TERRAIN CORRECTIONS FOR GRAVIMETER STATIONS*

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ABSTRACT

In this paper the correction for the gravitational attraction of the topography on a gravity station is considered as consisting of two parts; (1) the restricted but conventional "Bouguer correction" which postulates as a convenient approximation that the topography consists of an infinite horizontal plain, and (2) the "Terrain correction" which is a supplementary correction taking into account the gravitational effect of the undulations of the terrain about the plane through the gravity station. The paper illustrates the necessity of making terrain corrections if precise gravity surveys are desired in hilly country and presents terrain correction tables with which this quantity may be determined to a relative accuracy of one-tenth milligal. This accuracy is required to fully utilize the high instrumental precision of modern gravimeters.

INTRODUCTION

Applications of the gravity method of geophysical prospecting in the search for small, local, geological structures require very precise data. To supply this need gravimeters have been developed or improved to the point where the probable error of the observed gravity values is of the order of 1/10 mg. or even less[1,2,3,4] To fully utilize this high instrumental precision it is obvious that corrections for the several non-significant influences, which are present in the observed gravity values, must be of the same order of accuracy. One of the important non-significant influences is the gravitational attraction of the topography in the vicinity of a gravity station. This so-called "topographic correction" will be considered, by definition, in the present paper to consist of two parts; (1) the "Bouguer correction" which postulates as a convenient approximation, that the topography consists of an infinite horizontal plain, and (2) the "Terrain correction" which evaluates the error in the Bouguer correction due to undulations of the terrain. The purposes of this paper are (a) to illustrate the necessity of making terrain corrections if precise gravity surveys are desired in hilly country, and (b) to present

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[1] E. A. Eckhardt, Geophysics *1*, pp. 292–293 (1936) (abstract).

[2] D. C. Barton and W. T. White, Trans. Am. Geophys. Union, Part I, pp. 106–107 (1937).

[3] A. Graf. Zeits f. Geophys. *14*, pp. 152–172 (1938).

[4] H. Hedstrom, A.I.M.E. Tech. Publ. No. 953 (1938).

tables with which the terrain correction may be determined to the accuracy merited by the precision of modern gravimeters.

THE CORRECTION FOR TOPOGRAPHY

A common procedure in gravity prospecting is to correct for the attraction of the topography by use of an approximate value "B" calculated from the simple Bouguer formula

$$B = 2\pi\gamma\sigma(H - H_0) \tag{1}$$

where $H-H_0$ is the elevation of the gravity station above the level datum
σ is the density of the surface soil or rock
γ is the gravitational constant.

This formula calculates the gravity effect of the matter between a horizontal plane through the field station and the horizontal elevation-datum-plane on the assumption that the space between these two planes out to an infinite radius is uniformly filled with matter if the station is above the datum plane, or is uniformly empty if the station is below this plane.[5] (See Fig. 1). Subtracting this quantity (algebraically) from the observed gravity value amounts to removing the *assumed* amount of matter in the infinite flat plate if the station is above the datum plane or to completely filling up this space if the station is below the datum plane so as to reduce, in effect, the surface of the ground to the plane through the elevation datum.

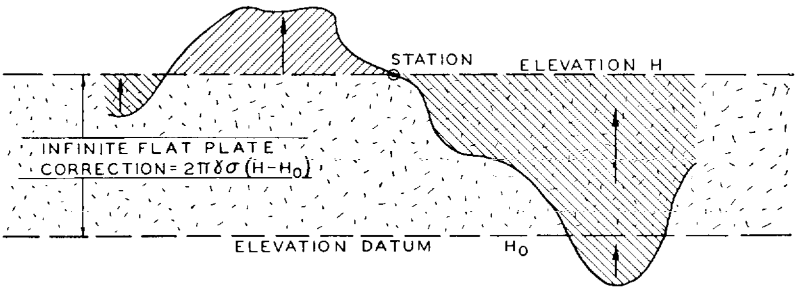

FIG. 1. Schematic diagram of the Bouguer correction, illustrating residual gravitational effects due to undulating terrain.

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[5] For an excellent brief statement of the nature of the Bouguer reduction see Hayford and Bowie, U.S.C. & G.S. Special Publication No. 10, p. 75 (1912). These authors define the "Bouguer Correction" in the more general sense which is equivalent to our "Topographic Correction."

Undulations of the terrain are completely ignored in this Bouguer correction. Let us consider the error involved. First we see in Fig. 1 that the Bouguer formula overestimates the gravitational attraction of the actual mass below the station because it ignores the voids in this space. Therefore the corrected gravity value obtained by subtracting the approximate Bouguer correction will tend to be too low. Second, the upward component of the gravitational attraction of the mass above the plane through the station, which tends to lower the observed gravity value, is not removed by the Bouguer correction. Thus, topographic elevations above and depressions below the station both act in the same sense and the gravity value corrected by the Bouguer formula will always be too low. Corrections for the effect of undulations of the terrain with respect to the station elevation plane will therefore always be positive.

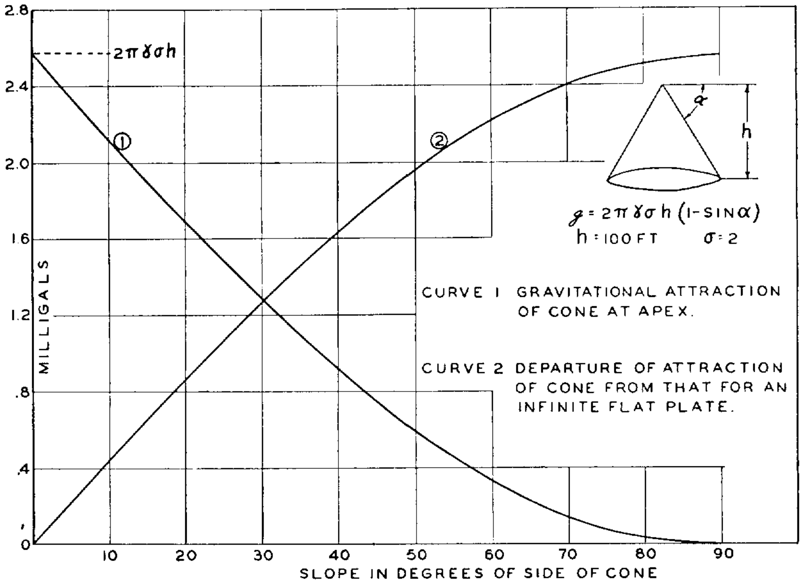

FIG. 2. Gravitational attraction of an idealized hill, illustrating the magnitude of the error in the Bouguer correction.

Possible magnitudes of the terrain correction (as defined in this paper) are illustrated in Figs. 2 and 3. Figure 2 shows an idealized hypothetical case in which a gravity station is situated at the apex of a conical hill, which in turn is located upon a flat plain. Curve 1 shows the decrease in the gravitational attraction of the hill as the sides become steeper. The attraction calculated by the Bouguer formula is the ordinate of curve 1 at $\alpha = 0$. Curve 2 shows the error in the Bouguer formula, that is the terrain correction, as a function of the steepness of the hill. For example, if the slope of the hillside is 30° the Bouguer correction is twice too large.

Fig. 3 shows an actual example of a gravimeter profile across a very steep topographic feature, Sierra Madera (also known as Madera Mountain), in Western Pecos County, Texas, which is known to be a sharply uplifted geological structure. Curve 1 depicts the topography. Curve 2 is the gravity profile (Bouguer anomaly) as reduced with the ordinary Bouguer correction, and curve 3 is the gravity profile after applying terrain corrections. The terrain corrections in this case amount to several milligals. It is clear that the interpretations of the two gravity profiles will differ fundamentally. Admittedly, this example is an extreme one for ordinary gravity prospecting but we must conclude that terrain corrections cannot be disregarded if precise gravity surveys are desired in hilly country.

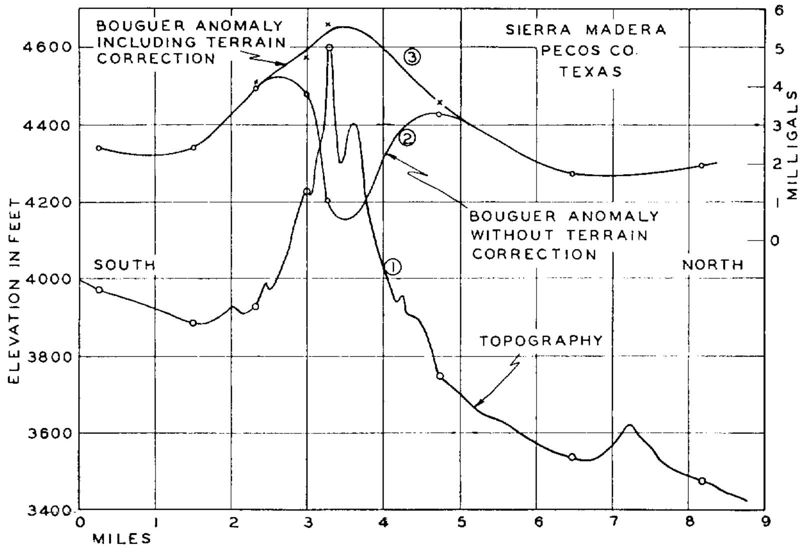

FIG. 3. Gravimeter profile across Sierra Madera, Pecos County, Texas, illustrating the importance of terrain corrections.

Having demonstrated the importance of the terrain correction to gravity stations in hilly country, we consider next the method of evaluating it. By its very definition it is clear that the terrain correction depends upon the details of the topography. Therefore, a topographic map, or the equivalent, of the vicinity of the gravity station is required. The area about the station is divided into zones and compartments, for example, by laying a transparent terrain correction zone chart, such as that shown in Fig. 4, upon the map and centering it at the gravity station. The average departure without regard to sign of the topography in each compartment from the plane through the station is then determined* and the terrain cor-

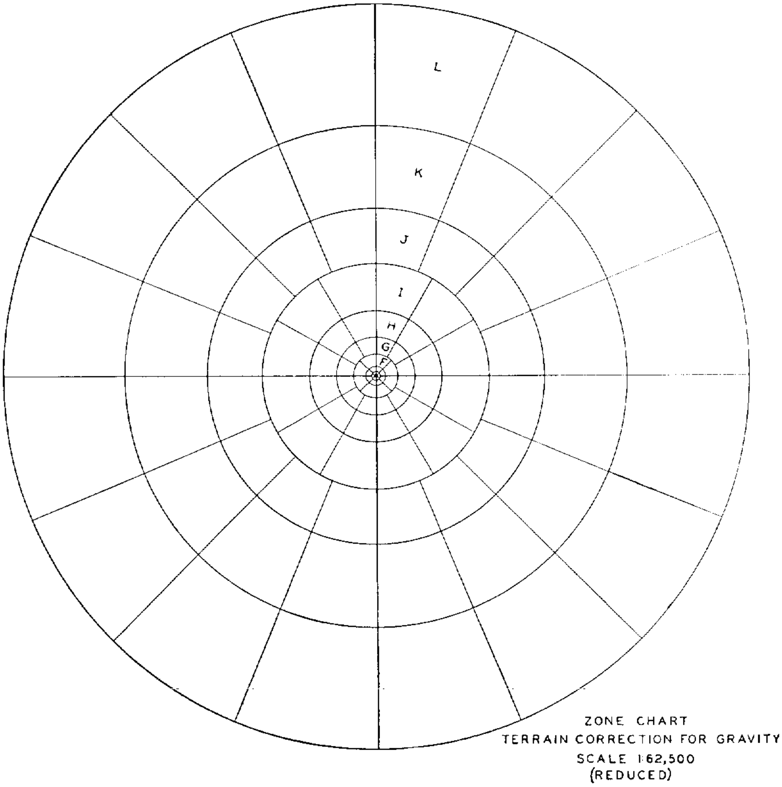

Fig. 4. Zone chart for use in evaluating terrain corrections at gravity stations.

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* Since the sign of the terrain correction (as defined in this paper) depends only on the absolute value of the departure of an element of the topography from the plane through the station, and not on its sign (that is, whether it is above or below the plane) it follows that the difference between the simple average elevation in each compartment and the station elevation cannot be used, *in principle,* if the terrain within any one com-

rection corresponding to this average departure is evaluated for each compartment by means of tables calculated for that purpose. Finally these terrain corrections are summed over all the zones in which there are appreciable effects.

TERRAIN CORRECTION TABLES

A number of terrain correction tables have been published[6,7,8] for geodetic and regional gravity work. However, the author knows of no previously published tables which are sufficiently precise for modern gravity prospecting needs. The tables presented in this paper have been designed especially for this use after experience extending over a number of years with terrain correction calculations.

The present tables were modelled after those by Hayford and Bowie[6,7] but are more precise and have been modified in principle to evaluate, not the total topographic correction, but only the error in the Bouguer correction as discussed above.* This modification permits relatively larger compartments in the distant zones, for a given precision, and thus reduces the labor involved in the use of the tables. However, to attain the precision desired the compartments in the zones near the station are smaller in the present tables than in those by Hayford and Bowie. Another important advantage of the modification in the principle is that it is necessary to use the tables in moder-

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partment consists of elements both above and below the station. Thus, theoretically, if the topography within one compartment consists of a hill above the station and a valley below the station, the average departure must be determined as if the topography consisted of two hills or two valleys. However, in practice, it turns out that the terrain effect in such a compartment is ordinarily zero. Therefore, the mathematically rigorous but cumbersome procedure outlined above may usually be replaced by the much more rapid procedure of estimating the average elevations in all the compartments (irrespective of whether the topography in any one compartment is above and below the station) and then calculating the departures of these simple average elevations from the station elevation. This would not be true, in general, in extremely rugged topography of large relief but even then the complication may usually be avoided by rotating the zone chart to a different azimuth. The azimuth must, of course, be maintained while reading the elevations in all the compartments of any one zone.

[6] Hayford and Bowie, *loc. cit.*, pp. 13–53.

[7] Hayford and Bowie, U.S.C. & G.S. Spec. Publ. 40, pp. 11–18 (1917).

[8] Bullard, Philos. Trans. Roy. Soc. London A235, pp. 486–491 (1936).

* Bullard's tables (*loc. cit.*) are also of this type. His paper contains a detailed discussion of the advantages of this modification. A recent search in the literature reveals the historically interesting fact that this method was described by F. R. Helmert in 1884 ("Theorien der höheren Geodäsie," Vol. II pp. 169–172) and was called "the ordinary method" by F. A. Venning-Meinesz in 1923 ("Observations de Pendule dans les Pays-Bas." Publ. de la comm. géod. néerlandaise, p. 141).

TERRAIN CORRECTION TABLES FOR GRAVITY

Each zone is a circular ring of given radii (in feet) divided into 4, 6, 8, 12, or 16 compartments of arbitrary azimuth. "h" is the mean topographic elevation in feet (without regard to sign) in each compartment with respect to the elevation of the station. The tables give the correction "T" for each compartment due to undulations of the terrain in units of 1/100 mg. for density $(\sigma)=2.0$. This correction, when applied to Bouguer anomaly values which have been calculated with the simple Bouguer correction, is always positive.

Zone B 4 compartments 6.56 to 54.6*		Zone C 6 compartments 54.6 to 175		Zone D 6 compartments 175 to 558		Zone E 8 compartments 558 to 1280		Zone F 8 compartments 1280 to 2936		Zone G 12 compartments 2936 to 5018	
$\pm h$ (ft.)	T	$\pm h$ (ft.)	T	$\pm h$ (ft.)	T	$\pm h$ (ft.)	T	$\pm h$ (ft.)	T	$\pm h$ (ft.)	T
0 to 1.1	0	0 to 4.3	0	0 to 7.7	0	0 to 18	0	0 to 27	0	0 to 58	0
1.1- 1.9	0.1	4.3- 7.5	0.1	7.7- 13.4	0.1	18- 30	0.1	27- 46	0.1	58- 100	0.1
1.9- 2.5	0.2	7.5- 9.7	0.2	13.4- 17.3	0.2	30- 39	0.2	46- 60	0.2	100- 129	0.2
2.5- 2.9	0.3	9.7- 11.5	0.3	17.3- 20.5	0.3	39- 47	0.3	60- 71	0.3	129- 153	0.3
2.9- 3.4	0.4	11.5- 13.1	0.4	20.5- 23.2	0.4	47- 53	0.4	71- 80	0.4	153- 173	0.4
3.4- 3.7	0.5	13.1- 14.5	0.5	23.2- 25.7	0.5	53- 58	0.5	80- 88	0.5	173- 191	0.5
3.7- 7	1	14.5- 24	1	25.7- 43	1	58- 97	1	88-146	1	191- 317	1
7 - 9	2	24 - 32	2	43 - 56	2	97-126	2	146-189	2	317- 410	2
9 -12	3	32 - 39	3	56 - 66	3	126-148	3	189-224	3	410- 486	3
12 -14	4	39 - 45	4	66 - 76	4	148-170	4	224-255	4	486- 552	4
14 -16	5	45 - 51	5	76 - 84	5	170-189	5	255-282	5	552- 611	5
16 -19	6	51 - 57	6	84 - 92	6	189-206	6	282-308	6	611- 666	6
19 -21	7	57 - 63	7	92 -100	7	206-222	7	308-331	7	666- 716	7
21 -24	8	63 - 68	8	100 -107	8	222-238	8	331-353	8	716- 764	8
24 -27	9	68 - 74	9	107 -114	9	238-252	9	353-374	9	764- 809	9
27 -30	10	74 - 80	10	114 -120	10	252-266	10	374-394	10	809- 852	10
		80 - 86	11	120 -127	11	266-280	11	394-413	11	852- 894	11
		86 - 91	12	127 -133	12	280-293	12	413-431	12	894- 933	12
		91 - 97	13	133 -140	13	293-306	13	431-449	13	933- 972	13
		97 -104	14	140 -146	14	306-318	14	449-466	14	972-1009	14
		104 -110	15	146 -152	15	318-331	15	466-483	15	1009-1046	15

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* Radii of the zone in feet.

Zone H 12 compartments 5018 to 8578		Zone I 12 compartments 8578 to 14,662		Zone J 16 compartments 14,662 to 21,826		Zone K 16 compartments 21,826 to 32,490		Zone L 16 compartments 32,490 to 48,365		Zone M 16 compartments 48,365 to 71,996	
$\pm h$ (ft.)	T	$\pm h$ (ft.)	T	$\pm h$ (ft.)	T	$\pm h$ (ft.)	T	$\pm h$ (ft.)	T	$\pm h$ (ft.)	T
0 to 75	0	0 to 99	0	0 to 167	0	0 to 204	0	0 to 249	0	0 to 304	0
75– 131	0.1	99– 171	0.1	167– 290	0.1	204– 354	0.1	249– 431	0.1	304– 526	0.1
131– 169	0.2	171– 220	0.2	290– 374	0.2	354– 457	0.2	431– 557	0.2	526– 680	0.2
169– 200	0.3	220– 261	0.3	374– 443	0.3	457– 540	0.3	557– 659	0.3	680– 804	0.3
200– 226	0.4	261– 296	0.4	443– 502	0.4	540– 613	0.4	659– 747	0.4	804– 912	0.4
226– 250	0.5	296– 327	0.5	502– 555	0.5	613– 677	0.5	747– 826	0.5	912–1008	0.5
250– 414	1	327– 540	1	555– 918	1	677–1119	1	826–1365	1	1008–1665	1
414– 535	2	540– 698	2	918–1185	2	1119–1445	2	1365–1763	2	1665–2150	2
535– 633	3	698– 827	3	1185–1403	3	1445–1711	3	1763–2086	3	2150–2545	3
633– 719	4	827– 938	4	1403–1592	4	1711–1941	4	2086–2366	4	2545–2886	4
719– 796	5	938–1038	5	1592–1762	5	1941–2146	5	2366–2617	5	2886–3191	5
796– 866	6	1038–1129	6	1762–1917	6	2146–2335	6	2617–2846	6	3191–3470	6
866– 931	7	1129–1213	7	1917–2060	7	2335–2509	7	2846–3058	7	3470–3728	7
931– 992	8	1213–1292	8	2060–2195	8	2509–2672	8	3058–3257	8	3728–3970	8
992–1050	9	1292–1367	9	2195–2322	9	2672–2826	9	3257–3444	9	3970–4198	9
1050–1105	10	1367–1438	10	2322–2443	10	2826–2973	10	3444–3622	10	4198–4414	10
1105–1158	11	1438–1506	11	2443–2558	11						
1158–1209	12	1506–1571	12	2558–2669	12						
1209–1257	13	1571–1634	13	2669–2776	13						
1257–1305	14	1634–1694	14	2776–2879	14						
1305–1350	15	1694–1753	15	2879–2978	15						

FIG. 5. Tables for precise evaluation of terrain corrections at gravity stations.

ately hilly country for only a relatively small number of stations, and the labor involved in their use when needed is materially shortened. Also, the present tables are arranged in a form which eliminates interpolations and thus facilitates their use. With the present tables the office calculations of the terrain correction can be carried through in one-half to one hour per station.

The calculation of the tables was based upon the well known formula for the gravitational attraction of a vertical hollow cylinder at a point on the axis and in the plane of one end of the cylinder

$$g = 2\pi\gamma\sigma\left[R_2 - R_1 + \sqrt{R_1^2 + h^2} - \sqrt{R_2^2 + h^2}\right] \tag{2}$$

where R_1 and R_2 are the inner and outer radii and h is the height of the cylinder (representing the average height of the terrain). The calculations were carried out by solving this equation for h in terms of the radii and an adopted unit gravitational attraction for one compartment.* To obtain the most nearly "square" compartments the ratio of the outer and inner radii (i.e., the radial length) of a zone was related to the width of the compartments in that zone by the condition $R_2/R_1 = (n+\pi)/(n-\pi)$ where n is the number of compartments in the zone.

Finally, the author believes, on the basis of experience, that the areas of the various compartments in the present tables are maximum (i.e., that the total number of compartments is a minimum) consistent with practical accuracy in the determinations of the mean elevation of the terrain in the compartments. This last qualification depends, of course, upon the nature of the topography but tests indicate that the compartments in the present tables are small enough to give satisfactory accuracy even in very hilly country.

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* For the convenience of anyone who might wish to make similar calculations, this formula, in the most convenient form, is

$$h_j = R_1\left[2a\left(\frac{a-1}{c}\right)\right]^{1/2} \times \left[j + \left\{\frac{a^2-3a+1}{2a\left(\frac{a-1}{c}\right)}\right\}j^2 - \left\{\frac{c^2}{2a}\right\}j^3 + \left\{\frac{c^2}{8a\left(\frac{a-1}{c}\right)}\right\}j^4\right]^{1/2} \div \left[\frac{a-1}{c} - j\right] \tag{3}$$

where R_1 = inner radius of the zone, a = ratio of outer to inner radius, $c = ng_0/2\pi\gamma\sigma R_1$, g_0 = unit gravity effect, n = number of compartments in the zone, h_j = height of the cylindrical segment in one compartment which produces a gravity effect $j\times g_0$ and j is any number ≥ 0. In the present calculations $j = 1/2, 3/2, 5/2 \ldots$ The formula is exact and not the first few terms of a series expansion.

The tables are given in Fig. 5 and comprise 12 zones "*B*" to "*M*" which in turn are subdivided into a total of 132 compartments. Zone *B* has an inner radius of 2 meters (6.56 feet) and zone *M* has an outer radius of 13½ miles. Terrain at greater distances is not significant locally, except in extreme cases where adjacent stations are at greatly different elevations. In the present tables the area of a compartment in zone *B* is 2300 square feet (equivalent to 48 feet square) and the area of a compartment in zone *M* is 20 square miles (about 4½ miles square). Zone *A*, the area within two meters of the station, is not given because only very extreme terrain conditions will give appreciable effects within this small area.* Curvature of the earth is ignored as its effect on the tabulated elevations is practically negligible within the area covered by the tables. (At the center of zone *M* the curvature amounts to 87 feet and is just on the verge of becoming appreciable).

The density of the surface material has been assumed, for the purpose of calculating the tables, to be 2.0. For another density σ the total terrain correction as evaluated by the tables must be multiplied by the factor $\sigma/2$. Gravity effects are tabulated to 0.001 and 0.01 mg. The reason for the two tabular intervals will be discussed later.

The precision with which the terrain correction may be obtained with the present tables is, of course, fundamentally dependent upon the adequacy of the topographic information available. The first elevation entries in the tables indicate roughly the required accuracy of the mean elevation in each compartment in the various zones. U.S.C.&G.S. topographic maps, where available, are generally adequate for the zones beyond one-fourth to one-half mile from the station, but in hilly country special level data near the station are ordinarily required. In moderately hilly country the tables may serve as a basis for judgment in the selection of station sites so as to avoid much of this special levelling.

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* The only important case we need consider is that in which the station is located on a hillside. The terrain effect within a circle of radius R of a plain inclined at an angle θ from the horizontal and passing through the gravity station is

$$T_R = 2\gamma\sigma R[\pi - 2\cos\theta\, K(\sin\theta)] \tag{4}$$

where $K(\sin\theta)$ is the complete elliptic integral of the first kind. Thus a terrain effect of 0.01 mg. (for $\sigma = 2.0$) within zone A requires a slope of 27½°. A common modification of this case is one in which the station is located at the edge of an embankment or road cut. Then the sloping plain occupies only half the area of the first zone and the terrain effect is one-half of that given by equation 4. In this case a terrain effect of 0.01 mg. within zone *A* requires a slope of 38°.

Assuming, for the purpose of discussing the precision of the tables themselves, that completely adequate topographic maps are available and that the correct average elevation in each compartment is known, we note that a complete determination of a terrain correction involves 132 entries from the tables, each of which is subject to a maximum error of 1/200 mg. These reading errors are of a random nature, hence the Theory of Errors predicts a "probable" accumulative reading error in the terrain correction of 0.02 mg. For statistical reasons in individual cases the actual accumulative reading error will range both larger and smaller than 0.02 mg. but in only rare cases will it exceed 0.1 mg. which is the desired precision.

The 1/100 mg. tabular interval considered above is not adequate in topography of low relief however. Consider, for example, an hypothetical case in which the terrain in each of the compartments has an average height relative to the station elevation just short of that producing a terrain effect of 0.005 mg. A table in which the first entry corresponds to 1/100 mg. effect would then yield a zero terrain correction whereas, actually, since the effects accumulate without cancellation, the correction should be $0.005 \times 132 = 0.7$ mg. To overcome this circumstance the first entries in the present tables include terrain effects down to 0.0005 mg. per compartment so that the maximum terrain correction which may escape evaluation by the tables is 0.07 mg. The error which arises from the tables themselves is thus less than 1/10 mg. and therefore the terrain correction, for the area covered by the tables, may be determined to this accuracy if adequate topographic information is available.

CONCLUSION

Gravimeter surveys in hilly country are subject to errors which may be much larger than the probable errors of the unreduced gravity values unless accurate corrections are made for the non-significant gravitational effects of the undulating topography. These corrections may be evaluated with an accuracy comparable to the instrumental precision of modern gravimeters by use of the terrain correction tables presented in this paper in conjunction with adequate topographic information about the vicinity of the gravity stations. With this additional technique it should be possible to extend gravimeter prospecting into hilly country without appreciable loss in precision.

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