GEOPHYSICS, VOL. XXVII, NO. 4 (AUGUST, 1962), PP. 455–462, 5 FIGS.

A COMPREHENSIVE SYSTEM OF TERRAIN CORRECTIONS USING A DIGITAL COMPUTER*

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The system of terrain corrections uses an electronic digital computer for much of the calculation. A new method using a desk calculator is suggested for terrain effects arising from close-in topography. The present system covers a square area 40 km by 40 km, with the station at the center. A selected group of corrections that were compared with conventionally computed ones agree within 0.1 mgal. The speed of the operation using the Datatron 220 computer is considerably faster than the conventional method of computing.

INTRODUCTION

The computation of a gravity terrain correction by desk-calculator methods (Hammer, 1939; Swick, 1942) is a simple but time-consuming operation. Recently, Bott (1959) reported a method in which a substantial part of the correction was made by a high-speed digital computer. The results proved comparable in accuracy to those made by conventional methods and the speed of the operation was greatly increased. The paragraphs below describe a system (Kane, 1960) which, though developed independently, parallels closely the procedures outlined by Bott. The principal difference is that the system discussed here limits the correction to a definite area around the station, thereby simplifying the task of combining the computer correction with the correction for very near and very distant terrain. At present it seems easier to compute the correction for these latter areas by other means.

The particular applications and problems of terrain effects are discussed in the literature (Hammer, 1939). In general, because of its complex form, terrain is divided into a series of smaller elements whose gravity attraction can be readily computed. A template similar to that shown in Figure 1A is used to divide the area around the gravity station into compartments. Average elevations are estimated for the compartments and then referred to appropriate tables; the sum of the effects of the compartments is the terrain correction. The attraction of a particular type of terrain, such as an inclined slope, can often be computed by simpler methods (Sandberg, 1958).

The range in terrain effects arising in areas close to the station is large, and to achieve a consistent accuracy, it is necessary to describe topography close to the station with greater detail than the more distant topography. The terrain effect of topography beyond 15 to 25 km from the station is often small or varies only slightly over large distances. In many places the terrain effect of distant topography is proportional to elevation over an area wide enough to include many stations (depending on station spacing).

THE DIGITAL TERRAIN MODEL

In order to make use of the computer, it is necessary to convert the terrain data, as provided by topographic maps, into digital form. Miller and Laflamme (1958) analyzed the problem and proposed the "digital terrain model" for use with computers. The model is comprised of a series of elevations recorded along with their horizontal location on punched cards. The most simple and efficient model for computer use is that defined by elevations recorded on a regular or square grid. This model was selected for the terrain corrections and is made up of the average elevations of squares formed by the grid. Figure 2 illustrates the conversion of topographic contours (simulated) to a digital terrain model.

The choice of spacing for the terrain model is

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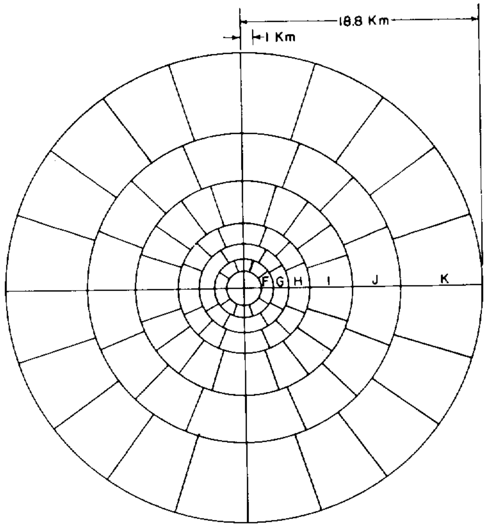
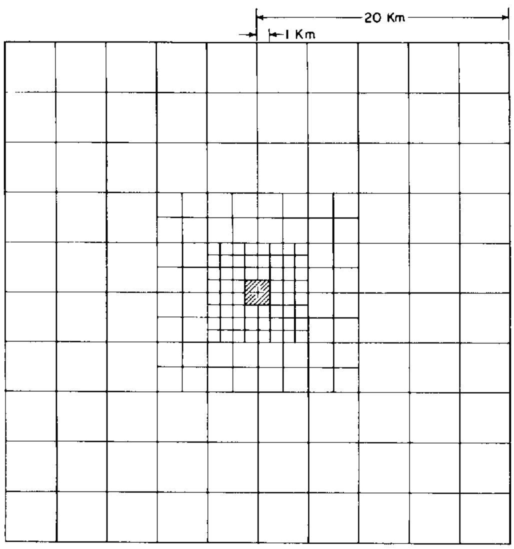


(A) Hayford-Bowie terrain correction template, zones F through K



(B) Pattern used in selecting elevation data from digital terrain model

Central square not included in computer terrain correction

FIG. 1. Comparison between conventional terrain correction template and pattern used in selecting elevation data for computer terrain corrections.

critical, as it determines the accuracy of the model and the proximity of the closest terrain for which an accurate correction can be made. On the other hand, the spacing must be large enough to keep the problem of data compilation within reasonable limits. A spacing of a few thousand feet seemed to best satisfy both needs. A spacing of one km was finally chosen because the Army Map Service "Universal Transverse Mercator Grid" (1955) provides a ready-made reference system. The Mercator grid ticks are printed on recent editions of U. S. Geological Survey topographic maps, thereby reducing grid lay-out problems. It should be noted that the choice of spacing is somewhat arbitrary and a smaller or larger unit might be selected, depending on the accuracy requirements.

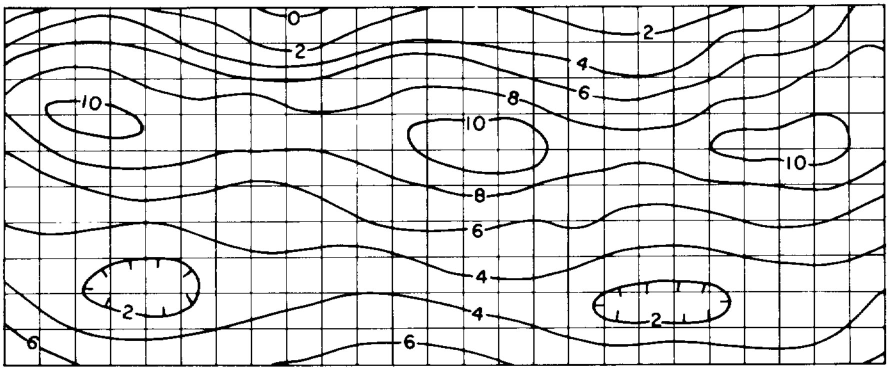
THE PATTERN

The pattern (Figure 1B) is designed to select elevation data around the gravity station from the terrain model. It is similar in concept to a conventional graticule (Figure 1A) except that it is comprised of squares. A side of the smallest square is one km, the same as the spacing of the terrain model. The central 2×2-km square is not included in the pattern because a terrain model with one-km spacing is not sufficiently precise for this zone; each of the four outer sides is 20 km from the station. The area covered by the pattern corresponds approximately to zones F through K of the Hayford-Bowie system (Swick, 1942); and to zones G through M of Hammer (1939).

Computation of pattern elevations

In selecting elevation data from the terrain model, the pattern is aligned with its sides parallel to the sides of the model. The pattern squares will rarely coincide exactly with the terrain model squares so that it is necessary to compute the pattern elevations from the model.

If the pattern is superimposed on the terrain model, it can be seen that the centers of the smallest squares of the pattern (first three rows; same size as terrain model squares) will nearly always fall between the centers of four terrain model squares (Figure 4A). Less commonly the center of the smallest pattern square may fall directly between the centers of two model squares. On very rare occasions it may fall directly on the center of the model square. If it is assumed that topography varies linearly between adjacent squares of the terrain model, the elevation of any square in the first three rows of the pattern may be computed from the digital terrain model in the



Topography

X → 1 Km; Y ↓

3.0	4.0	4.5	4.8	3.0	2.5	1.5	1.0	0.5	1.5	2.0	3.0	3.0	3.0	3.0	2.5	2.0	1.5	1.5	2.0	3.0	3.5	4.5	5.5	7.0
6.5	7.5	7.5	7.0	6.5	5.5	4.5	4.0	4.0	4.0	6.0	6.5	6.5	5.5	5.0	4.5	4.0	3.5	3.5	4.0	4.5	6.0	6.5	7.5	8.5
9.0	9.5	9.5	9.0	8.5	8.0	8.0	8.0	7.0	7.5	7.5	8.5	8.5	8.5	8.0	7.0	6.0	6.0	6.0	6.5	7.5	8.0	8.5	9.0	9.5
8.0	9.0	10.0	10.0	9.5	9.0	9.0	8.5	8.5	8.5	9.0	10.0	10.5	10.5	10.5	9.0	8.5	8.0	8.5	9.0	10.0	10.0	10.5	10.5	9.5
5.5	6.5	7.5	8.0	7.5	7.5	7.0	7.0	7.0	7.5	8.0	9.0	9.5	10.0	10.0	9.0	8.5	8.0	8.0	8.5	9.0	9.0	9.5	9.5	8.5
4.5	5.0	4.5	5.5	5.5	5.5	5.5	5.5	5.5	6.0	6.5	7.0	7.5	7.5	7.0	7.0	6.5	6.5	6.0	6.5	6.5	7.0	7.5	7.0	6.0
3.5	3.5	3.5	3.0	3.0	3.5	4.5	4.5	4.5	4.5	4.5	5.0	5.5	5.5	5.5	5.5	5.0	4.5	4.5	4.5	4.5	5.0	5.5	6.0	4.5
3.5	3.0	2.0	1.5	1.5	2.0	3.0	3.0	3.0	3.5	3.5	3.5	4.0	4.0	4.0	4.0	3.5	2.5	2.5	2.5	3.0	3.5	3.5	3.5	3.5
4.5	3.5	2.5	2.0	2.0	2.5	3.5	3.5	4.0	4.5	4.5	4.5	4.0	4.0	3.5	3.5	2.5	1.5	1.5	1.5	2.0	3.0	3.0	3.5	4.0
6.5	5.5	5.0	4.5	4.5	4.5	5.0	5.5	5.5	6.0	6.0	6.5	6.0	5.5	5.0	4.5	4.0	4.0	4.0	4.0	4.0	4.0	4.0	3.5	2.5

Digital terrain model

FIG. 2. Simulated topography and corresponding digital terrain model.

following manner. If (Figure 4A):

Zo = elevation to be computed,

(Xo, Yo) = coordinates of Zo,

(Xa, Ya), (Xa, Yb), (Xb, Ya), (Xb, Yb) = coordinates of the centers of the terrain model squares nearest (Xo, Yo),

Za, Zb, Zc, Zd = average elevations of the respective terrain model squares, and we let (Figure 4):

$$Yo - Ya = \Delta y,$$

$$Xo - Xa = \Delta x,$$

$$Yb - Ya = Xb - Xa$$
$$= \text{terrain model spacing} = \text{unity},$$

then

$$\begin{aligned} Zo &= \Delta x\{[(Zd - Zc)\Delta y + Zc] \\ &\qquad - [(Zb - Za)\Delta y + Za]\} \\ &\quad + [(Zb - Za)\Delta y + Za] \\ &= Za + \Delta x(Zc - Za) + \Delta y(Zb - Za) \\ &\quad + \Delta x \Delta y(Zd - Zc - Zb + Za). \end{aligned}$$

The elevation of any square in the outer five rows of the pattern is determined by averaging the elevations of the terrain-model squares whose centers are enclosed by the pattern squares. Figure 4B illustrates the case for the fourth and fifth rows where

$$Zo = \frac{Za + Zb + Zc + Zd}{4}.$$

Elevations for the sixth, seventh, and eighth rows are computed similarly except that the pattern square will usually encompass sixteen terrain-model squares. Occasionally the squares of the fourth and fifth rows will include six or nine centers and those of the sixth, seventh, and eighth rows will include twenty-four or thirty-six centers.

GRAVITY ATTRACTION OF A PRISM

The formula for the gravity attraction of a prism contains twenty-four terms and proved long, even for the digital computer. The attraction of a prism can be approximated by that of an annular ring with the same height (difference in the attraction of two vertical cylinders with the same height but differing radii) times the ratio of the area of a horizontal section of the prism to that of the horizontal section of the ring (Figure 3). This formula is:

$$g = 2GDA^2 \frac{(R_2 - R_1 + \sqrt{R_1^2 + H^2} - \sqrt{R_2^2 + H^2})}{(R_2^2 - R_1^2)},$$

where

- g = gravity attraction,
- G = gravitational constant,
- D = density,
- A = length of horizontal side of prism,
- R_1 = radius of inner circle of annular ring,
- R_2 = radius of outer circle of annular ring, and
- H = height of annular ring or prism.

R_1 and R_2 may be replaced by $(R-C)$ and $(R+C)$, where R is the distance from the gravity station to the center of the square and C is a constant which can be determined by comparing the results of the formula above with those computed by the rigorous formula. C was determined as $0.63A$. Hence, $R_1 = R - 0.63A$, and $R_2 = R + 0.63A$. Therefore,

$$g = \frac{GDA(1.26A + \sqrt{(R - 0.63A)^2 + H^2} - \sqrt{(R + 0.63A)^2 + H^2})}{1.26R}.$$

This formula is accurate to within two percent or 0.1 mgal, whichever is larger, where H/R is less than one; it is usually accurate within one percent.

The geometric effect of the constant can be seen from Figure 3. A square is superimposed on the ring where the difference between R_2 and R_1 is equal to a side of the square (or where the constant would be $0.5A$). A larger proportion of the square is closer to the origin than the arcuate sector of the ring. A slight elongation of the ring in the radial direction while holding the areas of the square and sector equal tends to compensate this condition.

INNER ZONE CONSTRUCTIONS

A method of manual computation of inner zone correction, designed for use with the computer system, incorporates both the division of terrain into compartments and the inclined slope (Sandberg, 1958). Conventional methods may be used to approximate the terrain effects of the central 2×2-km square with only a small loss in accuracy. However, it is felt that the following method offers some practical advantages in speed and convenience.

The 2×2-km square is divided into octants (Figure 5A). The surface of each octant[1] is assumed to slope continuously from the apex to the outer edge. The gravity attraction of this solid (Figure 5B) can be approximated by that of a cylinder with an inverted cone removed. When the slopes and volumes of the respective solids are equal, the difference in gravity attraction is small for slopes of less than 45 degrees. The gravity attraction of the solid shown in Figure 5B

[1] Once a slope is assumed (Figure 5B), the use of the term octant is not precise. However, the term is convenient and the meaning should be clear.

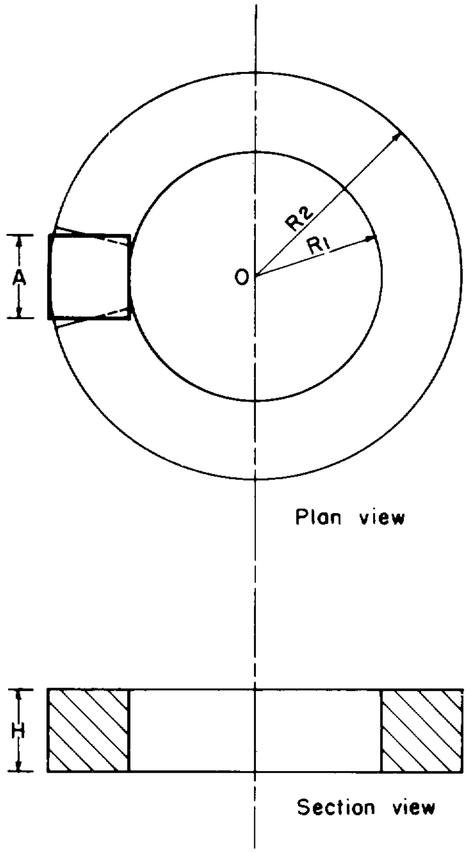


FIG. 3. Relationship of square to segment of ring both having an equal area.

is:

$$g = \frac{\pi G D}{4}\left(R - \sqrt{R^2 + H^2} + H \sin\beta\right),$$

where

g = gravity attraction,
G = gravitational constant,
D = density,
R and H = radius and height of cylinder, respectively, and
β = angle between the octant surface and a horizontal surface (Figure 5B) (also the complimentary angle of the inverted cone).

Where it is applicable, the octant can be used in the continuous slope sense by noting the difference in elevation, *Hd* (Figure 5B), and referring it to an appropriate table or curve. If the terrain does not conform to a continuous slope, the octant may be subdivided into segments by the equal distances, *OB* and *BD* (Figure 5C), or *OA*, *AB*, *BC*, and *CD* (Figure 5D). The gravity attraction of these segments may be derived from perspectivity (MacMillan, 1958, pp. 8–10), where the parallel, equi-spaced planes divide the solid into segments of equal gravity attraction. Appropriate tables may be computed in terms of *Hb* and *Hd* of Figure 5C, and *Ha*, *Hb*, *Hc*, and *Hd* of Figure 5D. An alternate method is to use the average heights of the compartments, *Ha*, *Hb*, *Hc*, and *Hd*, as in Figure 5E.

In most cases terrain in the inner zones slopes upward or downward from, or is level with, the station so that the inclined slope (as pointed out by Sandberg, 1958) is a more suitable approximation than a series of compartments of average elevation would be. The direction of slope is only critical in the innermost zones, and reversals in slope generally occur outside this area. For a majority of cases the inclined slope is applicable to the whole octant, thus replacing the twenty compartments of Hayford-Bowie and the thirty compartments of Hammer with eight octants. Rules of thumb may also be easily developed for areas with negligible correction. When topography becomes more variable, the octants can be subdivided according to the needs of individual stations.

COMPILATION AND COMPUTING

The survey area including a 21-km boundary is marked on appropriate topographic maps and the grid lines ruled in. The average elevations of the grid squares are estimated and noted directly on the map. These elevations and the relative coordinates of the squares are transferred to punched cards and stored in machine memory for use as field data. The numbers, elevations, geographic coordinates, and inner zone corrections of the stations are similarly punched on cards. The computer calculates the location of the station within the terrain model, the relative elevations of the pattern squares, and finally the corresponding terrain correction. The inner zone and computer terrain corrections are combined for the total correction.

RESULTS

Terrain corrections were made for 313 gravity stations in Las Vegas Valley, Clark County,

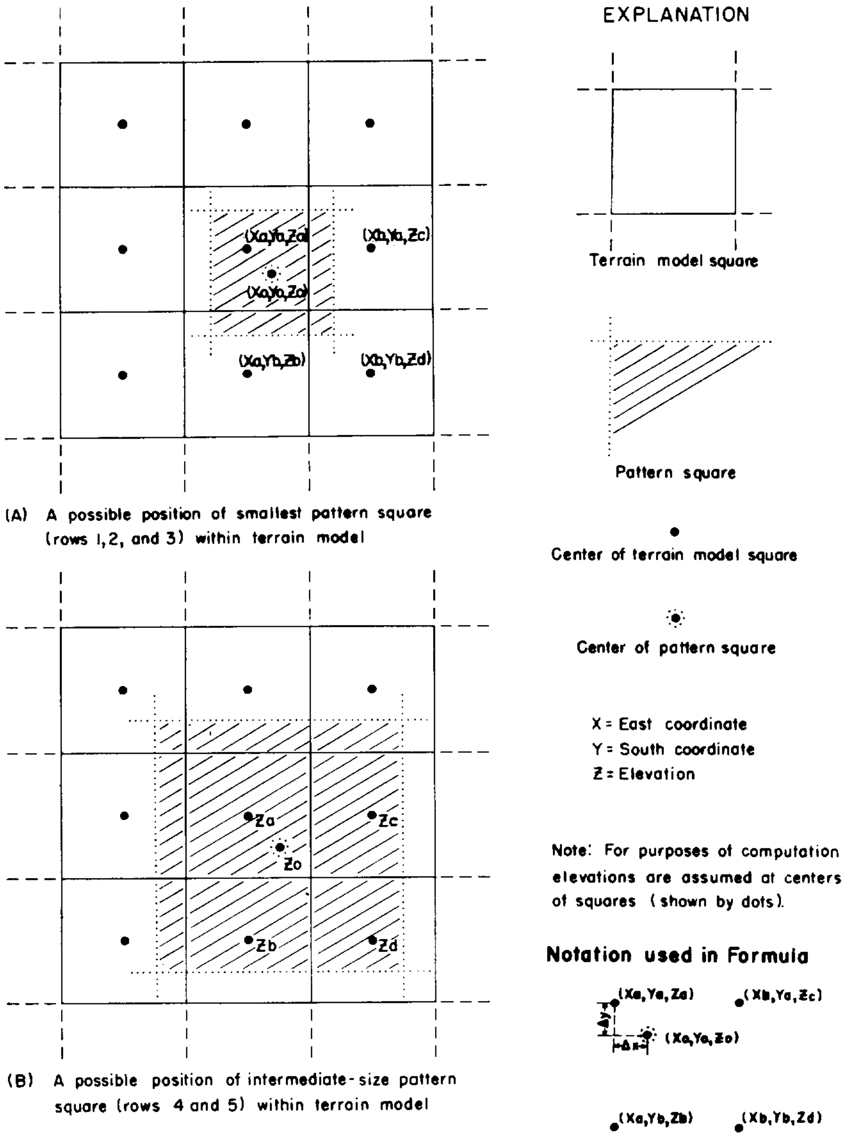


FIG. 4. Examples of relative horizontal positions of centers of terrain model squares and pattern squares illustrating general cases for computation.

Nevada. The stations were established over an area of about 1,200 square miles, giving a station density of about one station per 4 square miles. The terrain is moderately rugged and corrections range from less than 0.1 mgal to more than 4.0 mgal.

A rectangular grid 94 by 97 km was used to compile the terrain model. The estimation and card punching of 9,118 average elevations and their coordinates took about 40 man-hours. Computer time per station for the U. S. Geological Survey's Datatron 220 was 30 sec, so the entire body of corrections consumed $2\frac{1}{2}$ hours of computer time. Inner zone corrections were made by the method described above in about 16 hours. The total time consumption was therefore about 60 hours.

Table 1 is a comparison of Hayford-Bowie cor-

Table 1. Comparison of computer and conventional terrain corrections (in milligals).

Station	Digital computer	Hayford-Bowie	Difference
1	0.63	0.66	−0.03
40	1.60	1.69	−0.09
55	2.62	2.54	+0.08
77	2.50	2.44	+0.06
81	1.29	1.34	−0.05
165	0.66	0.63	+0.03
202	0.71	0.76	−0.05
285	0.61	0.67	−0.06
314	2.30	2.24	+0.06
316	3.86	3.81	+0.05

rections with the computer corrections. The stations were chosen to represent the greatest range in terrain conditions. The computer corrections are compensated for the small discrepancies in the areas that are covered. The average difference is 0.06 mgal and the largest is 0.09 mgal, which are well within the significant accuracy of terrain corrections as a whole.

SUMMARY

A system of digital-computer terrain corrections has been programmed which provides accurate terrain corrections at a substantial reduction in time over conventional methods. The economy of the system depends on the cost factor for computer time and man-hours, and for this case resulted in savings of a third over the cost of conventional corrections. The savings also depend on station spacing since the cost of compiling a terrain model for a few stations is greater per station than if the model were used for a large number of stations. For the Las Vagas Valley survey, the spacing could have been three times that used, or about one station per 12

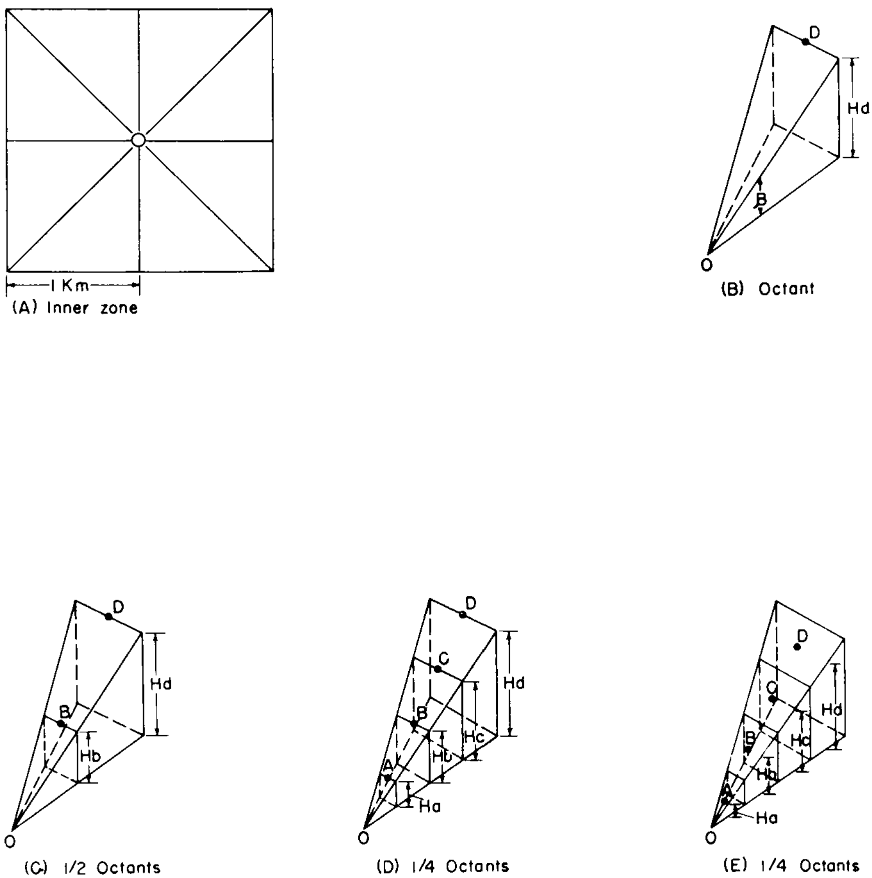


FIG. 5. Division of inner terrain correction zone into octants.

square miles, without an increase in cost over conventional methods. In some surveys additional cost might be justified by the better internal consistency provided by the use of the same terrain data for each correction.

Perhaps one of the principal advantages of the digital computer system is that the compilation of data is a simple operation and requires little experience or judgment on the part of the compiler. An efficient conventional system which takes advantage of the many time-saving steps that are possible usually requires a more experienced operator. Once a terrain model is compiled for a given area, corrections can be made for an additional number of stations almost instantaneously. Terrain models can also be compiled prior to or during a survey, thereby making possible completely corrected data soon after the completion of the fieldwork. The system does require a reasonably long-term use to overcome the original investment in computer programming. For regional surveys with a very wide station spacing, the compilation of the terrain model would probably entail more time and cost than conventional corrections.

The system described above covers the effect of terrain from the gravity station to a distance of 20 km. Corrections for more distant topography could be calculated by the electronic computer using a similar system coupled with a terrain model based on a 10-km spacing. A 10-km terrain model covering the United States south of Canada and including a sufficient fringe area would be comprised of less than 200,000 elements, and could be compiled in a few months time. Such a model might also be useful in studies concerning isostatic compensation and other geophysical problems.

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