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A 3-D gravity inversion tool based on exploration of model possibilities☆

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Abstract

A computational tool for the development and implementation of a recently published method of 3-D (three dimensional) inversion for gravity data is presented. This method seeks to determine the geometry of an indefinite number of anomalous bodies with prescribed (fixed or variable) density contrasts, positive and negative values being indiscriminately accepted in the model. The approach is based on a prismatic partition of the subsurface and attempts to determine the anomalous bodies by means of a "growth" sequence, analysing (systematically or randomly) the several model possibilities and from that choosing the best for the growth progress. Moreover, a regional trend for the gravity data can be simultaneously adjusted. The non-uniqueness of the gravity inversion is avoided by means of a mixed condition about the residuals and the whole body anomalous mass. This inversion method has been applied with good results to simulation tests and to several real examples. Here, we present a main program that realises the inversion according to several possibilities for general application (scale of the survey, fixed or variable density contrasts, optional smoothing, optional trend adjustment, systematic or random exploration, optional a priori information, weighting, etc.). This program is presented along with a previous program for selection of unknowns and parameters and another program for visual presentation of the results. All three programs are written in Fortran 77 and completes the inversion tool.

Keywords: Gravity inversion; Three-dimensional models; Model exploration; Gravity anomaly; Anomalous density contrast

1. Introduction

The gravity inversion problem usually deals with discrete, inaccurate and irregularly sampled data of some gravity anomaly with the aim of obtaining information about the distribution of the subsurface anomalous masses that are the source of the detected gravity anomaly. The non-uniqueness of solution is characteristic of this inversion problem. This demands a geological or mathematical hypothesis that constrain the problem and allow to obtain realistic results. For instance, a usual approach consists of constraining the possible density contrasts of the anomalous structures and then looking for the geometry of these anomalous bodies. The above procedure corresponds to a non-linear context. In this case, the traditional methods for non-linear approach work iteratively, for instance by means of gradient calculations (e.g., Farquharson and Oldenburg, 1998), starting from an approximate initial solution. These methods are dependent on the quality of the initial model to define the unknown geometrical parameters and to assure the convergence of the non-linear process. In Camacho et al. (2000), we propose a method of 3-D (three dimensional) gravity inversion inspired by the method of René (1986) and based on a "growth" process that works by means of exploration of the model possibilities (see Tarantola, 1987 for general

☆Code available from server at http://www.iamg.org/CGEditor/index.htm

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methodology for inverse problem and about exploratory methods) with constraints for the density contrasts. Moreover, instead of adjusting a small number of geometrical parameters, such as vertex coordinates for the anomalous body (e.g. Enmark, 1981), this method determines the geometry of the body as an accretion of filled cells defined from a general 3-D grid of the subsurface.

The determination of the regional trend of the gravity data is a rather polemic question. Usually, this component is calculated by means of a smoothing criteria, for instance a polynomial fit or a filtering technique, and removed prior to inversion (e.g. Beltrao et al., 1991). We propose to calculate this trend simultaneously with the inversion process.

2. Gravity inversion method

Consider a gravity data set observed at n gravity stations irregularly distributed (see Fig. 1). Let (x_i, y_i, z_i), $i = 1, \ldots, n$, be the planar coordinates (e.g., UTM coordinates) and the altitudes of the gravity stations and gravity anomaly (Bouguer) be P_i and Δg_i, respectively. We represent the uncertainty in the gravity data by a data covariance (n, n)-matrix, $\mathbf{Q}_D$ (Tarantola, 1987), where the elements $q_{ij} = 0$, for $i \neq j$, and $q_{ii} = e_i^2$, and e_i, $i = 1, \ldots, n$, are standard deviations of the gravity values.

To determine the geometry of the anomalous bodies, we decompose the neighbouring subsurface volume into a 3-D grid of m contiguous cells (see Fig. 1). The adopted basic cell j, $j = 1, \ldots, m$, is the parallelepiped, whose attraction, for unit mass density, at the survey point P_i, $i = 1, \ldots, n$, is represented by a_{ij} and can be calculated according to Pick et al. (1973).

$$a_{ij} = -G\left[\left[\left[x\ln(y + (x^2 + y^2 + z^2)^{1/2}) + y\ln(x + (x^2 + y^2 + z^2)^{1/2}) + z\arctan \times (z(x^2 + y^2 + z^2)^{1/2}x^{-1}y^{-1})\right]_{u_1^j - x_i}^{u_2^j - x_i}\right]_{v_1^j - y_i}^{v_2^j - y_i}\right]_{w_1^j - z_i}^{w_2^j - z_i}, \qquad (1)$$

where G is the gravitation constant, the edges of the jth prism are parallel to the reference axes, and the limiting coordinates for its volume are u_1^j, u_2^j for the x coordinate, v_1^j, v_2^j for the y coordinate and w_1^j, w_2^j for z (negative downwards). So, $\mathbf{A}$ will be the design (n, m)-matrix with elements a_{ij}. Usually, we suppose $m > n$. The anomalous bodies responsible for the observed gravity anomaly will be determined as a composition or accretion of prismatic cells with an assigned density contrast.

Apart from the data and the subsurface partition, the third characteristic element of the inversion method is the set of prescribed density contrasts for the anomalous bodies. In fact, we consider some a priori density contrasts to fill the cells and then build the anomalous bodies. An interesting advantage of this method is the possibility of considering both positive and negative density contrasts (excess and deficit of density anomaly from a non-anomalous subsurface). So, for each of the jth cell we accept three possible anomalous model density contrasts: $\Delta\rho_j^-$ (negative), $\Delta\rho_j^+$ (positive) and 0 (not anomalous). Filling the cells with the possible contrasts, the anomalous bodies are determined step by step in a "growth" process. Moreover, $\Delta\rho_j^-$, $\Delta\rho_j^+$, $j = 1, \ldots, m$, can represent constant values for the whole subsurface volume, and also different values spatially defined (for instance, according to prescribed zones or as a result of vertical stratification based upon a geological hypothesis, but with only three values, zero positive one and negative one, for each cell) or "temporally" variable values (across the growth process).

With these basic elements (division of the medium into 3-D prismatic cells and a priori contrast density values) we try to determine the anomalous bodies by an "expansion" or "growth" process, filling step by step cells of the 3-D grid with the prescribed density contrast (Camacho et al., 2000). If we consider $\mathbf{m}$ to be the column vector of model density contrasts $\Delta\rho_j$, $j = 1, \ldots, m$, for the 3-D grid cells (where $\Delta\rho_j$ can take only the values $\Delta\rho_j^-$, $\Delta\rho_j^+$ and 0, according to the constrains), initially, and without previous particular hypothesis, $\mathbf{m}$ is identically null. The algorithm works step by step. For the kth step, the "growing" anomalous body is

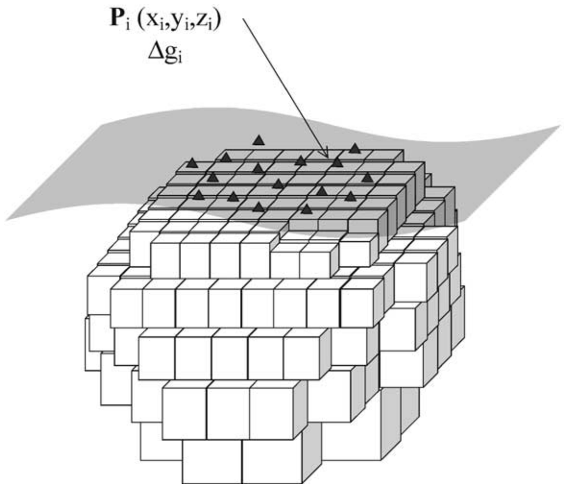

Fig. 1. Sketch of gravity stations P_i (irregularly located upon non-planar surface) and 3-D grid of parallelepiped representing neighbouring subsurface volume.

composed of $k-1$ cells, filled with the possible a priori density contrast. A new cell is searched (among the actually not filled cells) to be filled, with the prescribed density contrast, as new expansion of the anomalous body. The condition for choosing this cell for the kth expansion is that the new structure (composed by the previous $k-1$ cells plus the new considered cell) fit, with a positive scale factor, the observed gravity anomaly. The process continues obtaining further expansions of the anomalous body (corresponding to decreasing scale factors). When the anomalous body reaches its "real" size, the scale factor to fit the model anomaly with the observed anomaly is very close to 1, and then the algorithm stops.

The adoption of a priori contrasts drastically reduces the indeterminacy, but the fact of accepting both positive and negative density contrasts again produces a non-uniqueness problem, requiring a new general hypothesis that permits us to seek a definite solution for this gravity inversion. As a first step, the several possibilities of the model (each cell and each prescribed density contrast) are explored selecting a first cell (with its positive or negative prescribed contrast) so that the corresponding gravity effect upon the stations provided the best fit, by means of a suitable scale factor $f_1 \geqslant 1$, to the observed anomaly. Then, a second element is searched, again, by means of exploration, so that the model composed by these two elements fits, with a scale factor f_2 ($f_1 \geqslant f_2 \geqslant 1$), the observed anomaly. The "growth" process continues step by step. Nevertheless, already from the selection of the second element the positive–negative option introduces a non-uniqueness in the inversion. This question is avoided by means of a new general constraint that we shall describe with a generic kth step of the process.

Let us suppose that $k-1$ cells, named $j=\lambda_1, \ldots, \lambda_{k-1}$, have been filled with their density contrast values. The gravity value created by this $(k-1)$-th model upon the ith station is

$$g_i^{(k-1)} = \sum_{j=\lambda_1}^{\lambda_{k-1}} a_{ij}\,\Delta\rho_j. \tag{2}$$

A new cell is searched to be filled thus expanding the model. For greater power of the inversion method, we try to simultaneously adjust a simple regional trend, linear for instance, with unknown coefficients p_0, p_x, p_y ($p_0^{(k)}$, $p_x^{(k)}\, p_y^{(k)}$ for the k step). Then, we test each jth, $j \neq \lambda_1, \ldots, \lambda_{k-1}$, free cell and each respective possible density contrast $\Delta\rho_j$ (i.e., $\Delta\rho_j^+$, $\Delta\rho_j^-$) obtaining the residuals $v_i^{(k)}$ ($v_i^{(k)}(\Delta\rho_j^+)$, $v_i^{(k)}(\Delta\rho_j^-)$, respectively) for the gravity stations in the form:

$$\Delta g_i - (g_i^{(k-1)} + a_{ij}\,\Delta\rho_j)f_k - (p_o^k + p_x^{(k)}(x_i - x_{\mathrm{M}}) + p_y^{(k)}(y_i - y_{\mathrm{M}})) = v_i^{(k)} \quad i = 1, \ldots, n, \tag{3}$$

where x_{M}, y_{M} are the coordinates of an arbitrary central point for the survey (for example, mean coordinates). The k-model is constituted by the $k-1$ previous filled cells and the actual tested jth cell with one of its possible density contrasts. For this model, the unknown values f_k, $p_o^{(k)}$, $p_x^{(k)}$, $p_y^{(k)}$, for scale factor and linear trend, are determined by means of a least squares adjustment that fits the anomalies of this "actual" kth model to the observed gravity data (see Camacho et al., 2000, for further details).

Taking into account the positive and negative options and the trend unknown, the minimisation of residuals $v_i^{(k)}$ is not a sufficient condition to decide the suitability of this cell and its density contrast for this k step. To solve this question, we propose a mixed minimisation condition about the residuals of the adjustment ("fitness") and about the total anomalous mass ("smoothness"):

$$\mathbf{v}_k^{\mathrm{T}}\mathbf{Q}_{\mathrm{D}}^{-1}\mathbf{v}_k + \lambda f_k^2 \mathbf{m}_k^{\mathrm{T}}\mathbf{Q}_{\mathrm{M}}^{-1}\mathbf{m}_k = \min, \tag{4}$$

where $\mathbf{v}_k = (v_1^{(k)}, \ldots, v_n^{(k)})^{\mathrm{T}}$ (T for transposed) is the column vector of residuals for the kth step, $\mathbf{m}_k$ is the column vector of the density contrasts included in the kth model (the previously assigned $\Delta\rho_{\lambda_1}, \ldots, \Delta\rho_{\lambda_k}$ plus the new $\Delta\rho_j$ tested for the actual step), λ is a positive factor fixed to balance between model fitness and model smoothness, and $\mathbf{Q}_{\mathrm{M}}$ is a suitable covariance matrix corresponding to the determinability or sensibility of the 3-D grid cells from the gravity stations. For a problem without previous information about the model structure, we suggest (Camacho et al., 1997) to take a model covariance matrix $\mathbf{Q}_{\mathrm{M}}$ given by a diagonal normalising matrix of non-null elements that are the same as the diagonal elements of $\mathbf{A}^{\mathrm{T}}\mathbf{Q}_{\mathrm{D}}^{-1}\mathbf{A}$.

The first addend of the minimisation functional Eq. (4) corresponds to the adjustment residuals ($\mathbf{v}_k$) weighted with the data quality matrix. The second addend is a weighted addition of the model density contrasts. Given that the matrix $\mathbf{Q}_{\mathrm{M}}^{-1}$ contains the prism volumes as factor, this second addend is connected with the anomalous mass of the model. In this form, for high λ value the model keeps a small total anomalous mass (with small and smooth volume) but offers a poor fit; conversely, low λ value gives a good fit and introduces a lot of structures in the model.

The value given by Eq. (4) is chosen as the suitability parameter for this jth cell and its density contrast. After testing each jth, $j \neq \lambda_1, \ldots, \lambda_{k-1}$, free cell and each respective possible density contrast, the cell and the prescribed density contrast that minimise Eq. (4) are selected as the optimum expansion for this kth step. In this way, the "growth" of the anomalous bodies continues, and in each step a nearly decreasing scale factor $f_k \geqslant 1$ is obtained. The process stops when $f_k \cong 1$, providing a final model constituted derived from several cells filled with their (positive or negative) assigned density contrasts and the final values for the adjusted regional trend (p_0, p_x, p_y).

The proposed method of gravity inversion has the following advantages: (1) inaccurate, non planar and irregularly distributed data are accepted; (2) previous models are not necessary, but if they exist, they can be incorporated into the inversion process (see Camacho et al., 2000); (3) a 3-D context is supposed; (4) an indefinite number of anomalous bodies can be adjusted; (5) a regulation of the fit level and of the model complexity is conducted; (6) a simple regional trend can be simultaneously adjusted; and (7) positive and negative density contrasts are simultaneously accepted in the model. Moreover, the software presented in this paper incorporates several options (about weighting, exploration method, a priori densities, etc.) that offer a versatile tool for the gravity inversion.

In Camacho et al. (2000) and Araña et al. (2000), several application examples are included. Here we show some additional examples.

3. Program description

We present a tool for gravity inversion composed of three programs. The first is "PARAM" a preliminary program for selection of parameters, options and unknowns. The second is "GROWTH", the main program for gravity inversion. The third is, "SECTIONS", a simple program needed for visualisation of the results.

The previously described inversion method, that does not require matrix inversion, is not very concerned about memory size. Moreover, the vectors have been reduced, whenever possible, to Integer*2 form. The final memory size depends on the application parameters: number of gravity stations (400 for instance) and desired resolution level of the model (12,000 cells, for instance).

The basic input data for the application is an ASCII file, named "GRA.DAT". This file contains a line corresponding to each gravity station with the following values: x-coordinate (m), y-coordinate (m), altitude (m), gravity anomaly (μGal) and relative error estimation (μGal) ($1\,\mu\text{Gal} = 10^{-8}\,\text{m/s}^2$). The end of the data is indicated with a line of null values.

3.1. "PARAM" program

This program reads the data file GRA.DAT and requires, in a dialog form that offers default values or example values, several parameters to create an output file named "PRI.DAT". This file contains the 3-D grid of cells and other data that will be used for the rest of the inversion tool.

The values required by the PARAM program are the following:

1. A scale factor ($\delta \geqslant 0$) for planar and vertical coordinates.
2. Centre coordinates x_m, y_m and z_m for the model.

The previous parameters require the user to confirm (or modify) the suggested default values. These arbitrary mean values will be used to adapt the coordinates to an Integer*2 format and also to represent the regional gravity trend and the relative depths (see below).

3. Depth (m) from the surface of the 3-D grid bottom.
4. Side length (m) of the smallest cell.
5. Minimum margin (m) from the survey elevation to the top of the model cells.
6. Maximum relative error for the 3-D cells.

The value of relative error for each cell is calculated as being inversely proportional to the root mean gravity attraction for the whole of the gravity stations, further normalised by the error value of an arbitrary central cell. This relative value and the proximity of the cells to the previous model masses (if this model exists) are used to limit the volume of the 3-D grid for the model adjustment.

7. Coefficient $\varepsilon \geqslant 0$ for vertical enlargement of the cell sides as a function of depth.
8. Use data from previous model: Yes/No.

This last option permits the use of a previous model that is contained in a file named "MOD.DAT" with the same format as the final result of the inversion process. The previous model can proceed from a low-resolution pre-adjustment or from an external information as a priori model. With the use of this previous model we accomplish two tasks: first, we constrain the adjusted solution to be close to the previous solution, and, secondly, we can use this previous model to restrict the volume of the 3-D grid according to the volumes of pre-located anomalous bodies. From this previous model, the "PARAM" program assigns, by means of interpolation and smoothing, a priori density contrast values for the cells of the new 3-D grid. These assigned contrasts can be optionally considered as fixed values in the further gravity inversion. We suggest a process of the adjustment of successive models with an increasing resolution.

The parameters discussed above define the geometrical configuration of the 3-D grid. The number of resulting cells of the 3-D grid mainly depends on the side of the smallest cell and on the volume limits coming from the previous model and from the former question number 6. Once the 3-D grid has been established, the program prompts the user for other parameters that are related to the prescribed values for the gravity inversion:

9. Stratification of the anomalous mass: Yes/No.

This option permits us to consider the case of some application where we desire that the prescribed density contrast values for the growth process of the anomalous bodies present a determined vertical increase with the depth for the whole model. For instance the case corresponding to a compaction in sedimentary basins.

10. Stratification: exponential or by means of layers.

Actually, two possible stratification patterns are considered with this option: exponential and layered. Moreover, an arbitrary spatial variation of the a priori density contrasts can be introduced later directly on the file "PRI.DAT".

For the case of exponential stratification, we adopt the following expressions for the prescribed density contrast ($\Delta\rho_j^-$, $\Delta\rho_j^+$) corresponding to the jth cell located in a depth z_j

$$\Delta\rho_j^- = R_N \, e^{-\theta_N(Z_N - z_j)/\delta}, \quad \Delta\rho_j^+ = R_P \, e^{-\theta_P(Z_P - z_j)/\delta},$$

where R_N and R_P are the extreme density contrasts (N-negative and P-positive) for the model, Z_N and Z_P are selected heights close to the top of the model, and $\theta_N \geqslant 0$ and $\theta_P \geqslant 0$ are coefficients to characterise decreasing density contrasts with depth (the coefficient δ for normalising is introduced in step 1). So, for exponential stratification the following parameters are required:

10a. Extreme density contrasts (kg/m^3), negative R_N and positive R_P, for the further inversion. This is the only question that appears for the usual case of spatially uniform density contrast (no stratification of the anomalous mass).
10b. Depths (m), Z_N and Z_P, for the extreme density contrasts, negative and positive.
10c. Coefficients θ_N and θ_P for the exponential decreasing. (For instance, about 0.2.)

For the case of stratification by means of layers, the program asks about the prescribed density contrasts (negative and positive) for each layer of the cell distribution in the 3-D grid:

10d. Density contrast (kg/m^3), $\Delta\rho^-$ and $\Delta\rho^+$, for the cells located in each layer of the 3-D grid.

Once the 3-D grid is defined, the next questions proposed by PARAM consider the several parameters that will be used in the further gravity inversion. They will be read and transmitted on PRI.DAT for the application of the main inversion program.

11. Balance factor $\lambda > 0$ of the minimisation condition (4).
12. Smooth coefficient $\tau \geqslant 0$.

This last parameter allows us to prescribe a priori varying density contrasts $\Delta\rho^+$, and $\Delta\rho^-$, that decrease with the growth process in the form

$$\Delta\rho^- = R_N \, e^{-\tau f_k}, \quad \Delta\rho^+ = R_P \, e^{-\tau f_k},$$

where $f_k \geqslant 1$ is the variable scale factor across the inversion process. This option corresponds to a possibility of "temporal" variability of the prescribed density contrasts suggested in the method exposition. The extreme density contrast R_N and R_P will be used in the first step of the anomalous body construction, then corresponding to the main gravity anomalies. When the "growth" process continues the density contrast employed will be smaller, using small density contrast (negative and positive) for the periphery of the anomalous bodies and for the smallest gravity anomalies.

13. Random coefficient $\alpha \geqslant 1$.

This parameter permits us to substitute the systematic exploration of the model possibilities (for $\alpha = 1$) by a faster random exploration. Instead of exploring all the m cells, only m/α cells (randomly selected at each time) are explored for the model growth. As usual for heuristic techniques, the random exploration allows us to obtain good solutions but not necessarily the optimal one (e.g. Montesinos, 1999, presents several gravimetric applications with random searching). For instance, we can use $\alpha = 6$ for the first inversion tests. An advanced model could be obtained with $\alpha = 2$, and the final "good" model will correspond to $\alpha = 1$.

14. Further weighting of the gravity data: Yes/No ($k_w = 1, 0$).

The gravity inversion process can optionally consider the relative error values of the gravity data (if available) from the file "GRA.DAT" to an inversion with weighted data.

15. Further use of the a priori density values in the inversion calculus: Yes/No ($k_d = 1, 0$).

By means of this option, the gravity inversion accepts further a priori contrast values (obtained here by interpolating from the previous model) as definitive ones. It can serve to include previously known values from which a faster solution is obtained. If we answer affirmatively then we are prompted for the next question.

16. Maximum error for accepting density contrast values from the previous model.

This parameter permits us to consider, for the previous model, those cells with a relative error smaller than the input value (according to some normalised error values) so, as to eliminate the very peripheral or erroneous cells of the previous model.

17. Further use of the a priori model errors: Yes/No ($k_e = 1, 0$).

This option enables the user to consider an a priori model weighting system contained in the file "PRI.DAT" or, alternatively, to accept the usual model weighting system (matrix $\mathbf{Q}_M^{-1}$) included as the default in the "GROWTH" program.

18. Number $n_h \geqslant 0$ of a priori horizontal surfaces of discontinuity for the non-anomalous volume.

We know that the gravity survey is nearly insensitive to horizontal stratification. Therefore, the inversion model reflects only the anomalous structures independent of the existence of a general horizontal stratification. Nevertheless, we can introduce, by means of this option, an a priori general stratification constituted by several horizontal layers for the non-anomalous volume. The resulting inverse model is somewhat different because the approach keeps the anomalous density across the stratified subsurface. If the proposed number of layers is $n_h > 0$, then the program requests a priori information about these layers.

19. Depth (in meters, negative downwards) h(*j*) and density contrast (in kg/m^3, positive everywhere) d(*j*) for each *j*th discontinuity surface, for $j = 1, \ldots, n_h$.
20. Fixed values for regional trend of the gravity data: Yes/No.

As previously indicated, the inversion method is able to determine, simultaneously with the anomalous body adjustment, a linear trend. Nevertheless, for some particular applications we will desire to keep an a priori regional trend $(\Delta g)_R$ as

$$(\Delta g_i)_R = p_0 + p_x(x_i - x_M) + p_y(y_i - y_M),$$

(slopes p_x, p_y and/or base constant p_0), that is not subjected to further readjustment. For this case, the program next asks about the values of the trend parameters.

20a. Trend slopes: West–East (p_x) and South–North (p_y) (μGal/km), and base constant (p_0) (μGal) for the known regional trend of the gravity data. The input of a null value for some of these parameters is further interpreted as indicating that the parameter is an adjustable parameter and not a fixed parameter.

The 3-D grid of model cells, with the a priori values, and the other parameters for the gravity inversion appear in the output file "PRI.DAT".

3.2. "*GROWTH*" *Program*

This is the main program for the gravity inversion. The input files are "GRA.DAT" for the gravity data and "PRI.DAT" for the subsoil 3-D grid and the unknown parameters. No further information is required by the program. After the step by step growth, the results are written to an output file named "MOD.DAT". The format of this output file is the same as that of the input file "PRI.DAT":

(A) Three first lines showing the values of the several parameters and options of the gravity inversion:

m, λ, R_N, R_P, τ, k_w, k_d, α, k_e, ε,

δ, x_m, y_m, z_m, p_0, p_x, p_y,

n_h, (h(*j*), d(*j*), for $j = 1, \ldots, n_h$)

When R_N and R_P are null values, a spatial distribution of the prescribed density contrast values is considered, indicating the corresponding value for each cell in this file.

(B) *m* lines corresponding to the *m* cells of the model showing, for each cell, the following data:

1. The planar coordinates (m) x, y of the cell centre (referred to the coordinate centre x_m, y_m and taking into account the applied scale factor δ).
2. The horizontal sides Δx, Δy of the cell (once the scale factor has been applied).
3. The depths (m) (negative below sea level) of the top and bottom of the cell (referred to the values: z_m, origin, and δ, scale factor).
4. The assigned (or a priori in the case of the file "PRI.DAT") density contrast of the cell.
5. The calculated relative error that indicates the different solubility of the cells in the adjustment.
   Two additional values are included for the file "PRI.DAT":
6. The prescribed negative and positive density contrasts for the adjustment of this cell. These last values are used in the case of spatially variable distribution of density contrasts (for R_P and R_N equal to zero).

At the end of the file "MOD.DAT", some information about the resulting model is included. At this point we can see several results such as: mass and mean depth of the whole anomalous bodies, mass and mean depth of the positive/negative anomalous bodies, mean density contrast of the positive/negative anomalous bodies, top and bottom depths of the whole, positive and negative anomalous bodies, number of stations, total number of cells, number of cells with assigned positive/negative density contrast, maximum and minimum values of the side lengths of cells, regional trend, a priori stratification, and, as the most interesting values, the extreme and mean values of the gravity residuals produced by the inversion approach.

The calculated standard deviation of the inversion residuals and the visual identification of the geometry and distribution of the resulting anomalous bodies can help us to decide about a change of value for the λ parameter (and to carry out a new gravity inversion). For a low λ value, low residuals can be obtained but the model offers an overly complex geometry. On the contrary, for a high λ value a more simple model would be obtained but the fitness would be somewhat lower. For λ values significantly high or low the program stops pointing out this warning (nevertheless, the level of inversion residuals also depends on the cell sides, that is to say, on the resolution of the model). A possible criterion for the choice of λ is to select a value that allows one to obtain a standard deviation of the residuals that is close to the a priori suspected accuracy of the data. Nevertheless, this inversion approach is, itself, a good test of the data accuracy, identifying the very noisy stations as those with high inversion residual values that correspond to a non-invertible component of the data (noise).

The structure of the "GROWTH" program is comprised of a main program and two subroutines that carry out the calculus of the vertical attraction of a parallelepiped cell. The dimension of the vectors are connected to the number of stations and the number of cells (the total number of cells and the number of those cells that can be readjusted). These numbers are included in the first executable sentence (a "parameter" sentence) to be optionally changed. When the data size exceeds the adopted limits the program stops with a corresponding warning.

3.3. "*SECTIONS*" *program*

The program "SECTIONS" provides simple views of the models that take part in the inversion process (those corresponding to the files PRI.DAT and MOD.DAT). The data of GRA.DAT are also used to plot, on the screen, the gravity anomaly, the altitudes and the gravity data relative errors. A third file, named MAP.BLN, can contain an ASCII base map to superimpose upon the horizontal pictures (a first line with the total number of points and the following lines with the coordinates of the points defining the base map).

With the aid of "SECTIONS", the 3-D model of the anomalous bodies is visualised by means of horizontal sections at desired depths and vertical profiles. Two kinds of vertical profiles are displayed: those N–S, W–E, according to desired x, y values, and those oblique vertical profiles, corresponding to desired azimuths and centres. The basic screen for the model representation shows simultaneously one horizontal section and two vertical profiles (pointing out the topographical profile and the altitude level 0) with a colour scale for the density contrasts. In the next section, a typical view is shown. Optionally, output files can be obtained to export the figures (in an ASCII grid form) to another more sophisticated plotting device.

Additional views can be obtained for the model's relative errors, the regional trend of the gravity data, the inversion residuals and the model gravity anomaly (as generated by the inversion solution). A last image can show the evolution of the density contrast and the mass with depth. Corresponding to these options, ASCII output files (named "OUT.DAT" for default) can be optionally obtained.

4. Application examples

To give an idea of the results of this gravity inversion approach we consider two simulation examples referenced to the same distribution of gravity stations. An area of about 2000 m in diameter is covered with stations

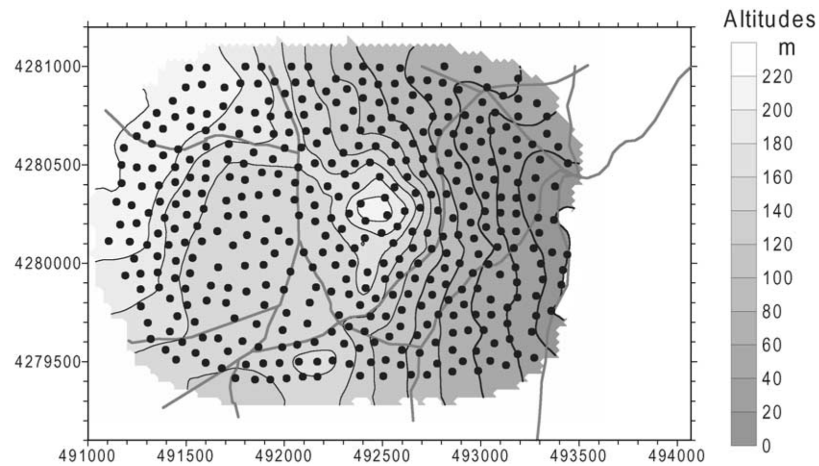

Fig. 2. Simulation example: contour map of altitudes ranging between 0 and 229 m and locations of 420 stations covering an area of 2000 m in diameter with a step of about 100 m. Axis coordinates are in m. Simulation base map is superimposed, that shows roads.

distributed at a mean increment of about 100 m. Fig. 2 represents the distribution of 420 stations, indicating their respective altitudes which range from 0 to 230 m. With this data distribution we consider two simulation examples.

4.1. Example 1

We suppose a buried anomalous structure is composed of two "L" shaped bodies. That is characterised by a density contrast of 400 kg/m^3 with respect to the non-anomalous subsurface matter and more or less aligned along the axis X. Fig. 3 gives a view of this structure (as given by means of the "SECTIONS" program, but using a grey scale instead of a coloured one). Several depths (always positive upwards) define the limiting horizontal surfaces for the geometrical bodies (100, 0, −50, −300, and −600 m, see Fig. 3). The anomalous masses and mass centres of these bodies are:

$$\text{body1}: \text{mass} = 264\times10^{11}\ \text{kg},$$
$$\text{centre}(X, Y, Z) = 4, 91, 559,\ 42{,}80{,}100,\ -134\ \text{m}$$

$$\text{body2}: \text{mass} = 360\times10^{11}\ \text{kg},$$
$$\text{centre}(X, Y, Z) = 4{,}92, 560,\ 42{,}80{,}100, -360\text{m}.$$

The whole anomalous mass is 624×10^{11} kg, with a mean Z value ("depth") of −264 m.

Additionally we shall suppose that, in the studied area, a linear regional trend is present, giving for the survey points an effect characterised by the following arbitrary values: $p_0 = 7000\ \mu\text{Gal}, p_x = 700\ \mu\text{Gal/km}$, $p_y = -700\ \mu\text{Gal/km}$. The "simulated" gravity field, obtained by adding the regional trend and the effect of the buried bodies is represented in Fig. 4. The contemplation of this map does not suggest directly the morphology of the bodies, moreover taking into account that there is a superimposed trend. The anomaly values range between 6003 and 8711 μGal. They are considered first as "exact" values.

At this point, we apply the gravity inversion programs as discussed above. The "PARAM" program suggests the default values $\delta = 0.240$, $x_m = 4{,}92{,}240$, $y_m = 42{,}80{,}160$ and $z_m = 120$. To get a more restricted volume and to adopt suitable parameters, we apply the inversion process starting from low resolution models (for instance with cells of sides greater than 100 m) that let us obtain (using a random exploration, for instance with $\alpha = 10$) a fast test of the suitable values for λ. After several tests with increasing resolution, we adopt the following final model: size of the smallest cells = 25 m, number of cells $m = 8500$, $\lambda = 1.4$ (a small value close to the operative limit, that lets us obtain very small final

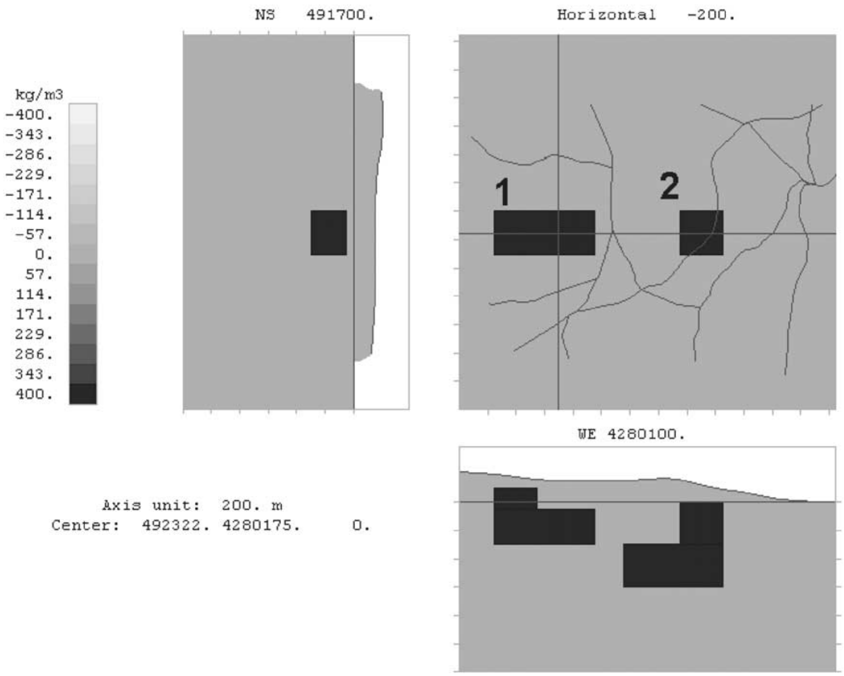

Fig. 3. Representation of original anomalous structure for first simulation test. Two geometrical bodies of positive density contrast 400 kg/m^3 appear at different depths This double "L" structure is shown by means of three sections (horizontal $z = -200$ m; vertical W–E; and vertical N–S, both vertical sections being indicated in horizontal one) as can be obtained by "SECTIONS" program (using here a grey scale). Axis unit is 200 m.

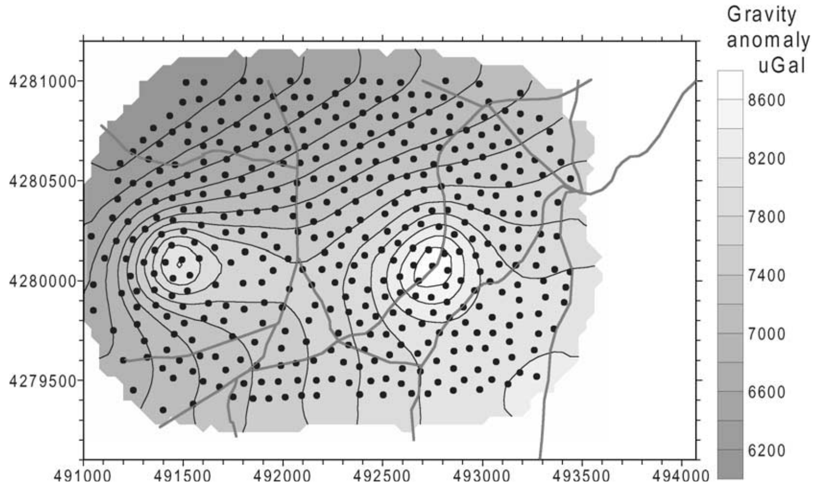

Fig. 4. Gravity anomaly due to anomalous bodies of Fig. 3, with anomalous density contrast of 400 kg/m^3, upon stations of Fig. 2, plus arbitrary linear trend with base constant 7000 μGal, and with slopes 700 μGal/km along the axis X and -700 μGal/km along axis Y.

residuals without clear distortions of the model), $R_N = -400$ (negative density contrast is also accepted), $R_P = 400$ (as known from the simulation), $\tau = 8$ (low smoothing corresponding to a non-smoothed simulation model), $k_w = 0$ (no weighting of the gravity data), $k_d = 0$ (a previously adjusted low resolution model is not used in the further process), $\alpha = 1$ (for the final model the slow and exact systematic exploration is used), $k_e = 0$ (no stratification of the anomalous mass), $\varepsilon = 0.07$ (small enlargement of the cells with depth), $n_h = 0$ (no additional stratification of the subsurface medium). For other questions and parameters the default values are always adopted.

The final model that is obtained with the gravity inversion method is shown in Fig. 5 by means of some smoothed sections. A good general agreement with the original structure (depths, size and shapes) is reached, especially for the top of the model (surfaces close to the depths 100, 0 and -50 m). The discrepancies result from the characteristic non-uniqueness of solution of the potential field and to the applied minimisation condition (4). A general tendency to smooth and round forms is observed, more clearly for the deep parts, where the inversion sensibility is much smaller than for the shallow parts (and where a partial contribution to the regional trend is suspected). Some very small negative bodies are obtained due to counterpoise effect. The total mass of the adjusted model is 613×10^{11} kg (607×10^{11} kg corresponding to positive density contrast and 6×10^{11} kg corresponding to negative density contrast) with a mean depth of 250 m, very close to the original value. In addition, for bodies 1 and 2 (slightly connected in this solution) we obtain characteristic values close to the original ones:

$$\text{body1}: \text{contrast} = 400\,\text{kg/m}^3: \text{mass} = 274\times10^{11}\,\text{kg}, \quad \text{centre}(X,Y,Z) = 4{,}91{,}582,\ 42{,}80{,}102,\ -147\,\text{m}$$

$$\text{body2}: \text{contrast} = 400\,\text{kg/m}^3: \text{mass} = 328\times10^{11}\,\text{kg}, \quad \text{centre}(X,Y,Z) = 4{,}92{,}576,\ 42{,}80{,}102,\ -336\,\text{m}.$$

The adjusted values for the regional trend are $p_0 = 6992$ μGal, $p_x = 703$ μGal/km and $p_y = -702$ μGal/km, which are very close to the original. Therefore, the inversion process is able to determine what part of the gravity data is due to local anomalous masses and what part corresponds to a lineal trend. Also, the program is able to show that the gravity anomaly corresponds in this case mainly to positive density contrast. The residuals resulting from the inversion model have a standard deviation of 1 μGal, close to the zero value supposed for the gravity data error.

The input and output files (GRA.DAT, MAP.BLN, PRI.DAT, MOD.DAT and FIL.DAT) of this former example and a file, GROWTH.TXT, for program documentation is also available in the server for public access.

To show the effect of inaccuracies on the gravity data, we now add to the gravity values, obtained by simulation of the double "L" structure (see Fig. 3), a simulated observational noise determined as Gaussian white noise of standard deviation 30 μGal (see Fig. 6a). Then we carry out the same process as for the former exact values.

First, we observe that now the inversion program does not consider λ values as low as that used (1.6) for the exact gravity values. This fact suggests the presence of

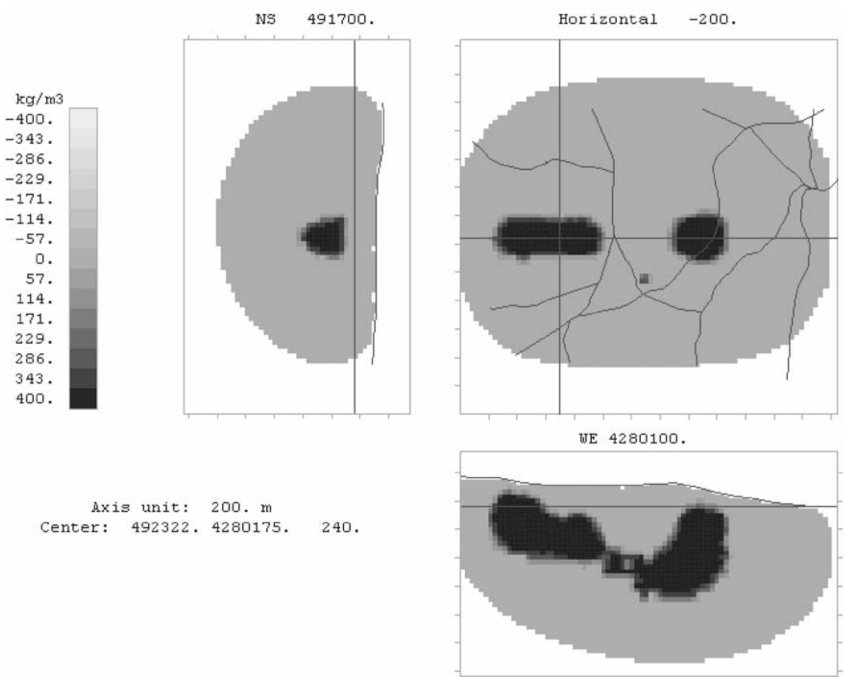

Fig. 5. View, drawn by means of "SECTIONS" program (using grey scale), of anomalous model resulting from gravity inversion approach. Axis unit is 200 m. This solution fits simulation body given in Fig. 3. Better fitness is observed for top zone of bodies than for bottom zone.

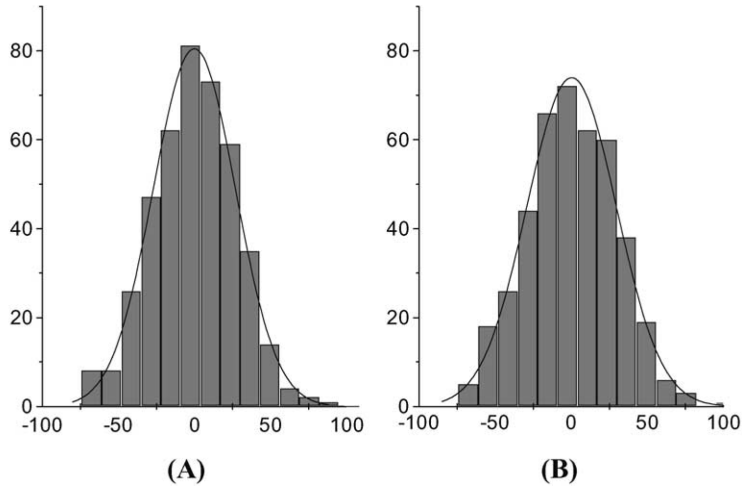

Fig. 6. (A) Distribution of Gaussian noise included in data as simulation of observational noise (standard deviation 30 μGal). (B) Distribution of resulting inversion residuals for second example.

some non-invertible component on the data. Finally, according to the proposed criteria, we adopt a value $\lambda = 12$ that give rise to the model represented in Fig. 7. This produces final inversion residuals with standard deviation of 27 μGal (see Fig. 6b).

The effect of the observational noise is evident in the obtained model (Fig. 7). A greater difference with the original structure is observable. This difference appears more clearly for the weakest part of the gravity inversion corresponding to the deepest parts of the model, giving rise

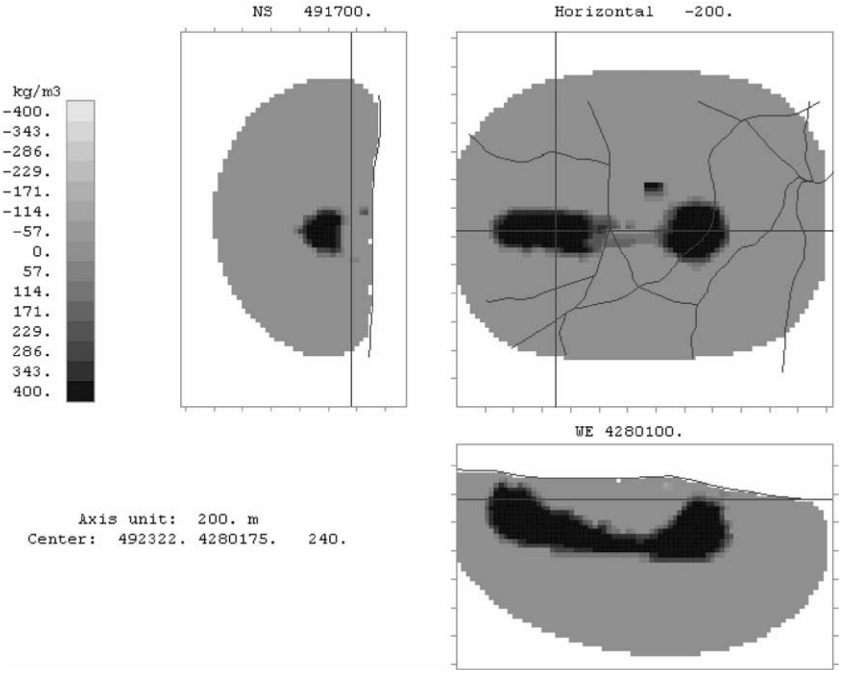

Fig. 7. Anomalous model resulting from gravity inversion of data corresponding to simulation structure of Fig. 3, plus observational noise of standard deviation 30 μGal added to data. Inclusion of this observational noise produces reduction and distortion of weakest part of model (mainly at bottom). Axis unit is 200 m.

to reduction and distortion of this part of the anomalous structure. The total anomalous mass of the model is 525×10^{11} kg with a mean depth of −185 m. Otherwise, the adjusted regional trend has the following values: $p_0 = 7025$ μGal, $p_x = 703$ μGal/km, $p_y = -707$ μGal/km.

4.2. Example 2

We present here (Fig. 8) a more complicated structure characterised by two anomalous bodies located within a stratified medium given by 3 discontinuity planes of depths −100, −300 and −600 m and density discontinuity of 100, 100 and 200 kg/m^3 respectively. The first body, with an intrusive appearance, is a homogeneous body with positive density contrast. This body cuts the stratified medium, then producing a different relative contrast with respect to the surrounding media (400 kg/m^3 for the top part, 300 kg/m^3 with respect to the second layer, 200 kg/m^3 with respect to the third layer, and 0 kg/m^3 with respect to the bottom matter). The second anomalous body consists of a basin with a negative density contrast characterised by an exponential stratification given by the parameters $R_N = -200$ kg/m^3, $\theta_N = 1$, $Z = 100$ m. (Then, connected to this second body, there are two stratifications: one, composed by layers of the external medium and another, that is exponential, of the anomalous matter itself.) Fig. 8 shows some profiles of this structure. The same regional trend of Example 1 ($p_0 = 7000$ μGal, $p_x = 700$ μGal/km and $p_y = -700$ μGal/km) is considered now. The resulting gravity data are shown in Fig. 9. This figure presents a simple feature, that does not suggest further structural information.

A suitable application of the inversion approach, introducing the stratification parameters for the media and for the anomalous negative body as known values, gives us the model shown in Fig. 10. We can again observe a good general agreement with the original model (Fig. 8), especially for the top of the model. In a similar way to example 1, the main distortions come from the tendency to rounded models, and are more accentuated for the deep areas. The adjusted values for the regional trend are: $p_0 = 7015$ μGal, $p_x = 700$ μGal/km and $p_y = -706$ μGal/km. They are close to the original values. In this case the adopted balance value is $\lambda = 1.7$, which produces final inversion residuals of standard deviation 1 μGal. Other parameters of the inversion are: $m = 12,200$ (number of cells), side of the smallest cells = 30 m, $\tau = 8$ (low smoothing corresponding to a non-smoothed simulation model), $k_w = 0$ (no weighting of the gravity data), $\alpha = 2$ (somewhat heavy random exploration: $m/2$ cells are investigated for each step), and $\varepsilon = 0.07$ (small enlargement of the cell sides with the depth).

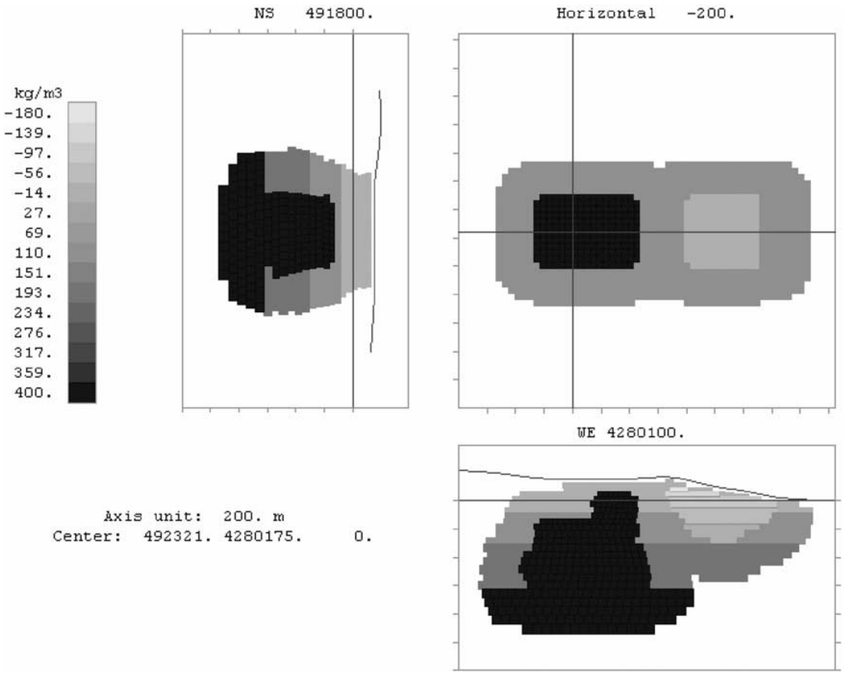

Fig. 8. Original anomalous structure for second simulation test. Positive density contrast body crosses homogeneously stratified media from bottom. Second anomalous body with negative density contrast is constituted by basin filled with exponentially stratified density contrast crossing stratified media. Axis unit is 200 m.

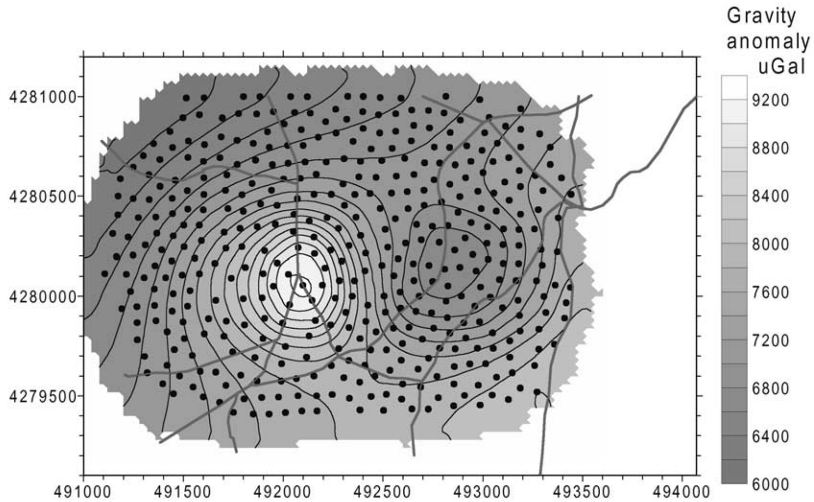

Fig. 9. Gravity anomaly due to anomalous structure represented in Fig. 8 upon simulation survey points, plus regional trend with base constant 7000 μGal and with slopes 700 μGal/km along the axis X and -700 μGal/km along the axis Y.

4.3. Real examples

The gravity inversion tool presented here has been already applied to some real surveys (mainly volcanic structures and archaeological rests) with notable success (e.g. Araña et al., 2000). The inversion approach, in a non-subjective way, obtains interesting structural conclusions starting from a lack of geological information.

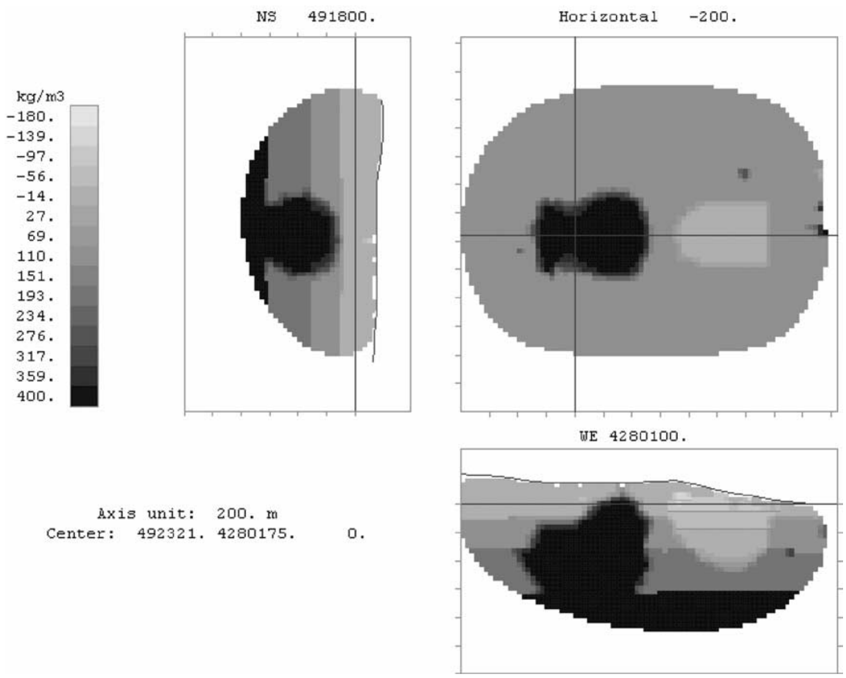

Fig. 10. Several sections (as produced by program SECTIONS) of obtained model from gravity inversion approach corresponding to simulation model of Fig. 8. Axis unit is 200 m. Usual tendency to rounded solutions is observed.

The question of the choice of the prescribed density contrast is not too decisive and can be solved satisfactorily by means of several tests, looking for coherent anomalous volumes (that are neither very "skeletal", nor very "inflated").

5. Conclusions

The software developed in this work can produce 3-D models of the geometrical distribution of anomalous density structures, which reproduce, with a desired accuracy level, the observed gravity data. By means of simulation tests, a good general agreement of the models with respect to the original bodies is observed. Also, a good adjustment of simulated linear trends is observed.

The main advantages of this method is the ability to incorporate negative and positive density contrasts in the model, the simultaneous determination of a regional trend, and the ability to search arbitrary structures starting from a possible lack of geological information or a priori model. Several options of the programs (optional application of weighting, random exploration, smoothing, iterative application, etc.) permit the user to obtain a versatile use of the method.

The disadvantages of this gravity inversion approach come from the nature of the gravity field. Firstly, the characteristic non-uniqueness of the solution requires the inclusion of constraints. Apart from the use of prescribed density contrasts, the main constraint is the mixed condition of data fitness and model smoothness. The method produces some smoothing in the deep part of the model, and, moreover, this deeper zone can be somewhat reduced from a partial contribution in the regional trend. Secondly, there is the question of resolution at different depths. This provides very good geometry for the top of the models, but it gives poor resolution in the deeper part of the models. Finally, we must take into account the effects of horizontal stratification. Therefore, the resulting model must be interpreted as anomalous structures with respect to a non-adjusted "normal" subsurface.

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