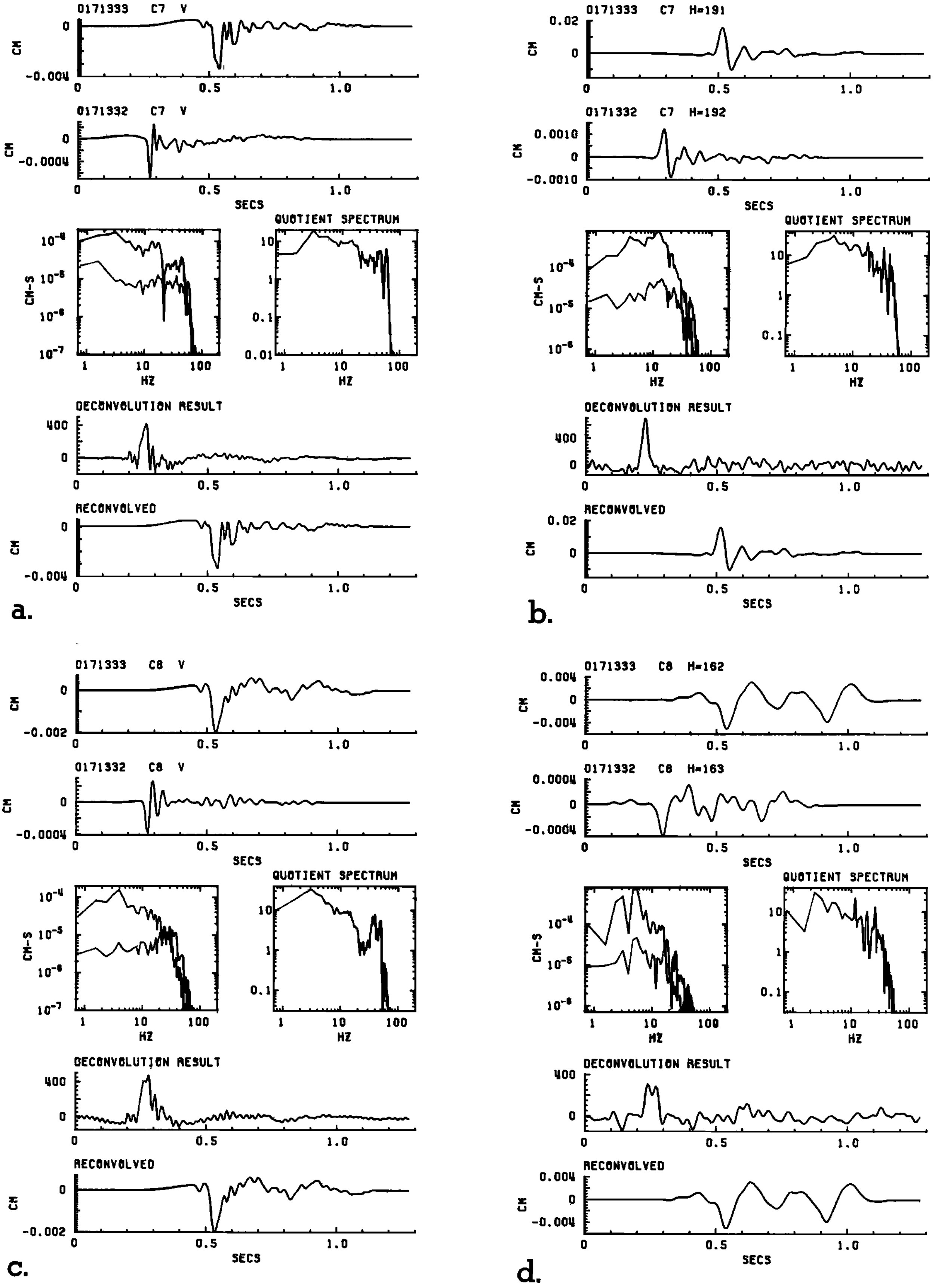

Fig. 2. Deconvolution of 13:33 seimograms using 13:32 as empirical Green's functions: (2a) C7-PZ, (2b) C7-SH, (2c) C8-PZ, (2d) C8-SH. The top of each figure shows the numerator and denominator traces. Spectral amplitudes are shown in the center. The quotient spectrum has been stabilized using waterlevel=0.001. The deconvolution result represents the far-field source-time function of the numerator event. The reconvolution closely matches the numerator in each case.

quotient spectra are near 15 Hz (best identified on smoothed spectra), consistent with those at C7. Considering each station alone, apparent P- and S-wave source durations are identical while both pulses are slightly broader at C8 than C7, perhaps due to source directivity.

Discussion and Conclusions

In one sense the goal of the deconvolution is to find a time series which, when convolved with the empirical Green's function, yields the original seismogram. The correct interpretation of the deconvolution result is that at frequencies below the corner frequency of the empirical-Green's-function source, it is a scaled version of the seismogram which the earthquake would have written in a homogeneous wholespace. The reconvolution can serve as a check on the severity of the manipulations employed to stabilize the deconvolution, but it should only be compared with a high-cut-filtered version of the original seismogram. This point has not been emphasized in other studies using empirical Green's functions.

The method will yield the most interesting results when the two earthquakes under study are widely separated in size, because of the assumption that the source of the empirical Green's function is a delta function. So practically speaking, the usefulness of the technique is limited by the dynamic range of the recording system. In practice, some form of automatic gain-ranging or manual gain adjustment would maximize dynamic range and thereby maximize the usefulness of the technique. However as the two earthquakes become widely separated in size, the method is self-limiting because a single Green's function may no longer be appropriate if the earthquake under study departs significantly from a point source. Part of a complex earthquake, for example the nucleation, could still be studied. It is also worth pointing out that some pathological cases exist, even when the deconvolution seems well posed in terms of hypocenters and focal mechanisms. For example, the method may not work if the large/complex earthquake source has significant directivity and a locally recorded seismogram contains significant reflected energy.

With high-quality data the method described in this paper should enhance seismic-source studies with locally recorded body waves. The divisional-deconvolution technique does not require any assumptions about the seismograms as time series [Clayton and Wiggins, 1976]. Detailed knowledge of path, recording site, and instrument responses is not required, only that they be similar for both earthquakes. Properly deconvolved seismograms will ideally be free of distortion from "deterministic" wave-propagation effects as well as "stochastic" scattering effects, distortion which has inevitably compromised past source studies (Helmberger and Malone, 1975; Aki, 1981). Ground-motion nonlinearity can also be studied by comparing apparent source-time functions at different stations for large earthquakes.

Acknowledgements. E. Cranswick and E. Sembera collected and processed the Miramichi aftershock data. I am grateful for the comments and encouragement of R. Archuleta, J. Boatwright, D. Boore, E. Cranswick, and P. Spudich. P. Spudich brought the pathological case mentioned in the text to my attention.

References

Aki, K., Source and scattering effects on the spectra of small local earthquakes, Bull. Seismol. Soc. Am., 71, 1687-1700, 1981.

Bakun, W.H., C.G. Bufe, and R.M. Stewart, Body-wave spectra of California earthquakes, Bull. Seismol. Soc. Am., 66, 363-384, 1976.

Boatwright, J., Preliminary body-wave analysis of the St. Elias, Alaska earthquake of February 28, 1979, Bull. Seismol. Soc. Am., 70, 419-436, 1980.

Clayton R.W. and R.A. Wiggins, Source shape estimation and deconvolution of teleseismic bodywaves, Geophys. J. R. Astron Soc., 47, 151-177, 1976.

Cranswick, E., C. Mueller, R. Wetmiller, and E. Sembera, Local multi-station digital recordings of aftershocks of the January 9th, 1982 New Brunswick earthquake, U.S. Geol. Surv. Open-File Rep. 82-777, 267 pp., 1982.

Frankel, A. and H. Kanamori, Determination of rupture duration and stress drop from earthquakes in southern California, Bull. Seismol. Soc. Am., 73, 1527-1551, 1983.

Hadley, D.M. and D.V. Helmberger, Simulation of strong ground motions, Bull. Seismol. Soc. Am., 70, 617-630, 1980.

Hartzell, S., Earthquake aftershocks as Green's functions, Geophys. Res. Lett, 5, 1-4, 1978.

Heaton, T.H., The 1971 San Fernando earthquake: a double event?, Bull. Seismol. Soc. Am., 72, 2037-2062, 1982.

Helmberger, D.V. and R.A. Wiggins, Upper mantle structure of midwestern United States, J. Geophys. Res., 76, 3229-3245, 1971.

Helmberger, D.V. and S.D. Malone, Modelling local earthquakes as shear dislocations in a layered half space, J. Geophys. Res., 80, 4881-4888, 1975.

Irikura, K., Semi-empirical estimation of strong ground motions during large earthquakes, Prev. Res. Inst. Kyoto Univ., 33, 63-104, 1983.

Kanamori, H., A semi-empirical approach to prediction of long-period ground motions from great earthquakes, Bull. Seismol. Soc. Am., 69, 1654-1670, 1979.

Kikuchi, M. and H. Kanamori, Inversion of complex body waves, Bull. Seismol. Soc. Am., 72, 491-506, 1982.

Mueller, C.S. and E. Cranswick, Source parameters from locally recorded aftershocks of the Jan. 9, 1982 Miramichi, New Brunswick earthquake, Bull. Seismol. Soc. Am., in press, 1984.

Ruff, L. and H. Kanamori, The rupture process of three great earthquakes from long-period diffracted P-waves, Phys. Earth Planet. Inter., 31, 202-230, 1983.

Wetmiller, R.J., J. Adams, F.M. Anglin, H.S. Hasegawa, and A.E. Stevens, Aftershock sequences of the 1982 Miramichi, New Brunswick, earthquake, Bull. Seismol. Soc. Am., 74, 621-653, 1984.

C. S. Mueller, U.S. Geological Survey, 345 Middlefield Rd. MS 977, Menlo Park, CA 94025.

(Received September 26, 1984;
accepted October 19, 1984.)